LESSON 1-1

Points, Lines, and Planes

Then

- You used basic geometric concepts and properties to solve problems.

Now

1. Identify and model points, lines, and planes.
2. Identify intersecting lines and planes.

Why?

- On a subway map, the locations of stops are represented by *points*. The route the train can take is modeled by a series of connected paths that look like *lines*. The flat surface of the map on which these points and lines lie is representative of a *plane*.

Brand X Pictures/Jupiterimages

NewVocabulary

undefined term
point
line
plane
collinear
coplanar
intersection
definition
defined term
space

Common Core State Standards

Content Standards

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Mathematical Practices

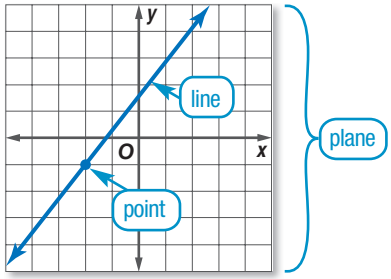
4 Model with mathematics.
6 Attend to precision.

1 Points, Lines, and Planes

Unlike the real-world objects that they model, shapes, points, lines, and planes do not have any actual size. In geometry, *point*, *line*, and *plane* are considered **undefined terms** because they are only explained using examples and descriptions.

You are already familiar with the terms point, line, and plane from algebra. You graphed on a coordinate *plane* and found ordered pairs that represented *points* on *lines*. In geometry, these terms have a similar meaning.

The phrase *exactly one* in a statement such as, "There is exactly one line through any two points," means that there is *one and only one*.



KeyConcept Undefined Terms

A **point** is a location. It has neither shape nor size.

Named by a capital letter

Example point *A*

A **line** is made up of points and has no thickness or width. There is exactly one line through any two points.

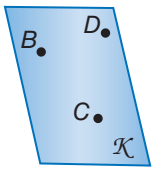
Named by the letters representing two points on the line or a lowercase script letter

Example line *m*, line *PQ* or $\overleftrightarrow{PQ}$, line *QP* or $\overleftrightarrow{QP}$

A **plane** is a flat surface made up of points that extends infinitely in all directions. There is exactly one plane through any three points not on the same line.

Named by a capital script letter or by the letters naming three points that are not all on the same line

Example plane $\mathcal{K}$, plane *BCD*, plane *CDB*, plane *DCB*, plane *DBC*, plane *CBD*, plane *BDC*



Collinear points are points that lie on the same line. *Noncollinear* points do not lie on the same line. **Coplanar** points are points that lie in the same plane. *Noncoplanar* points do not lie in the same plane.

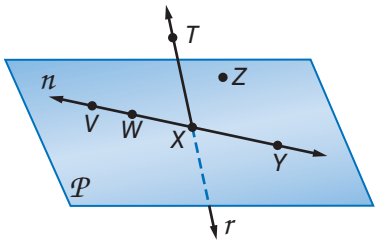
Example 1 Name Lines and Planes

Use the figure to name each of the following.

a. a line containing point *W*

The line can be named as line n, or any two of the four points on the line can be used to name the line.

$\overleftrightarrow{VW}$ $\overleftrightarrow{WV}$ $\overleftrightarrow{VX}$ $\overleftrightarrow{XV}$ $\overleftrightarrow{VY}$ $\overleftrightarrow{YV}$
$\overleftrightarrow{WX}$ $\overleftrightarrow{XW}$ $\overleftrightarrow{WY}$ $\overleftrightarrow{YW}$ $\overleftrightarrow{XY}$ $\overleftrightarrow{YX}$



b. a plane containing point *X*

One plane that can be named is plane $\mathcal{P}$. You can also use the letters of any three *noncollinear* points to name this plane.

plane XZY plane VZW plane VZX
plane VZY plane WZX plane WZY

The letters of each of these names can be reordered to create other acceptable names for this plane. For example, XZY can also be written as XYZ, ZXY, ZYX, YXZ, and YZX. In all, there are 36 different three-letter names for this plane.

StudyTip

Additional Planes Although not drawn in Example 1b, there is another plane that contains point *X*. Since points *W*, *T*, and *X* are noncollinear, point *X* is also in plane *WTX*.

GuidedPractice

1A. a plane containing points T and Z

1B. a line containing point T

Real-World Example 2 Model Points, Lines, and Planes

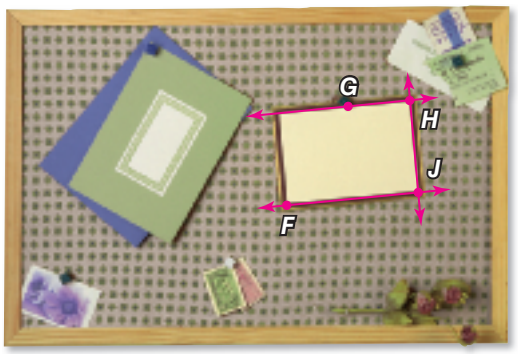
MESSAGE BOARD **Name the geometric terms modeled by the objects in the picture.**

The push pin models point G.

The maroon border on the card models line GH.

The edge of the card models line HJ.

The card itself models plane FGJ.



Real-WorldCareer

Drafter Drafters use perspective to create drawings to build everything from toys to school buildings. Drafters need skills in math and computers. They get their education at trade schools, community colleges, and some 4-year colleges. Refer to Exercises 50 and 51.

GuidedPractice

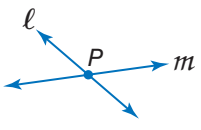
Name the geometric term modeled by each object.

2A. stripes on a sweater

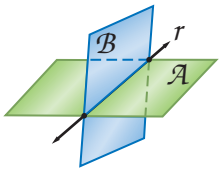
2B. the corner of a box

2 Intersections of Lines and Planes

The **intersection** of two or more geometric figures is the set of points they have in common. Two lines intersect in a point. Lines can intersect planes, and planes can intersect each other.



P represents the intersection of lines ℓ and m.



Line *r* represents the intersection of planes $\mathcal{A}$ and $\mathcal{B}$.

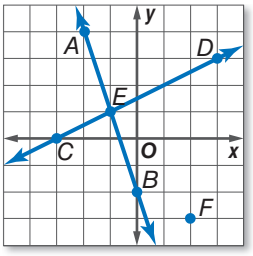
Digital Vision/Getty Images

Example 3 Draw Geometric Figures

Draw and label a figure for each relationship.

a. ALGEBRA Lines AB and CD intersect at E for $A(-2, 4)$, $B(0, -2)$, $C(-3, 0)$, and $D(3, 3)$ on a coordinate plane. Point F is coplanar with these points, but not collinear with $\overleftrightarrow{AB}$ or $\overleftrightarrow{CD}$.

Graph each point and draw $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

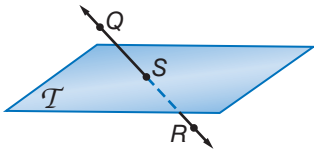


Label the intersection point as E.

An infinite number of points are coplanar with A, B, C, D and E but not collinear with $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. In the graph, one such point is $F(2, -3)$.

b. QR intersects plane $\mathcal{T}$ at point S.

Draw a surface to represent plane $\mathcal{T}$ and label it.



Draw a dot for point S anywhere on the plane and a dot that is not on plane $\mathcal{T}$ for point Q.

Draw a line through points Q and S. Dash the line to indicate the portion hidden by the plane. Then draw another dot on the line and label it R.

StudyTip

Three-Dimensional Drawings Because it is impossible to show an entire plane in a figure, edged shapes with different shades of color are used to represent planes.

GuidedPractice

3A. Points $J(-4, 2)$, $K(3, 2)$, and L are collinear.

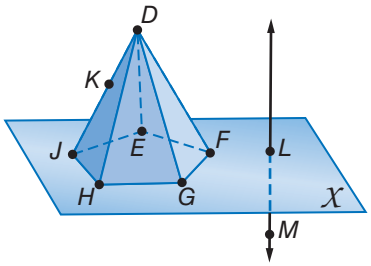
3B. Line p lies in plane $\mathcal{N}$ and contains point L.

Definitions or **defined terms** are explained using undefined terms and/or other defined terms. **Space** is defined as a boundless, three-dimensional set of all points. Space can contain lines and planes.

Example 4 Interpret Drawings

a. How many planes appear in this figure?

Six: plane $\mathcal{X}$, plane JDH, plane JDE, plane EDF, plane FDG, and plane HDG.



b. Name three points that are collinear.

Points J, K, and D are collinear.

c. Name the intersection of plane HDG with plane $\mathcal{X}$.

Plane HDG intersects plane $\mathcal{X}$ in $\overleftrightarrow{HG}$.

d. At what point do $\overleftrightarrow{LM}$ and $\overleftrightarrow{EF}$ intersect? Explain.

It does not appear that these lines intersect. $\overleftrightarrow{EF}$ lies in plane $\mathcal{X}$, but only point L of $\overleftrightarrow{LM}$ lies in $\mathcal{X}$.

StudyTip

CCSS Precision A point has no dimension. A line exists in one dimension. However, a circle is two-dimensional, and a pyramid is three-dimensional.

GuidedPractice

Explain your reasoning.

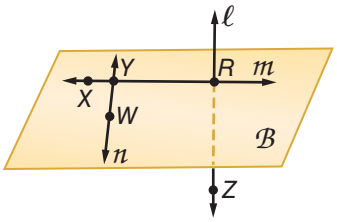
4A. Are points E, D, F, and G coplanar?

4B. At what point or in what line do planes JDH, JDE, and EDF intersect?

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **Use the figure to name each of the following.**



1. a line containing point X
2. a line containing point Z
3. a plane containing points W and R

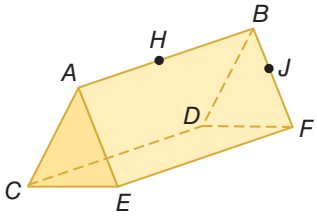
Example 2 **Name the geometric term modeled by each object.**

4. a beam from a laser
5. a floor

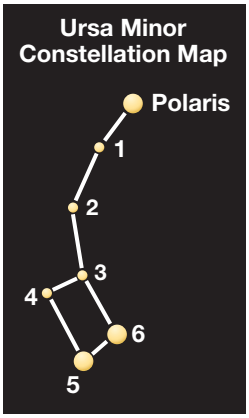
Example 3 **Draw and label a figure for each relationship.**

6. A line in a coordinate plane contains $A(0, -5)$ and $B(3, 1)$ and a point C that is not collinear with $\overleftrightarrow{AB}$.
7. Plane Z contains lines x, y, and w. Lines x and y intersect at point V and lines x and w intersect at point P.

Example 4 **Refer to the figure.**



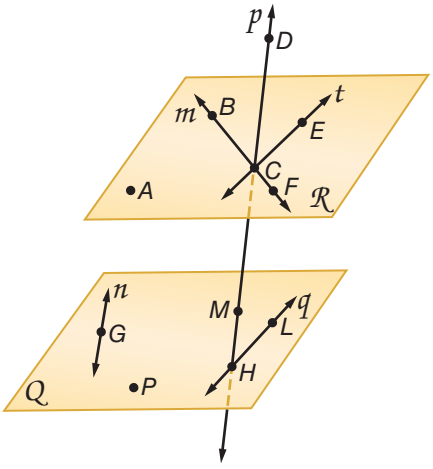
8. How many planes are shown in the figure?
9. Name three points that are collinear.
10. Are points A, H, J, and D coplanar? Explain.
11. Are points B, D, and F coplanar? Explain.
12. **ASTRONOMY** Ursa Minor, or the Little Dipper, is a constellation made up of seven stars in the northern sky including the star Polaris.
    - **a.** What geometric figures are modeled by the stars?
    - **b.** Are Star 1, Star 2, and Star 3 collinear on the constellation map? Explain.
    - **c.** Are Polaris, Star 2, and Star 6 coplanar on the map?



Practice and Problem Solving

Extra Practice is on page R1.

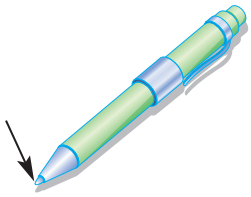
Example 1 **Refer to the figure.**



13. Name the lines that are only in plane Q.
14. How many planes are labeled in the figure?
15. Name the plane containing the lines m and t.
16. Name the intersection of lines m and t.
17. Name a point that is not coplanar with points A, B, and C.
18. Are points F, M, G, and P coplanar? Explain.
19. Name the points not contained in a line shown.
20. What is another name for line t?
21. Does line n intersect line q? Explain.

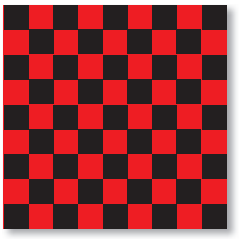
Example 2 **Name the geometric term(s) modeled by each object.**

22.

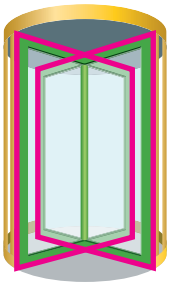


23.

24.



25.



26. a blanket

27. a knot in a rope

28. a telephone pole

29. the edge of a desk

30. two connected walls

31. a partially opened folder

Example 3 **Draw and label a figure for each relationship.**

32. Line m intersects plane $\mathcal{R}$ at a single point.

33. Two planes do not intersect.

34. Points X and Y lie on $\overleftrightarrow{CD}$.

35. Three lines intersect at point J but do not all lie in the same plane.

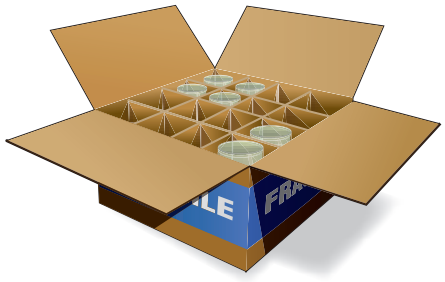
36. Points $A(2, 3)$, $B(2, -3)$, C and D are collinear, but A, B, C, D, and F are not.

37. Lines $\overleftrightarrow{LM}$ and $\overleftrightarrow{NP}$ are coplanar but do not intersect.

38. $\overleftrightarrow{FG}$ and $\overleftrightarrow{JK}$ intersect at $P(4, 3)$, where point F is at $(-2, 5)$ and point J is at $(7, 9)$.

39. Lines s and t intersect, and line v does not intersect either one.

Example 4 CCSS **MODELING** **When packing breakable objects such as glasses, movers frequently use boxes with inserted dividers like the one shown.**

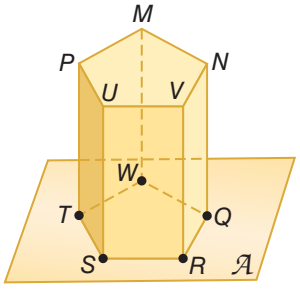


40. How many planes are modeled in the picture?

41. What parts of the box model lines?

42. What parts of the box model points?

Refer to the figure at the right.



43. Name two collinear points.

44. How many planes appear in the figure?

45 Do plane $\mathcal{A}$ and plane MNP intersect? Explain.

46. In what line do planes $\mathcal{A}$ and QRV intersect?

47. Are points T, S, R, Q, and V coplanar? Explain.

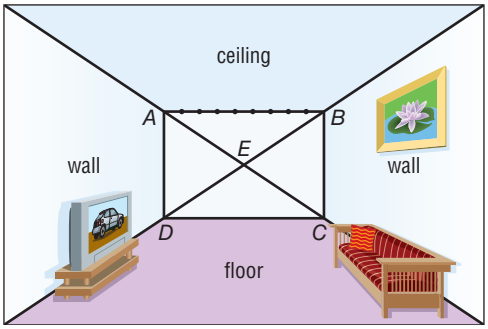
48. Are points T, S, R, Q, and W coplanar? Explain.

49 **FINITE PLANES** A *finite plane* is a plane that has boundaries, or does not extend indefinitely. The street signs shown are finite planes.

a. If the pole models a line, name the geometric term that describes the intersection between the signs and the pole.

b. What geometric term(s) describes the intersection between the two finite planes? Explain your answer with a diagram if necessary.

50. **ONE-POINT PERSPECTIVE** One-point perspective drawings use lines to convey depth. Lines representing horizontal lines in the real object can be extended to meet at a single point called the *vanishing point*. Suppose you want to draw a tiled ceiling in the room below with nine tiles across.

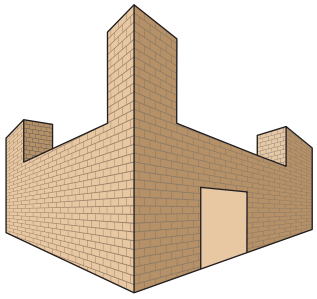


a. What point represents the vanishing point in the drawing?

b. Trace the figure. Then draw lines from the vanishing point through each of the eight points between *A* and *B*. Extend these lines to the top edge of the drawing.

c. How could you change the drawing to make the back wall of the room appear farther away?

51. **TWO-POINT PERSPECTIVE** Two-point perspective drawings use two vanishing points to convey depth.



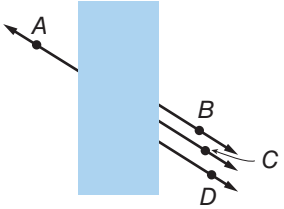
a. Trace the drawing of the castle shown. Draw five of the vertical lines used to create the drawing.

b. Draw and extend the horizontal lines to locate the vanishing points and label them.

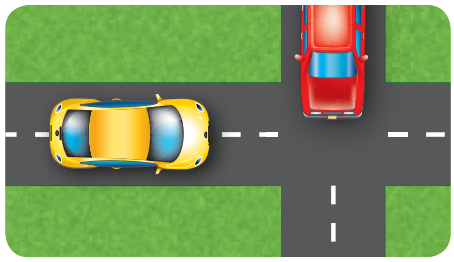
c. What do you notice about the vertical lines as they get closer to the vanishing point?

d. Draw a two-point perspective of a home or a room in a home.

52. CCSS **ARGUMENTS** Name two points on the same line in the figure. How can you support your assertion?



53. TRANSPORTATION When two cars enter an intersection at the same time on opposing paths, one of the cars must adjust its speed or direction to avoid a collision. Two airplanes, however, can cross paths while traveling in different directions without colliding. Explain how this is possible.



54. MULTIPLE REPRESENTATIONS Another way to describe a group of points is called a locus. A **locus** is a set of points that satisfy a particular condition. In this problem, you will explore the locus of points that satisfy an equation.

a. Tabular Represent the locus of points satisfying the equation $2 + x = y$ using a table of at least five values.

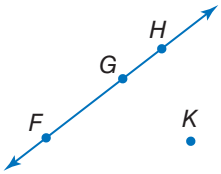
b. Graphical Represent this same locus of points using a graph.

c. Verbal Describe the geometric figure that the points suggest.

55 PROBABILITY Three of the labeled points are chosen at random.

a. What is the probability that the points chosen are collinear?

b. What is the probability that the points chosen are coplanar?



56. MULTIPLE REPRESENTATIONS In this problem, you will explore the locus of points that satisfy an inequality.

a. Tabular Represent the locus of points satisfying the inequality $y < -3x - 1$ using a table of at least ten values.

b. Graphical Represent this same locus of points using a graph.

c. Verbal Describe the geometric figure that the points suggest.

H.O.T. Problems Use Higher-Order Thinking Skills

57. OPEN ENDED Sketch three planes that intersect in a line.

58. ERROR ANALYSIS Camille and Hiroshi are trying to determine the most number of lines that can be drawn using any two of four random points. Is either correct? Explain.

Camille	Hiroshi
Since there are four points, 4 · 3 or 12 lines can be drawn between the points.	You can draw 3 · 2 · 1 or 6 lines between the points.

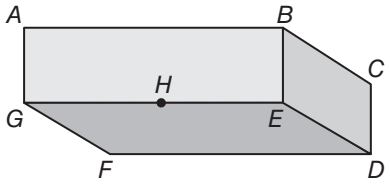
59. CCSS ARGUMENTS What is the greatest number of planes determined using any three of the points A, B, C, and D if no three points are collinear?

60. REASONING Is it possible for two points on the surface of a prism to be neither collinear nor coplanar? Justify your answer.

61. WRITING IN MATH Refer to Exercise 49. Give a real-life example of a finite plane. Is it possible to have a real-life object that is an infinite plane? Explain your reasoning.

Standardized Test Practice

62. Which statement about the figure below is *not* true?



A Point H lies in planes AGE and GED.

B Planes GAB, GFD and BED intersect at point E.

C Points F, E, and B are coplanar.

D Points A, H, and D are collinear.

63. ALGEBRA What is the value of x if $3x + 2 = 8$?

F -2 **G** 0 **H** 2 **J** 6

64. GRIDDED RESPONSE An ice chest contains 3 types of drinks: 10 apple juices, 15 grape juices, and 15 bottles of water. What is the probability that a drink selected randomly from the ice chest does *not* contain fruit juice?

65. SAT/ACT A certain school's enrollment increased 6% this year over last year's enrollment. If the school now has 1378 students enrolled, how many students were enrolled last year?

A 1295 **C** 1350 **E** 1500

B 1300 **D** 1460

Spiral Review

Simplify. (Lesson 0-9)

66. $\sqrt{72}$

67. $\sqrt{18} \cdot \sqrt{14}$

68. $\sqrt{44x^4y^3}$

69. $\dfrac{3}{\sqrt{18}}$

70. $\sqrt{\dfrac{28}{75}}$

71. $\dfrac{\sqrt{8a^6}}{\sqrt{108}}$

72. $\dfrac{5}{4 - \sqrt{2}}$

73. $\dfrac{4\sqrt{3}}{2 + \sqrt{5}}$

74. FINANCIAL LITERACY Suppose you buy 3 shirts and 2 pairs of slacks on sale at a clothing store for \$72. The next day, a friend buys 2 shirts and 4 pairs of slacks for \$96. If the shirts you each bought were all the same price and the slacks were also all the same price, then what was the cost of each shirt and each pair of slacks? (Lesson 0-8)

Graph the following points, and connect them in order to form a figure. (Lesson 0-7)

75. $A(-5, 3)$, $B(3, -4)$, and $C(-2, -3)$

76. $P(-2, 1)$, $Q(3, 4)$, $R(5, 1)$, and $S(0, -2)$

GROCERIES **Find an approximate metric weight for each item.** (Lesson 0-2)

77.

Net Wt: 15 oz

78.

Net Wt: 8.2 oz

79.

Net Wt: 2.5 lb

Skills Review

Replace each ● with >, <, or = to make a true statement.

80. $\frac{1}{4}$ in. ● $\frac{1}{2}$ in.

81. $\frac{3}{4}$ in. ● $\frac{5}{8}$ in.

82. $\frac{3}{8}$ in. ● $\frac{6}{16}$ in.

83. 18 mm ● 2 cm

84. 32 mm ● 3.2 cm

85. 0.8 m ● 8 cm

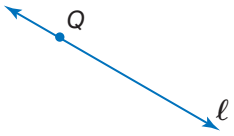
EXTEND

1-1 Geometry Lab
Describing What You See

When you are learning geometric concepts, it is critical to have accurate drawings to represent the information. It is helpful to know what words and phrases can be used to describe figures. Likewise, it is important to know how to read a geometric description and be able to draw the figure it describes.

Common Core State Standards
Content Standards
G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★
Mathematical Practices 6

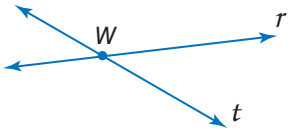
The figures and descriptions below help you visualize and write about points, lines, and planes.



Point Q is on ℓ.

Line ℓ contains Q.

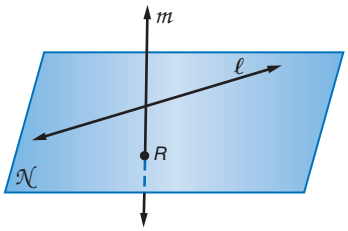
Line ℓ passes through Q.



Lines r and t intersect at W.

Point W is the intersection of r and t.

Point W is on r. Point W is on t.



Line ℓ and point R are in $\mathcal{N}$.

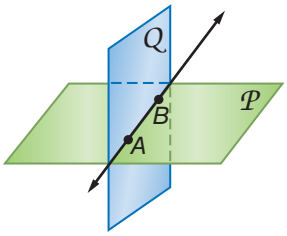
Point R lies in $\mathcal{N}$.

Plane $\mathcal{N}$ contains R and ℓ.

Line m intersects $\mathcal{N}$ at R.

Point R is the intersection of m with $\mathcal{N}$.

Lines ℓ and m do not intersect.



$\overleftrightarrow{AB}$ is in $\mathcal{P}$ and Q.

Points A and B lie in both $\mathcal{P}$ and Q.

Planes $\mathcal{P}$ and Q both contain $\overleftrightarrow{AB}$.

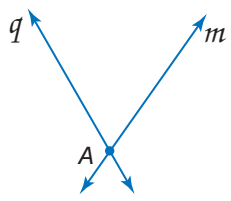
Planes $\mathcal{P}$ and Q intersect in $\overleftrightarrow{AB}$.

$\overleftrightarrow{AB}$ is the intersection of $\mathcal{P}$ and Q.

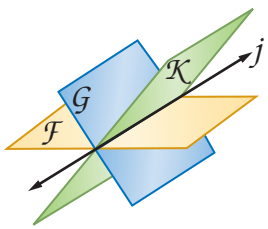
Exercises

Write a description for each figure.

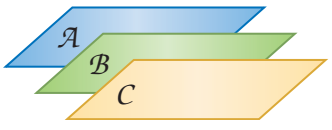
1.



2.



3.



4. Draw and label a figure for the statement Planes $\mathcal{N}$ and $\mathcal{P}$ contain line a.