

LESSON 1-2 Linear Measure

Then

- You identified and modeled points, lines, and planes.

Now

- 1 Measure segments.
- 2 Calculate with measures.

Why?

- When the ancient Egyptians found a need for a measurement system, they used the human body as a guide. The cubit was the length of an arm from the elbow to the fingertips. Eventually the Egyptians standardized the length of a cubit, with ten *royal cubits* equivalent to one *rod*.



New Vocabulary

line segment
betweenness of points
between
congruent segments
construction



Common Core State Standards

Content Standards

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

Mathematical Practices

6 Attend to precision.

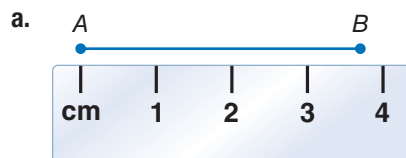
1 Measure Line Segments Unlike a line, a **line segment**, or *segment*, can be measured because it has two endpoints. A segment with endpoints A and B can be named as \overline{AB} or \overline{BA} . The *measure* of \overline{AB} is written as AB . The length or measure of a segment always includes a unit of measure, such as meter or inch.

All measurements are approximations dependent upon the smallest unit of measure available on the measuring instrument.



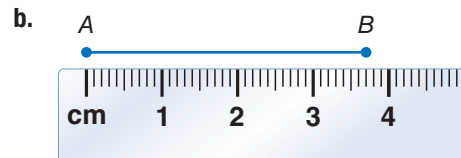
Example 1 Length in Metric Units

Find the length of \overline{AB} using each ruler.



The ruler is marked in centimeters. Point B is closer to the 4-centimeter mark than to 3 centimeters.

Thus, \overline{AB} is about 4 centimeters long.



The long marks are centimeters, and the shorter marks are millimeters. There are 10 millimeters for each centimeter.

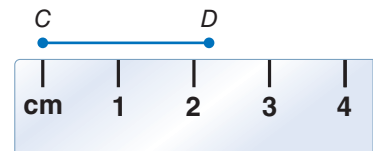
Thus, \overline{AB} is about 3.7 centimeters long.

Guided Practice

1A. Measure the length of a dollar bill in centimeters.

1B. Measure the length of a pencil in millimeters.

1C. Find the length of \overline{CD} .



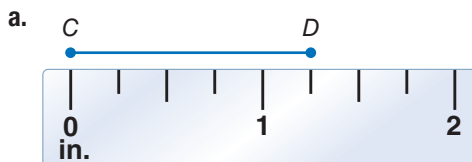


StudyTip

Using a Ruler The zero point on a ruler may not be clearly marked. For some rulers, zero is the left edge of the ruler. On others, it may be a fine line farther in on the scale. If it is not clear where the zero is, align one endpoint on 1 and subtract 1 from the measurement at the other endpoint.

Example 2 Length in Standard Units

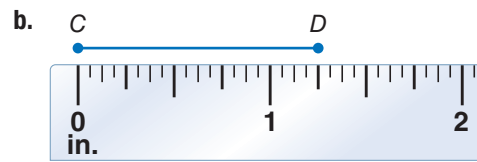
Find the length of \overline{CD} using each ruler.



Each inch is divided into fourths.

Point D is closer to the $1\frac{1}{4}$ -inch mark.

\overline{CD} is about $1\frac{1}{4}$ inches long.



Each inch is divided into sixteenths.

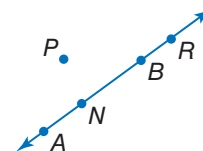
Point D is closer to the $1\frac{4}{16}$ -inch mark.

\overline{CD} is about $1\frac{4}{16}$ or $1\frac{1}{4}$ inches long.

GuidedPractice

- 2A. Measure the length of a dollar bill in inches.
- 2B. Measure the length of a pencil in inches.

2 Calculate Measures Recall that for any two real numbers a and b , there is a real number n that is *between* a and b such that $a < n < b$. This relationship also applies to points on a line and is called **betweenness of points**. In the figure, point N is between points A and B , but points R and P are not.



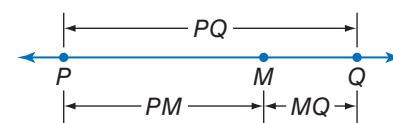
Measures are real numbers, so all arithmetic operations can be used with them. You know that the whole usually equals the sum of its parts. That is also true of line segments in geometry.

KeyConcept Betweenness of Points

Words

Point M is **between** points P and Q if and only if P , Q , and M are collinear and $PM + MQ = PQ$.

Model



StudyTip

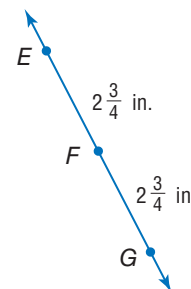
Comparing Measures Because measures are real numbers, you can compare them. If points X , Y , and Z are collinear in that order, then one of these statements is true: $XY = YZ$, $XY > YZ$, or $XY < YZ$.

Example 3 Find Measurements by Adding

Find EG . Assume that the figure is not drawn to scale.

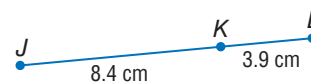
EG is the measure of \overline{EG} . Point F is between E and G . Find EG by adding EF and FG .

$$\begin{aligned} EF + FG &= EG && \text{Betweenness of points} \\ 2\frac{3}{4} + 2\frac{3}{4} &= EG && \text{Substitution} \\ 5\frac{1}{2} \text{ in.} &= EG && \text{Add.} \end{aligned}$$



GuidedPractice

3. Find JL . Assume that the figure is not drawn to scale.

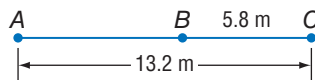




Example 4 Find Measurements by Subtracting

Find AB . Assume that the figure is not drawn to scale.

Point B is between A and C .



$$AB + BC = AC \quad \text{Betweenness of points}$$

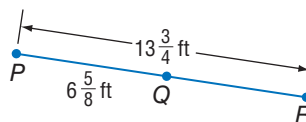
$$AB + 5.8 = 13.2 \quad \text{Substitution}$$

$$AB + 5.8 - 5.8 = 13.2 - 5.8 \quad \text{Subtract 5.8 from each side.}$$

$$AB = 7.4 \text{ m} \quad \text{Simplify.}$$

Guided Practice

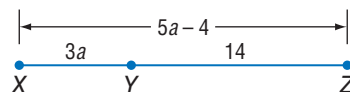
4. Find QR . Assume that the figure is not drawn to scale.



Example 5 Write and Solve Equations to Find Measurements

ALGEBRA Find the value of a and XY if Y is between X and Z , $XY = 3a$, $XZ = 5a - 4$, and $YZ = 14$.

Draw a figure to represent this information.



$$XZ = XY + YZ \quad \text{Betweenness of points}$$

$$5a - 4 = 3a + 14 \quad \text{Substitution}$$

$$5a - 4 - 3a = 3a + 14 - 3a \quad \text{Subtract } 3a \text{ from each side.}$$

$$2a - 4 = 14 \quad \text{Simplify.}$$

$$2a - 4 + 4 = 14 + 4 \quad \text{Add 4 to each side.}$$

$$2a = 18 \quad \text{Simplify.}$$

$$\frac{2a}{2} = \frac{18}{2} \quad \text{Divide each side by 2.}$$

$$a = 9 \quad \text{Simplify.}$$

Now find XY .

$$XY = 3a \quad \text{Given}$$

$$= 3(9) \text{ or } 27 \quad a = 9$$

Guided Practice

5. Find x and BC if B is between A and C , $AC = 4x - 12$, $AB = x$, and $BC = 2x + 3$.

Segments that have the same measure are called **congruent segments**.

WatchOut!

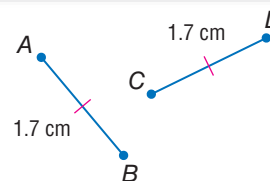
Equal vs. Congruent Lengths are equal and segments are congruent. It is correct to say that $AB = CD$ and $\overline{AB} \cong \overline{CD}$. However, it is *not* correct to say that $\overline{AB} = \overline{CD}$ or that $AB \cong CD$.

Key Concept Congruent Segments

Words Congruent segments have the same measure.

Symbols \cong is read *is congruent to*. Red slashes on the figure also indicate congruence.

Example $\overline{AB} \cong \overline{CD}$



Real-World Example 6 Congruent Segments

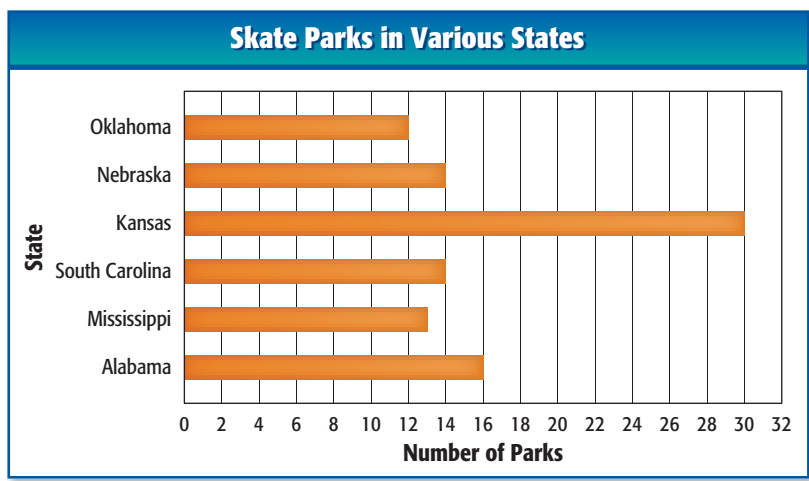
SKATE PARKS In the graph, suppose a segment was drawn along the top of each bar. Which states would have segments that are congruent? Explain.



Real-WorldLink

The first commercial skateboard was introduced in 1959. Now there are more than 500 skate parks in the United States.

Source: Encyclopaedia Britannica



Source: SITE Design Group, Inc.

The segments on the bars for Nebraska and South Carolina would be congruent because they both represent the same number of skate parks.

GuidedPractice

- 6A. Suppose Oklahoma added another skate park. The segment drawn along the bar representing Oklahoma would be congruent to which other segment?
- 6B. Name the congruent segments in the sign shown.

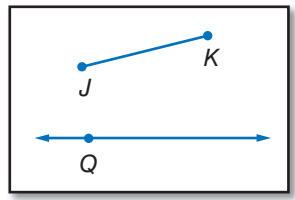


Drawings of geometric figures are created using measuring tools such as a ruler and protractor. **Constructions** are methods of creating these figures without the benefit of measuring tools. Generally, only a pencil, straightedge, and compass are used in constructions. *Sketches* are created without the use of any of these tools.

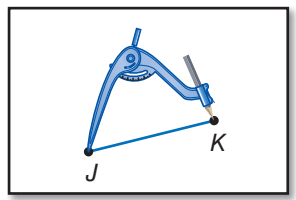
You can construct a segment that is congruent to a given segment.

Construction Copy a Segment

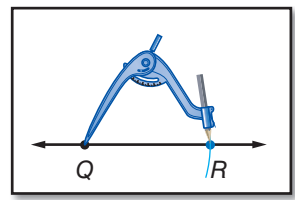
Step 1 Draw a segment \overline{JK} . Elsewhere on your paper, draw a line and a point on the line. Label the point Q .



Step 2 Place the compass at point J and adjust the compass setting so that the pencil is at point K .



Step 3 Using that setting, place the compass point at Q and draw an arc that intersects the line. Label the point of intersection R . $\overline{JK} \cong \overline{QR}$

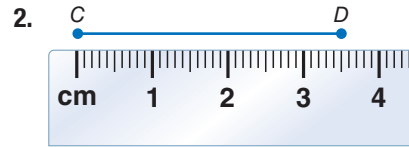
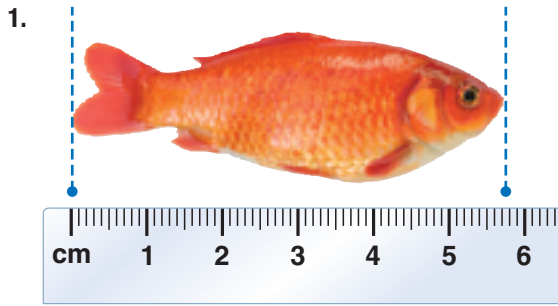


Matt Strohane/Getty Images

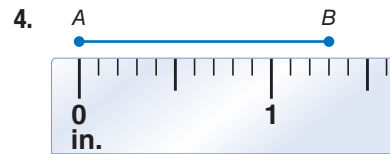
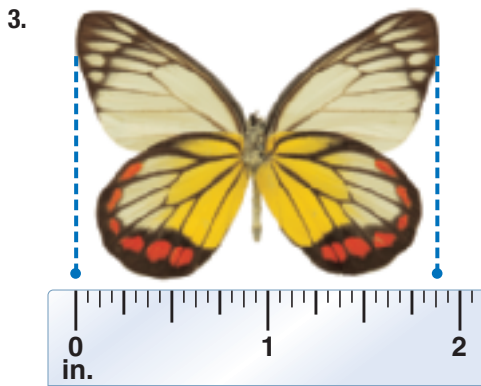




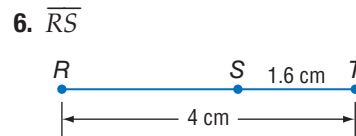
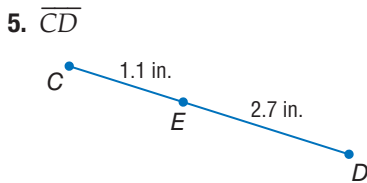
Example 1 Find the length of each line segment or object.



Example 2



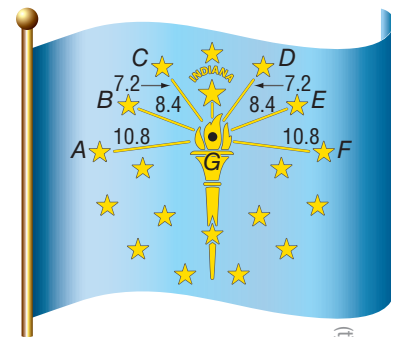
Examples 3–4 Find the measurement of each segment. Assume that each figure is not drawn to scale.



Example 5 **ALGEBRA** Find the value of x and BC if B is between C and D .

- 7. $CB = 2x$, $BD = 4x$, and $BD = 12$
- 8. $CB = 4x - 9$, $BD = 3x + 5$, and $CD = 17$

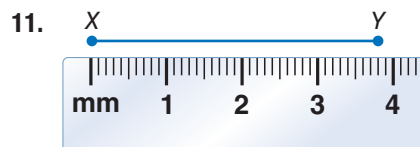
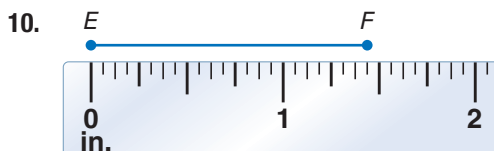
Example 6 9. **CCSS STRUCTURE** The Indiana State Flag was adopted in 1917. The measures of the segments between the stars and the flame are shown on the diagram in inches. List all of the congruent segments in the figure.

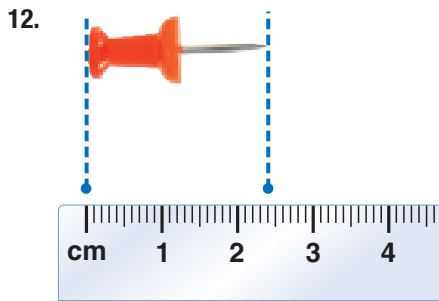


Practice and Problem Solving

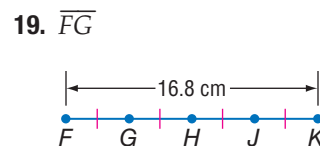
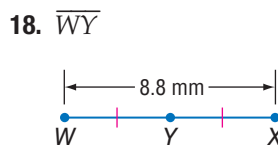
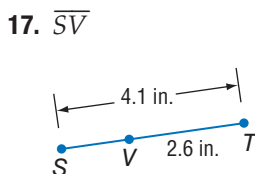
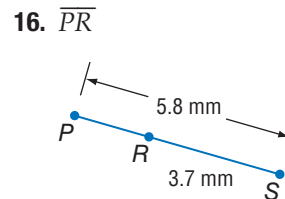
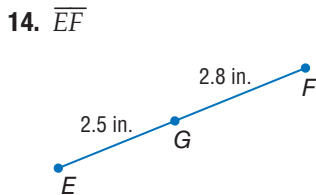
Extra Practice is on page R1.

Examples 1–2 Find the length of each line segment.

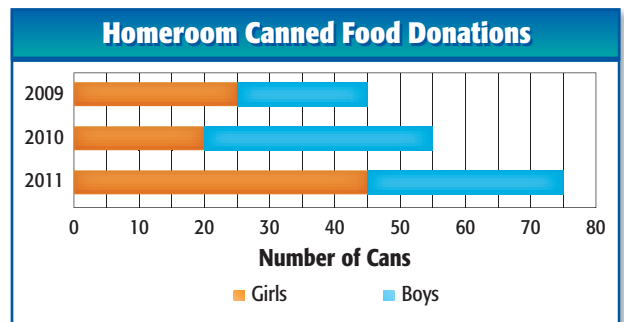




Examples 3–4 Find the measurement of each segment. Assume that each figure is not drawn to scale.



20. **CCSS SENSE-MAKING** The stacked bar graph shows the number of canned food items donated by the girls and the boys in a homeroom class over three years. Use the concept of betweenness of points to find the number of cans donated by the boys for each year. Explain your method.



Example 5 **ALGEBRA** Find the value of the variable and YZ if Y is between X and Z .

21. $XY = 11$, $YZ = 4c$, $XZ = 83$

22. $XY = 6b$, $YZ = 8b$, $XZ = 175$

23. $XY = 7a$, $YZ = 5a$, $XZ = 6a + 24$

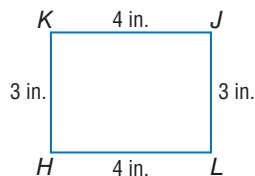
24. $XY = 11d$, $YZ = 9d - 2$, $XZ = 5d + 28$

25. $XY = 4n + 3$, $YZ = 2n - 7$, $XZ = 22$

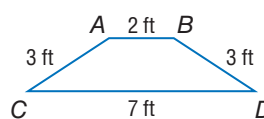
26. $XY = 3a - 4$, $YZ = 6a + 2$, $XZ = 5a + 22$

Example 6 Determine whether each pair of segments is congruent.

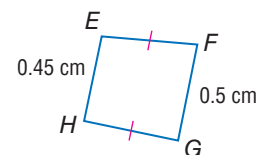
27. \overline{KJ} , \overline{HL}



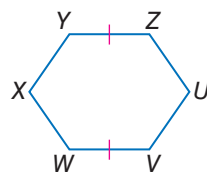
28. \overline{AC} , \overline{BD}



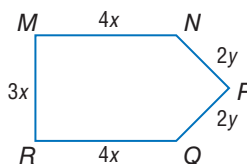
29. \overline{EH} , \overline{FG}



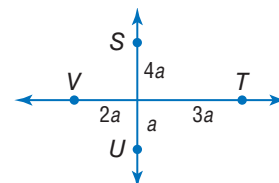
30. \overline{VW} , \overline{UZ}



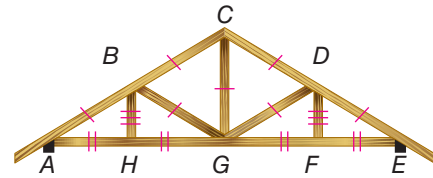
31. \overline{MN} , \overline{RQ}



32. \overline{SU} , \overline{VT}



- 33. TRUSSES** A truss is a structure used to support a load over a span, such as a bridge or the roof of a house. List all of the congruent segments in the figure.

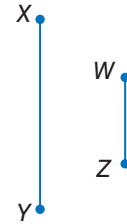


- 34. CONSTRUCTION** For each expression:

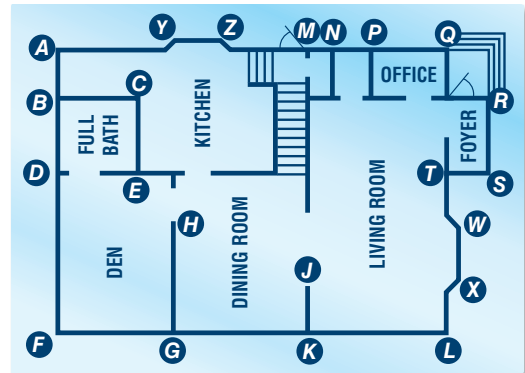
- construct a segment with the given measure,
- explain the process you used to construct the segment, and
- verify that the segment you constructed has the given measure.

a. $2(XY)$

b. $6(WZ) - XY$



- 35. BLUEPRINTS** Use a ruler to determine at least five pairs of congruent segments with labeled endpoints in the blueprint at the right.



- 36. MULTIPLE REPRESENTATIONS** Betweenness of points ensures that a line segment may be divided into an infinite number of line segments.
- Geometric** Use a ruler to draw a line segment 3 centimeters long. Label the endpoints A and D . Draw two more points along the segment and label them B and C . Draw a second line segment 6 centimeters long. Label the endpoints K and P . Add four more points along the line and label them L , M , N , and O .
 - Tabular** Use a ruler to measure the length of the line segment between each of the points you have drawn. Organize the lengths of the segments in \overline{AD} and \overline{KP} into a table. Include a column in your table to record the sum of these measures.
 - Algebraic** Give an equation that could be used to find the lengths of \overline{AD} and \overline{KP} . Compare the lengths determined by your equation to the actual lengths.

H.O.T. Problems Use Higher-Order Thinking Skills

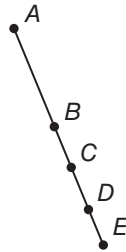
- WRITING IN MATH** If point B is between points A and C , explain how you can find AC if you know AB and BC . Explain how you can find BC if you know AB and AC .
- OPEN ENDED** Draw a segment \overline{AB} that measures between 2 and 3 inches long. Then sketch a segment \overline{CD} congruent to \overline{AB} , draw a segment \overline{EF} congruent to \overline{AB} , and construct a segment \overline{GH} congruent to \overline{AB} . Compare your methods.
- CHALLENGE** Point K is between points J and L . If $JK = x^2 - 4x$, $KL = 3x - 2$, and $JL = 28$, write and solve an equation to find the lengths of JK and KL .
- CCSS REASONING** Determine whether the statement *If point M is between points C and D , then CD is greater than either CM or MD is sometimes, never, or always true.* Explain.
- WRITING IN MATH** Why is it important to have a standard of measure?



Standardized Test Practice

- 42. SHORT RESPONSE** A 36-foot-long ribbon is cut into three pieces. The first piece of ribbon is half as long as the second piece of ribbon. The third piece is 1 foot longer than twice the length of the second piece of ribbon. How long is the longest piece of ribbon?

- 43.** In the figure, points A , B , C , D , and E are collinear. If $AE = 38$, $BD = 15$, and $\overline{BC} \cong \overline{CD} \cong \overline{DE}$, what is the length of \overline{AD} ?



- A 7.5 C 22.5
B 15 D 30.5

- 44. SAT/ACT** If $f(x) = 7x^2 - 4x$, what is the value of $f(2)$?

- F -8 J 17
G 2 K 20
H 6

- 45. ALGEBRA**

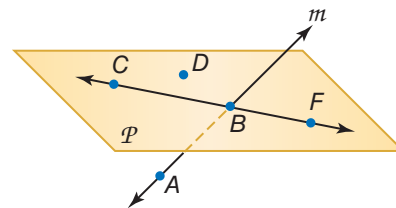
Simplify $(3x^2 - 2)(2x + 4) - 2x^2 + 6x + 7$.

- A $4x^2 + 14x - 1$
B $4x^2 - 14x + 15$
C $6x^3 + 12x^2 + 2x - 1$
D $6x^3 + 10x^2 + 2x - 1$

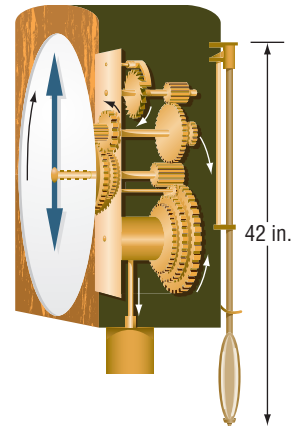
Spiral Review

Refer to the figure. (Lesson 1-1)

46. What are two other names for \overleftrightarrow{AB} ?
47. Give another name for plane \mathcal{P} .
48. Name the intersection of plane \mathcal{P} and \overleftrightarrow{AB} .
49. Name three collinear points.
50. Name two points that are not coplanar.



- 51. CLOCKS** The period of a pendulum is the time required for it to make one complete swing back and forth. The formula of the period P in seconds of a pendulum is $P = 2\pi\sqrt{\frac{\ell}{32}}$, where ℓ is the length of the pendulum in feet. (Lesson 0-9)
- What is the period of the pendulum in the clock shown to the nearest tenth of a second?
 - About how many inches long should the pendulum be in order for it to have a period of 1 second?



Solve each inequality. (Lesson 0-6)

52. $-14n \geq 42$ 53. $p + 6 > 15$
54. $-2a - 5 < 20$ 55. $5x \leq 3x - 26$

Skills Review

Evaluate each expression if $a = -7$, $b = 4$, $c = -3$, and $d = 5$.

56. $b - c$ 57. $|a - d|$ 58. $|d - c|$
59. $\frac{b-a}{2}$ 60. $(a - c)^2$ 61. $\sqrt{(a - b)^2 + (c - d)^2}$

