

LESSON 1-4 Angle Measure

Then

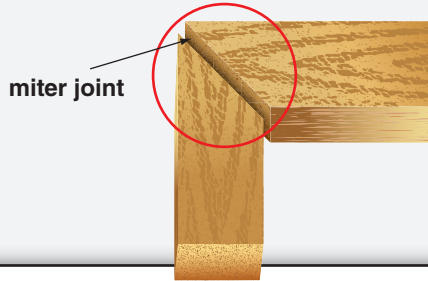
- You measured line segments.

Now

- 1 Measure and classify angles.
- 2 Identify and use congruent angles and the bisector of an angle.

Why?

- One of the skills Dale must learn in carpentry class is how to cut a *miter* joint. This joint is created when two boards are cut at an angle to each other. He has learned that one miscalculation in angle measure can result in mitered edges that do not fit together.



abc New Vocabulary

- ray
- opposite rays
- angle
- side
- vertex
- interior
- exterior
- degree
- right angle
- acute angle
- obtuse angle
- angle bisector

CCSS Common Core State Standards

Content Standards

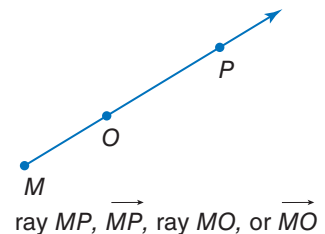
G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

Mathematical Practices

- 5 Use appropriate tools strategically.
- 6 Attend to precision.

1 Measure and Classify Angles A **ray** is a part of a line. It has one endpoint and extends indefinitely in one direction. Rays are named by stating the endpoint first and then any other point on the ray. The ray shown cannot be named as \overrightarrow{OM} because O is not the endpoint of the ray.

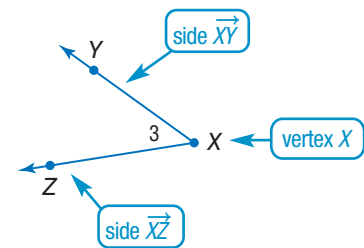


If you choose a point on a line, that point determines exactly two rays called **opposite rays**. Since both rays share a common endpoint, opposite rays are collinear



\overrightarrow{JH} and \overrightarrow{JK} are opposite rays.

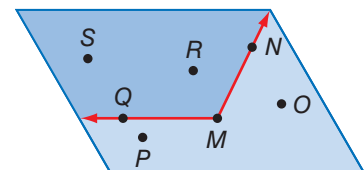
An **angle** is formed by two *noncollinear* rays that have a common endpoint. The rays are called **sides** of the angle. The common endpoint is the **vertex**.



When naming angles using three letters, the vertex must be the second of the three letters. You can name an angle using a single letter only when there is exactly one angle located at that vertex. The angle shown can be named as $\angle X$, $\angle YXZ$, $\angle ZXY$, or $\angle 3$.

An angle divides a plane into three distinct parts.

- Points Q , M , and N lie on the angle.
- Points S and R lie in the **interior** of the angle.
- Points P and O lie in the **exterior** of the angle.

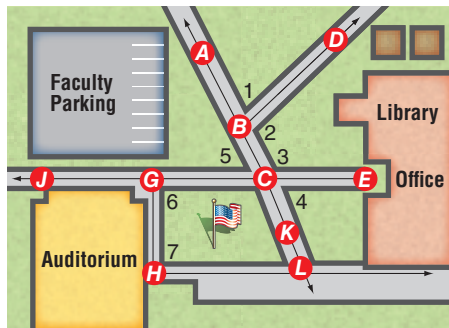


Dennis Hallinan/Hulton Archive/Getty Images



Real-World Example 1 Angles and Their Parts

MAPS Use the map of a high school shown.



a. Name all angles that have B as a vertex.

$\angle 1$ or $\angle ABD$, and $\angle 2$ or $\angle DBC$

b. Name the sides of $\angle 3$.

\overrightarrow{CA} and \overrightarrow{CE} or \overrightarrow{CB} and \overrightarrow{CE}

c. What is another name for $\angle GHL$?

$\angle 7$, $\angle H$, or $\angle LHG$

d. Name a point in the interior of $\angle DBK$.

Point E

StudyTip

Segments as Sides Because a ray can contain a line segment, the side of an angle can be a segment.

GuidedPractice

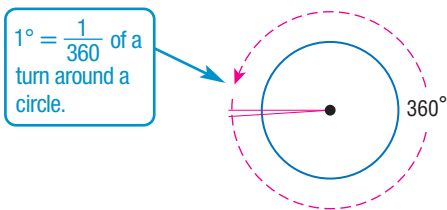
1A. What is the vertex of $\angle 5$?

1B. Name the sides of $\angle 5$.

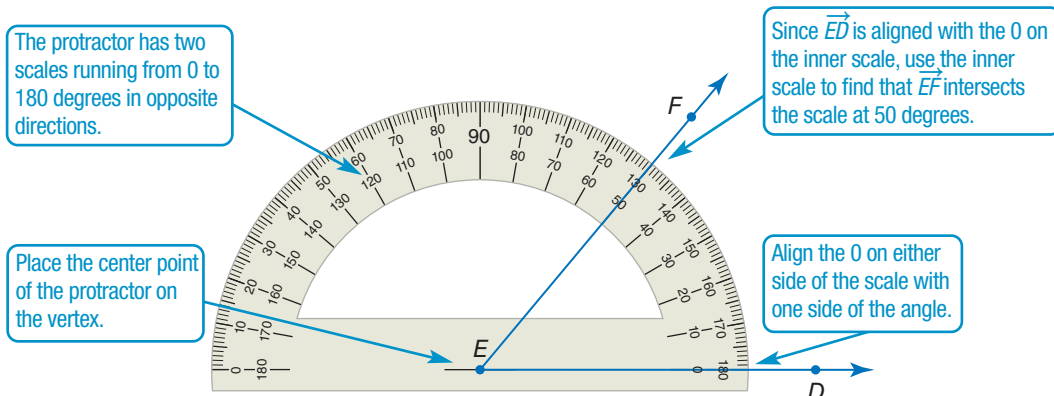
1C. Write another name for $\angle ECL$.

1D. Name a point in the exterior of $\angle CLH$.

Angles are measured in units called degrees. The **degree** results from dividing the distance around a circle into 360 parts.



To measure an angle, you can use a **protractor**. Angle DEF below is a 50 degree (50°) angle. We say that the *degree measure* of $\angle DEF$ is 50, or $m\angle DEF = 50$.

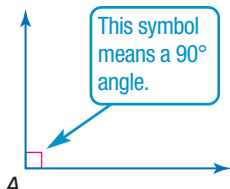
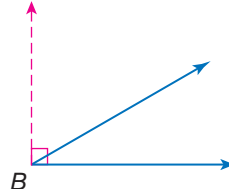
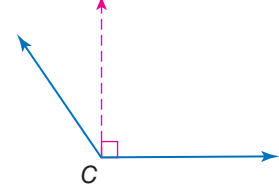


Angles can be classified by their measures as shown below.

ReadingMath

Straight Angle Opposite rays with the same vertex form a *straight angle*. Its measure is 180. Unless otherwise specified in this book, however, the term *angle* means a nonstraight angle.

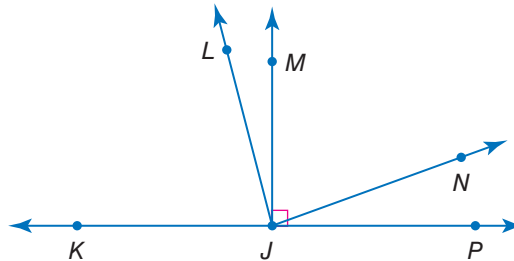
KeyConcept Classify Angles

right angle	acute angle	obtuse angle
 <p>$m\angle A = 90$</p>	 <p>$m\angle B < 90$</p>	 <p>$180 > m\angle C > 90$</p>

Example 2 Measure and Classify Angles



Copy the diagram below, and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree.



a. $\angle MJP$

$\angle MJP$ is marked as a right angle, so $m\angle MJP = 90$.

b. $\angle LJP$

Point L on angle $\angle LJP$ lies on the exterior of right angle $\angle MJP$, so $\angle LJP$ is an obtuse angle. Use a protractor to find that $m\angle LJP = 105$

CHECK Since $105 > 90$, $\angle LJP$ is an obtuse angle. ✓

c. $\angle NJP$

Point N on angle $\angle NJP$ lies on the interior of right angle $\angle MJP$, so $\angle NJP$ is an acute angle. Use a protractor to find that $m\angle NJP = 20$.

CHECK Since $20 < 90$, $\angle NJP$ is an acute angle. ✓

WatchOut!

Classify Before Measuring

Classifying an angle before measuring it can prevent you from choosing the wrong scale on your protractor. In Example 2b, you must decide whether $\angle LJP$ measures 75 or 105. Since $\angle LJP$ is an obtuse angle, you can reason that the correct measure must be 105.

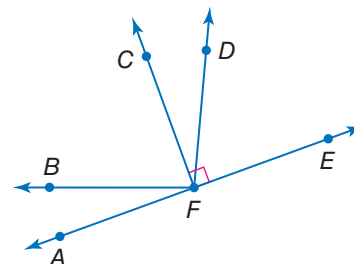
GuidedPractice

2A. $\angle AFB$

2B. $\angle CFA$

2C. $\angle AFD$

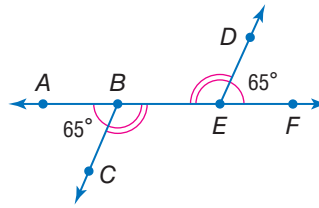
2D. $\angle CFD$



2 Congruent Angles

Just as segments that have the same measure are congruent segments, angles that have the same measure are *congruent angles*.

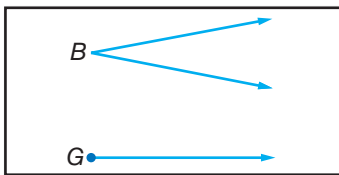
In the figure, since $m\angle ABC = m\angle FED$, then $\angle ABC \cong \angle FED$. Matching numbers of arcs on a figure also indicate congruent angles, so $\angle CBE \cong \angle DEB$.



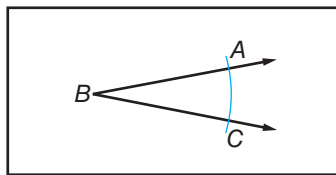
You can produce an angle congruent to a given angle using a construction.

Construction Copy an Angle

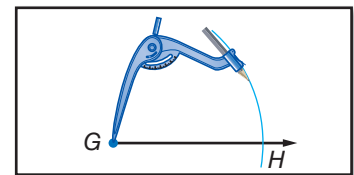
Step 1 Draw an angle like $\angle B$ on your paper. Use a straightedge to draw a ray on your paper. Label its endpoint G .



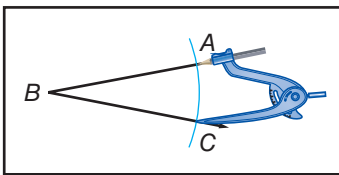
Step 2 Place the tip of the compass at point B and draw a large arc that intersects both sides of $\angle B$. Label the points of intersection A and C .



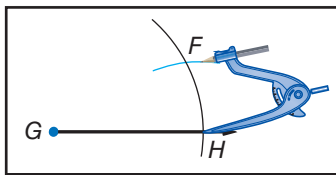
Step 3 Using the same compass setting, put the compass at point G and draw a large arc that starts above the ray and intersects the ray. Label the point of intersection H .



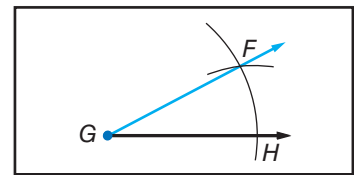
Step 4 Place the point of your compass on C and adjust so that the pencil tip is on A .



Step 5 Without changing the setting, place the compass at point H and draw an arc to intersect the larger arc you drew in Step 4. Label the point of intersection F .



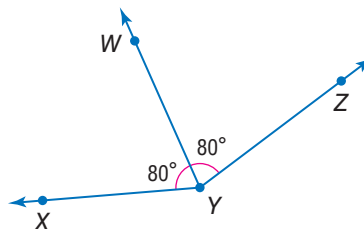
Step 6 Use a straightedge to draw \overrightarrow{GF} . $\angle ABC \cong \angle FGH$



StudyTip

Segments A line segment can also bisect an angle.

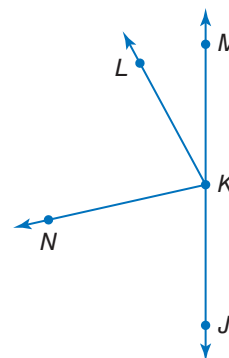
A ray that divides an angle into two congruent angles is called an **angle bisector**. If \overrightarrow{YW} is the angle bisector of $\angle XYZ$, then point W lies in the interior of $\angle XYZ$ and $\angle XYW \cong \angle WYZ$.



Just as with segments, when a line, segment, or ray divides an angle into smaller angles, the sum of the measures of the smaller angles equals the measure of the largest angle. So in the figure, $m\angle XYW + m\angle WYZ = m\angle XYZ$.

Example 3 Measure and Classify Angles

ALGEBRA In the figure, \overrightarrow{KJ} and \overrightarrow{KM} are opposite rays, and \overrightarrow{KN} bisects $\angle JKL$. If $m\angle JKN = 8x - 13$ and $m\angle NKL = 6x + 11$, find $m\angle JKN$.



Step 1 Solve for x .

Since \overrightarrow{KN} bisects $\angle JKL$, $\angle JKN \cong \angle NKL$.

$$m\angle JKN = m\angle NKL \quad \text{Definition of congruent angles}$$

$$8x - 13 = 6x + 11 \quad \text{Substitution}$$

$$8x = 6x + 24 \quad \text{Add 13 to each side.}$$

$$2x = 24 \quad \text{Subtract 6x from each side.}$$

$$x = 12 \quad \text{Divide each side by 2.}$$

Step 2 Use the value of x to find $m\angle JKN$.

$$m\angle JKN = 8x - 13 \quad \text{Given}$$

$$= 8(12) - 13 \quad x = 12$$

$$= 96 - 13 \text{ or } 83 \quad \text{Simplify.}$$

StudyTip

Checking Solutions Check that you have computed the value of x correctly by substituting the value into the expression for $\angle NKL$. If you don't get the same measure as $\angle JKN$, you have made an error.

GuidedPractice

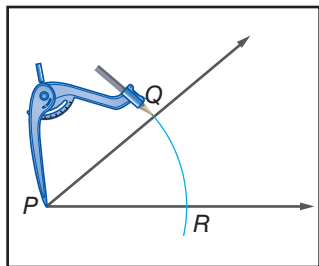
3. Suppose $m\angle JKL = 9y + 15$ and $m\angle JKN = 5y + 2$. Find $m\angle JKL$.

You can produce the angle bisector of any angle without knowing the measure of the angle.

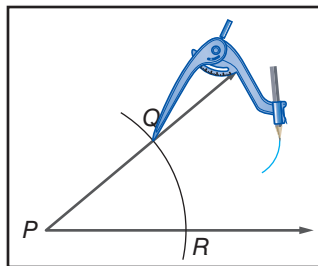
Construction Bisect an Angle



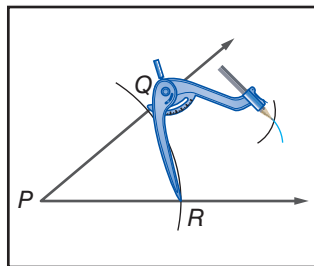
Step 1 Draw an angle on your paper. Label the vertex as P . Put your compass at point P and draw a large arc that intersects both sides of $\angle P$. Label the points of intersection Q and R .



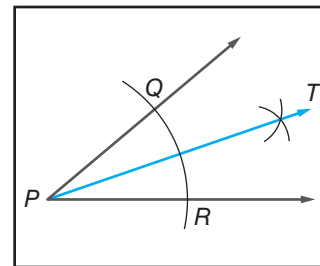
Step 2 With the compass at point Q , draw an arc in the interior of the angle.



Step 3 Keeping the same compass setting, place the compass at point R and draw an arc that intersects the arc drawn in Step 2. Label the point of intersection T .



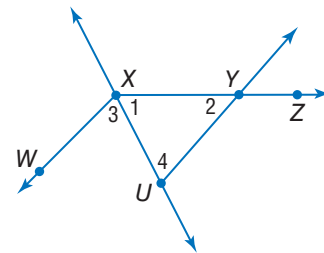
Step 4 Draw \overrightarrow{PT} . \overrightarrow{PT} is the bisector of $\angle P$.





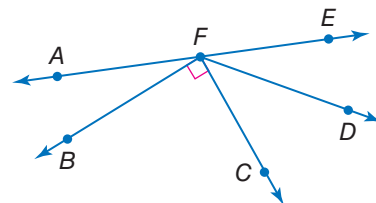
Example 1 Use the figure at the right.

1. Name the vertex of $\angle 4$.
2. Name the sides of $\angle 3$.
3. What is another name for $\angle 2$?
4. What is another name for $\angle UXY$?



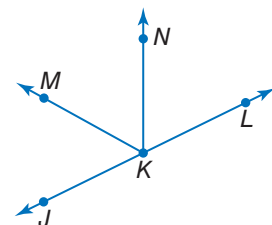
Example 2 Copy the diagram shown, and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree.

5. $\angle CFD$
6. $\angle AFD$
7. $\angle BFC$
8. $\angle AFB$

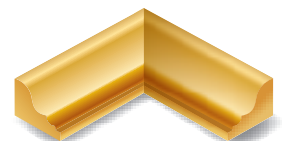


Example 3 **ALGEBRA** In the figure, \overrightarrow{KJ} and \overrightarrow{KL} are opposite rays. \overrightarrow{KN} bisects $\angle LKM$.

9. If $m\angle LKM = 7x - 5$ and $m\angle NKM = 3x + 9$, find $m\angle LKM$.
10. If $m\angle NKL = 7x - 9$ and $m\angle JKM = x + 3$, find $m\angle JKN$.



11. **CCSS PRECISION** A miter cut is used to build picture frames with corners that meet at right angles.
 - a. José miters the ends of some wood for a picture frame at congruent angles. What is the degree measure of his cut? Explain and classify the angle.
 - b. What does the joint represent in relation to the angle formed by the two pieces?



Practice and Problem Solving

Extra Practice is on page R1.

Example 1 For Exercises 12–29, use the figure at the right.

Name the vertex of each angle.

12. $\angle 4$
13. $\angle 7$
14. $\angle 2$
15. $\angle 1$

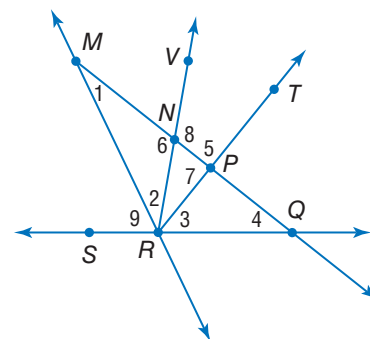
Name the sides of each angle.

16. $\angle TPQ$
17. $\angle VNM$
18. $\angle 6$
19. $\angle 3$

Write another name for each angle.

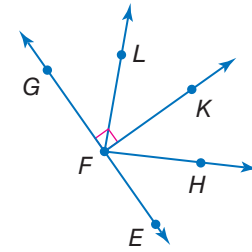
20. $\angle 9$
21. $\angle QPT$
22. $\angle MQS$
23. $\angle 5$

24. Name an angle with vertex N that appears obtuse.
25. Name an angle with vertex Q that appears acute.
26. Name a point in the interior of $\angle VRQ$.
27. Name a point in the exterior of $\angle MRT$.
28. Name a pair of angles that share exactly one point.
29. Name a pair of angles that share more than one point.



Example 2

Copy the diagram shown, and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree.



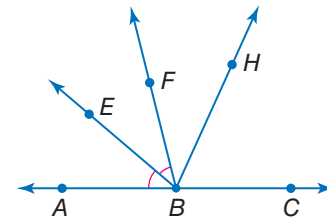
- 30. $\angle GFK$
- 31. $\angle EFK$
- 32. $\angle LFK$
- 33. $\angle EFH$
- 34. $\angle GFH$
- 35. $\angle EFL$

36. **CLOCKS** Determine at least three different times during the day when the hands on a clock form each of the following angles. Explain.
- a. right angle
 - b. obtuse angle
 - c. congruent acute angles



Example 3

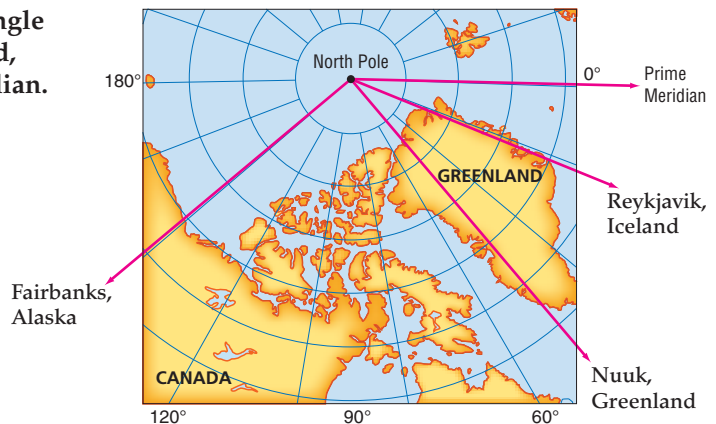
ALGEBRA In the figure, \overrightarrow{BA} and \overrightarrow{BC} are opposite rays. \overrightarrow{BH} bisects $\angle EBC$.



- 37. If $m\angle ABE = 2n + 7$ and $m\angle EBF = 4n - 13$, find $m\angle ABE$.
- 38. If $m\angle EBH = 6x + 12$ and $m\angle HBC = 8x - 10$, find $m\angle EBH$.
- 39. If $m\angle ABF = 7b - 24$ and $m\angle ABE = 2b$, find $m\angle EBF$.
- 40. If $m\angle EBC = 31a - 2$ and $m\angle EBH = 4a + 45$, find $m\angle HBC$.
- 41. If $m\angle ABF = 8s - 6$ and $m\angle ABE = 2(s + 11)$, find $m\angle EBF$.
- 42. If $m\angle EBC = 3r + 10$ and $m\angle ABE = 2r - 20$, find $m\angle EBF$.

43. **MAPS** Estimate the measure of the angle formed by each city or location listed, the North Pole, and the Prime Meridian.

- a. Nuuk, Greenland
- b. Fairbanks, Alaska
- c. Reykjavik, Iceland
- d. Prime Meridian



44. **CCSS TOOLS** A compass rose is a design on a map that shows directions. In addition to the directions of north, south, east, and west, a compass rose can have as many as 32 markings.

- a. With the center of the compass as its vertex, what is the measure of the angle between due west and due north?
- b. What is the measure of the angle between due north and north-west?
- c. How does the north-west ray relate to the angle in part a?



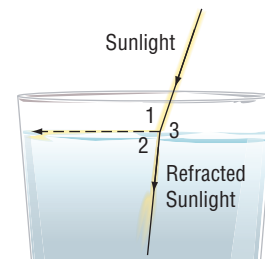
Plot the points in a coordinate plane and sketch $\triangle XYZ$. Then classify it as *right*, *acute*, or *obtuse*.

45. $X(5, -3), Y(4, -1), Z(6, -2)$

46. $X(6, 7), Y(2, 3), Z(4, 1)$

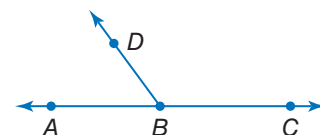
47. PHYSICS When you look at a pencil in water, it looks bent. This illusion is due to *refraction*, or the bending of light when it moves from one substance to the next.

- What is $m\angle 1$? Classify this angle as *acute*, *right*, or *obtuse*.
- What is $m\angle 2$? Classify this angle as *acute*, *right*, or *obtuse*.
- Without measuring, determine how many degrees the path of the light changes after it enters the water. Explain your reasoning.



48. MULTIPLE REPRESENTATIONS In this problem, you will explore the relationship of angles that compose opposite rays.

- Geometric** Draw four lines, each with points $A, B,$ and C . Draw \overrightarrow{BD} for each line, varying the placement of point D . Use a protractor to measure $\angle ABD$ and $\angle DBC$ for each figure.
- Tabular** Organize the measures for each figure into a table. Include a row in your table to record the sum of these measures.
- Verbal** Make a conjecture about the sum of the measures of the two angles. Explain your reasoning.
- Algebraic** If x is the measure of $\angle ABD$ and y is the measure of $\angle DBC$, write an equation that relates the two angle measures.



H.O.T. Problems Use Higher-Order Thinking Skills

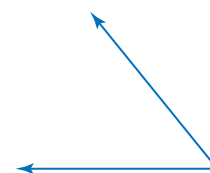
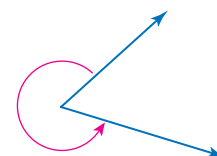
49. OPEN ENDED Draw an obtuse angle named $\angle ABC$. Measure $\angle ABC$. Construct an angle bisector \overrightarrow{BD} of $\angle ABC$. Explain the steps in your construction and justify each step. Classify the two angles formed by the angle bisector.

50. CHALLENGE Describe how you would use a protractor to measure the angle shown.

51. CCSS ARGUMENTS The sum of two acute angles is *sometimes*, *always*, or *never* an obtuse angle. Explain.

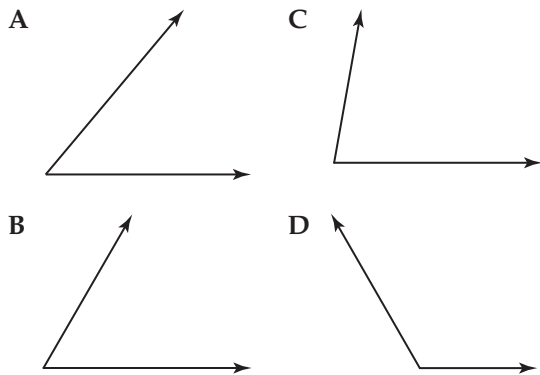
52. CHALLENGE \overrightarrow{MP} bisects $\angle LMN$, \overrightarrow{MQ} bisects $\angle LMP$, and \overrightarrow{MR} bisects $\angle QMP$. If $m\angle RMP = 21$, find $m\angle LMN$. Explain your reasoning.

53. WRITING IN MATH Rashid says that he can estimate the measure of an acute angle using a piece of paper to within six degrees of accuracy. Explain how this would be possible. Then use this method to estimate the measure of the angle shown.



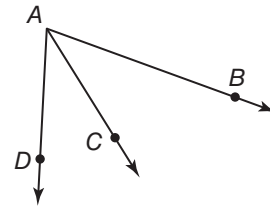
Standardized Test Practice

54. Which of the following angles measures closest to 60° ?



55. **SHORT RESPONSE** Leticia surveyed 50 English majors at a university to see if the school should play jazz music in the cafeteria during lunch. The school has 75 different majors and a total of 2000 students. Explain why the results of Leticia's survey are or are not representative of the entire student body.

56. In the figure below, if $m\angle BAC = 38$, what must be the measure of $\angle BAD$ in order for \overline{AC} to be an angle bisector?



- F 142
G 76
- H 52
J 38

57. **SAT/ACT** If n is divisible by 2, 5, and 14, which of the following is also divisible by these numbers?

- A $n + 7$
B $n + 10$
C $n + 14$
- D $n + 20$
E $n + 70$

Spiral Review

Find the distance between each pair of points. Round to the nearest hundredth.

(Lesson 1-3)

58. $A(-1, -8), B(3, 4)$
59. $C(0, 1), D(-2, 9)$
60. $E(-3, -12), F(5, 4)$
61. $G(4, -10), H(9, -25)$
62. $J\left(1, \frac{1}{4}\right), K\left(-3, \frac{7}{4}\right)$
63. $L\left(-5, \frac{8}{5}\right), M\left(5, \frac{2}{5}\right)$

Find the value of the variable and ST if S is between R and T . (Lesson 1-2)

64. $RS = 7a, ST = 12a, RT = 76$
65. $RS = 12, ST = 2x, RT = 34$

66. **PHOTOGRAPHY** Photographers often place their cameras on tripods. In the diagram, the tripod is placed on an inclined surface, and the length of each leg is adjusted so that the camera remains level with the horizon. Are the feet of the tripod coplanar? Explain your reasoning. (Lesson 1-1)



Complete each sentence. (Lesson 0-1)

67. 54 in. = ft
68. 275 mm = m
69. 7 gal = pt

Skills Review

Solve each equation.

70. $(90 - x) - x = 18$
71. $(5x + 3) + 7x = 180$
72. $(13x + 10) + 2x = 90$
73. $(180 - x) - 4x = 56$
74. $(4n + 17) + (n - 2) = 180$
75. $(8a - 23) + (9 - 2a) = 90$

