Angle Relationships

- You measured and classified angles.

1 Identify and use special pairs of angles.

2 Identify perpendicular lines.

Cheerleaders position their arms and legs at specific angles to create various formations when performing at games and at competitions. Certain pairs of angles have special names and share specific relationships.

NewVocabulary
adjacent angles
linear pair
vertical angles
complementary angles
supplementary angles
perpendicular

Common Core State Standards

Content Standards
Preparation for G.SRT.7
Explain and use the relationship between the sine and cosine of complementary angles.

Mathematical Practices
2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.

1 **Pairs of Angles** Some pairs of angles are special because of how they are positioned in relationship to each other. Three of these angle pairs are described below.

KeyConcept Special Angle Pairs

Adjacent angles are two angles that lie in the same plane and have a common vertex and a common side, but no common interior points.
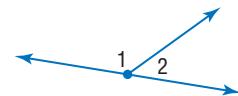
Examples ∠1 and ∠2 are adjacent angles.

Nonexamples ∠3 and ∠ABC are nonadjacent angles
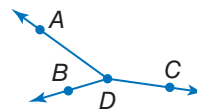
A **linear pair** is a pair of adjacent angles with noncommon sides that are opposite rays.
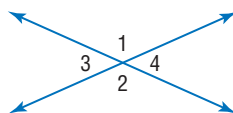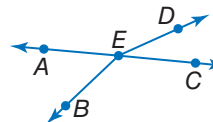
Example ∠1 and ∠2

Nonexample ∠ADB and ∠ADC

Vertical angles are two nonadjacent angles formed by two intersecting lines.

Examples ∠1 and ∠2; ∠3 and ∠4

Nonexample ∠AEB and ∠DEC

🌐 Real-World Example 1 Identify Angle Pairs

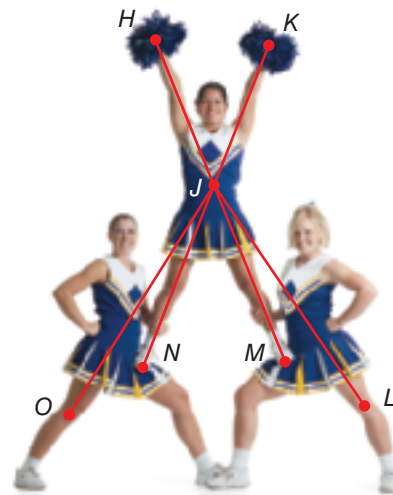**CHEERLEADING** Name an angle pair that satisfies each condition.

a. two acute adjacent angles

∠HJK, ∠LJM, ∠MJN, and ∠NJO are acute angles.

∠LJM and ∠MJN are acute adjacent angles, and ∠MJN and ∠NJO are acute adjacent angles.

b. two obtuse vertical angles

∠HJN and ∠KJM are obtuse vertical angles.

▶ **Guided**Practice
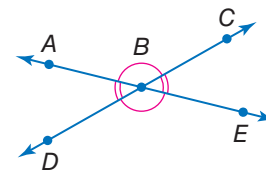
1A. a linear pair

1B. two acute vertical angles

Some pairs of angles are special because of the relationship between their angle measures.
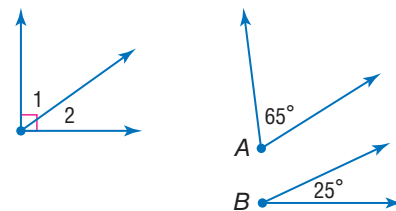

KeyConcept Angle Pair Relationships

Vertical angles are congruent.

Examples ∠ABC ≅ ∠DBE and ∠ABD ≅ ∠CBE

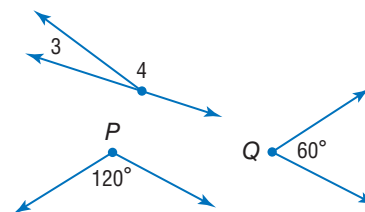
Complementary angles are two angles with measures that have a sum of 90.

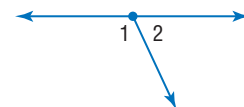**Examples** ∠1 and ∠2 are complementary.
∠A is complementary to ∠B.

65°

25°

Supplementary angles are two angles with measures that have a sum of 180.

Examples ∠3 and ∠4 are supplementary.
∠P and ∠Q are supplementary.

120°

60°

The angles in a linear pair are supplementary.

Example $m∠1 + m∠2 = 180$

StudyTip

Linear Pair vs. Supplementary Angles
While the angles in a linear pair are always supplementary, some supplementary angles do not form a linear pair.

Remember that angle measures are real numbers. So the operations for real numbers and algebra can be used with angle measures.

Example 2 Angle Measure

ALGEBRA Find the measures of two supplementary angles if the difference in the measures of the two angles is 18.

Understand The problem relates the measures of two supplementary angles. You know that the sum of the measures of supplementary angles is 180. You need to find the measure of each angle.

Plan Draw two figures to represent the angles. Let the measure of one angle be x. If $m\angle A = x$, then because $\angle A$ and $\angle B$ are supplementary, $m\angle B + x = 180$ or $m\angle B = 180 - x$.
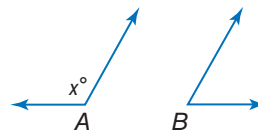
The problem states that the difference of the two angle measures is 18, or $m\angle B - m\angle A = 18$.

Solve

$m\angle B - m\angle A = 18$	Given
$(180 - x) - x = 18$	$m\angle A = x,\ m\angle B = 180 - x$
$180 - 2x = 18$	Simplify.
$-2x = -162$	Subtract 180 from each side.
$x = 81$	Divide each side by −2.

Use the value of x to find each angle measure.

$m\angle A = x \qquad\qquad m\angle B = 180 - x$

$\qquad = 81 \qquad\qquad\qquad = 180 - 81$ or 99

Check Add the angle measures to verify that the angles are supplementary.

$m\angle A + m\angle B \overset{?}{=} 180$

$81 + 99 = 180$ ✔

> **Problem-Solving Tip**
>
> **Write an Equation** While you could use the guess-and-check strategy to find two measures with a sum of 180 and a difference of 18, writing an equation is a more efficient approach to this problem.
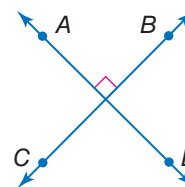
▶ **Guided Practice**

2. Find the measures of two complementary angles if the measure of the larger angle is 12 more than twice the measure of the smaller angle.

2 Perpendicular Lines Lines, segments, or rays that form right angles are **perpendicular.**

> **Key Concept** Perpendicular Lines
>
> • Perpendicular lines intersect to form four right angles.
>
> • Perpendicular lines intersect to form congruent adjacent angles.
>
> • Segments and rays can be perpendicular to lines or other line segments and rays.
>
> • The right angle symbol in the figure indicates that the lines are perpendicular.
>
> **Symbol** ⊥ is read *is perpendicular to.* **Example** $\overleftrightarrow{AD} \perp \overleftrightarrow{CB}$

Example 3 Perpendicular Lines

ALGEBRA Find x and y so that $\overleftrightarrow{PR}$ and $\overleftrightarrow{SQ}$ are perpendicular.

If $\overleftrightarrow{PR} \perp \overleftrightarrow{SQ}$, then $m\angle STR = 90$ and $m\angle PTQ = 90$.

To find x, use $\angle STW$ and $\angle WTR$.

$m\angle STR = m\angle STW + m\angle WTR$	Sum of parts = whole
$90 = 2x + (5x + 6)$	Substitution
$90 = 7x + 6$	Combine like terms.
$84 = 7x$	Subtract 6 from each side.
$12 = x$	Divide each side by 7.
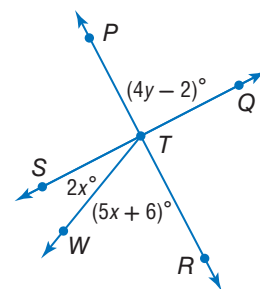
To find y, use $m\angle PTQ$.

|	
$m\angle PTQ = 4y - 2$	Given
$90 = 4y - 2$	Substitution
$92 = 4y$	Add 2 to each side.
$23 = y$	Divide each side by 4.

▶ **Guided**Practice

3. Suppose $m\angle D = 3x - 12$. Find x so that $\angle D$ is a right angle.

In the figure at the right, it *appears* that $\overleftrightarrow{FG} \perp \overleftrightarrow{JK}$. However, you cannot assume this is true unless other information, such as $m\angle FHJ = 90$, is given.

In geometry, figures are sketches used to depict a situation. They are not drawn to reflect total accuracy. There are certain relationships that you can assume to be true, but others you cannot. Study the figure and the lists below.
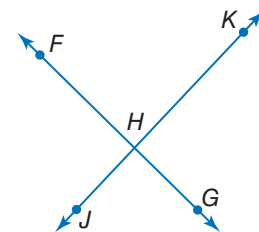
KeyConcept Interpreting Diagrams

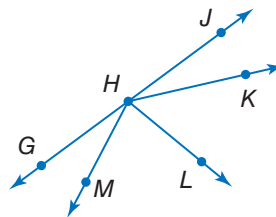| CAN be Assumed	CANNOT be Assumed
All points shown are coplanar.	Perpendicular lines: $\overrightarrow{HM} \perp \overrightarrow{HL}$
G, H, and J are collinear.	Congruent angles: $\angle JHK \cong \angle GHM$
$\overrightarrow{HM}$, $\overrightarrow{HL}$, $\overrightarrow{HK}$, and $\overleftrightarrow{GJ}$ intersect at H.	$\angle JHK \cong \angle KHL$
H is between G and J.	$\angle KHL \cong \angle LHM$
L is in the interior of $\angle MHK$.	Congruent segments: $\overline{GH} \cong \overline{HJ}$
$\angle GHM$ and $\angle MHL$ are adjacent angles.	$\overline{HJ} \cong \overline{HK}$
$\angle GHL$ and $\angle LHJ$ are a linear pair.	$\overline{HK} \cong \overline{HL}$
$\angle JHK$ and $\angle KHG$ are supplementary.	$\overline{HL} \cong \overline{HG}$

The list of statements that can be assumed is not a complete list.
There are more special pairs of angles than those listed.

StudyTip

Additional Information
Additional information for a figure may be given using congruent angle markings, congruent segment markings, or right angle symbols.

Example 4 Interpret Figures

Determine whether each statement can be assumed from the figure. Explain.

a. ∠KHJ and ∠GHM are complementary.
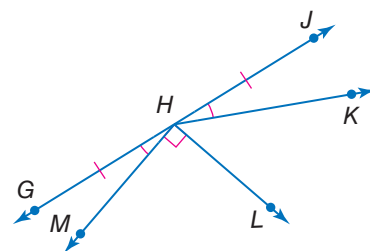
No; they are congruent, but we do not know anything about their exact measures.

b. ∠GHK and ∠JHK are a linear pair.

Yes; they are adjacent angles whose noncommon sides are opposite rays.

c. $\overrightarrow{HL}$ is perpendicular to $\overrightarrow{HM}$.

Yes; the right angle symbol in the figure indicates that $\overrightarrow{HL} \perp \overrightarrow{HM}$.

GuidedPractice

4A. ∠GHL and ∠LHJ are supplementary.

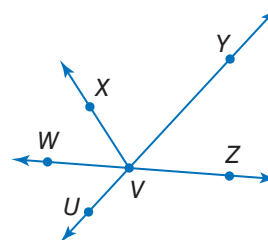**4B.** ∠GHM and ∠MHK are adjacent angles.

Check Your Understanding   = Step-by-Step Solutions begin on page R14.
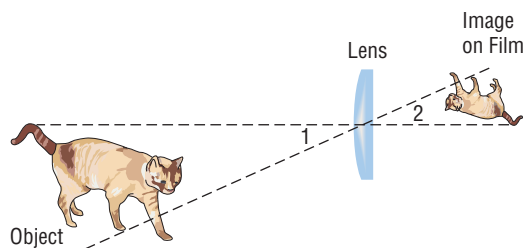
Example 1   **Name an angle pair that satisfies each condition.**

   1. two acute vertical angles

   2. two obtuse adjacent angles

Examples 1–2 **3. CAMERAS** Cameras use lenses and light to capture images.

   a. What type of angles are formed by the object and its image?

   **b.** If the measure of ∠2 is 15, what is the measure of ∠1?

Examples 2–3 **4. ALGEBRA** The measures of two complementary angles are $7x + 17$ and $3x - 20$. Find the measures of the angles.

5 **ALGEBRA** Lines x and y intersect to form adjacent angles 2 and 3. If $m\angle 2 = 3a - 27$ and $m\angle 3 = 2b + 14$, find the values of a and b so that x is perpendicular to y.

Example 4 Determine whether each statement can be assumed from the figure. Explain.

   6. ∠CAD and ∠DAB are complementary.

   7. ∠EDB and ∠BDA are adjacent, but they are neither complementary nor supplementary.
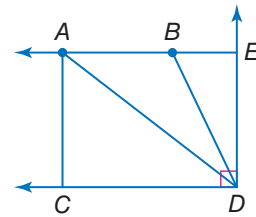
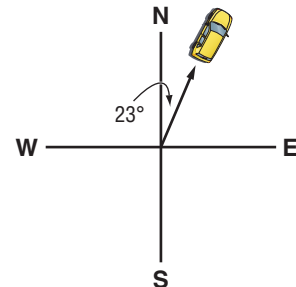## Practice and Problem Solving

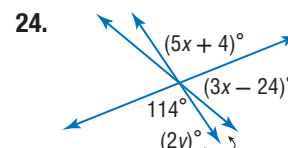**Examples 1–2** Name an angle or angle pair that satisfies each condition.

   8. two adjacent angles

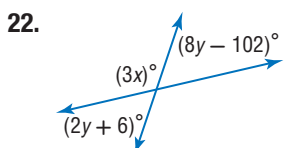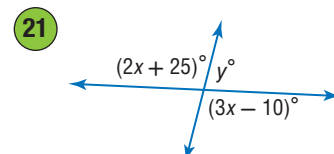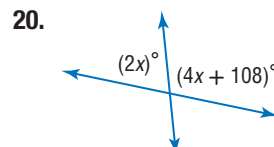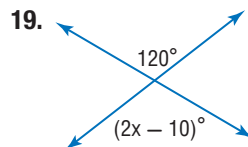   **9.** two acute vertical angles

   10. two obtuse vertical angles

   11. two complementary adjacent angles

   12. two complementary nonadjacent angles

   13. two supplementary adjacent angles

   14. a linear pair whose vertex is F

   15. an angle complementary to ∠FDG

   16. an angle supplementary to ∠CBF

   17. an angle supplementary to ∠JAE
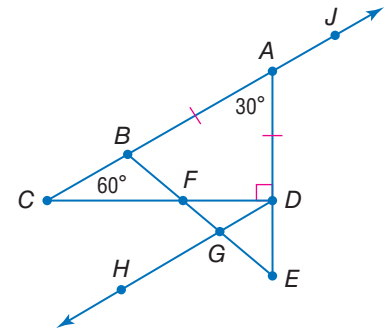
   18. **CCSS REASONING** You are using a compass to drive 23° east of north. Express your direction in another way using an acute angle and two of the four directions: north, south, east, and west. Explain your reasoning.

Example 2 Find the value of each variable.

   19.

   $120°$
   $(2x - 10)°$

   20.

   $(2x)°$
   $(4x + 108)°$

   21.

   $(2x + 25)°$ $y°$
   $(3x - 10)°$

   22.

   $(8y - 102)°$
   $(3x)°$
   $(2y + 6)°$

   23.

   $(2y + 50)°$ $(7x - 248)°$
   $(5y - 17)°$ $(x + 44)°$

   24.

   $(5x + 4)°$
   $(3x - 24)°$
   $114°$
   $(2y)°$
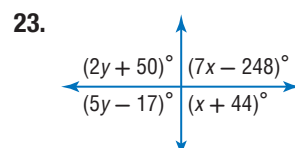
   25. **ALGEBRA** ∠E and ∠F are supplementary. The measure of ∠E is 54 more than the measure of ∠F. Find the measures of each angle.

   26. **ALGEBRA** The measure of an angle's supplement is 76 less than the measure of the angle. Find the measure of the angle and its supplement.

27. ALGEBRA The measure of the supplement of an angle is 40 more than two times the measure of the complement of the angle. Find the measure of the angle.

28. ALGEBRA $\angle 3$ and $\angle 4$ form a linear pair. The measure of $\angle 3$ is four more than three times the measure of $\angle 4$. Find the measure of each angle.

Example 3   **ALGEBRA Use the figure at the right.**

29 If $m\angle KNL = 6x - 4$ and $m\angle LNM = 4x + 24$, find the value of x so that $\angle KNM$ is a right angle.

30. If $m\angle JNP = 3x - 15$ and $m\angle JNL = 5x + 59$, find the value of x so that $\angle JNP$ and $\angle JNL$ are supplements of each other.

31. If $m\angle LNM = 8x + 12$ and $m\angle JNL = 12x - 32$, find $m\angle JNP$.

32. If $m\angle JNP = 2x + 3$, $m\angle KNL = 3x - 17$, and $m\angle KNJ = 3x + 34$, find the measure of each angle.
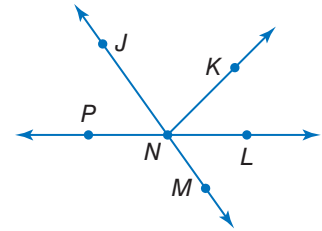
33. PHYSICS As a ray of light meets a mirror, the light is reflected. The angle at which the light strikes the mirror is the *angle of incidence*. The angle at which the light is reflected is the *angle of reflection*. The angle of incidence and the angle of reflection are congruent. In the diagram at the right, if $m\angle RMI = 106$, find the angle of reflection and $m\angle RMJ$.
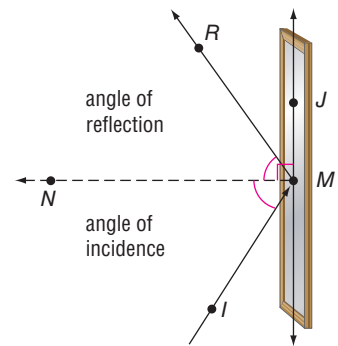
34. ALGEBRA Rays AB and BC are perpendicular. Point D lies in the interior of $\angle ABC$. If $m\angle ABD = 3r + 5$ and $m\angle DBC = 5r - 27$, find $m\angle ABD$ and $m\angle DBC$.

35. ALGEBRA $\overleftrightarrow{WX}$ and $\overleftrightarrow{YZ}$ intersect at point V. If $m\angle WVY = 4a + 58$ and $m\angle XVY = 2b - 18$, find the values of a and b so that $\overleftrightarrow{WX}$ is perpendicular to $\overleftrightarrow{YZ}$.

Example 4   **Determine whether each statement can be assumed from the figure. Explain.**

36. $\angle 4$ and $\angle 7$ are vertical angles.

37. $\angle 4$ and $\angle 8$ are supplementary.

38. $p \perp t$

39. $\angle 3 \cong \angle 6$

40. $\angle 5 \cong \angle 3 + \angle 6$

41. $\angle 5$ and $\angle 7$ form a linear pair.
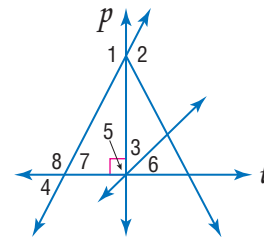
42. CCSS ARGUMENTS In the diagram of the pruning shears shown, $m\angle 1 = m\angle 3$. What conclusion can you reach about the relationship between $\angle 4$ and $\angle 2$? Explain.
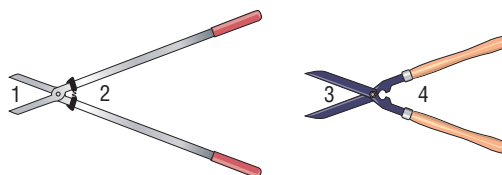
FLIGHT The wing of the aircraft shown can pivot up to 60° in either direction from the perpendicular position.
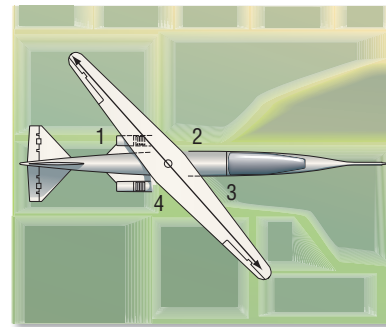
43. Identify a pair of vertical angles.

44. Identify two pairs of supplementary angles.

45 If $m\angle 1 = 110$, what is $m\angle 3$? $m\angle 4$?

46. What is the minimum possible value for $m\angle 2$? the maximum?

47. Is there a wing position in which none of the angles are obtuse? Explain.

48. 🔀 **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship between the sum of the interior angles of a triangle and the angles vertical to them.

a. Geometric Draw three sets of three intersecting lines and label each as shown.

b. Tabular For each set of lines, measure and record $m\angle 1$, $m\angle 2$, and $m\angle 3$ in a table. Record $m\angle 1 + m\angle 2 + m\angle 3$ in a separate column.

c. Verbal Explain how you can find $m\angle 4$, $m\angle 5$, and $m\angle 6$ when you know $m\angle 1$, $m\angle 2$, and $m\angle 3$.

d. Algebraic Write an equation that relates $m\angle 1 + m\angle 2 + m\angle 3$ to $m\angle 4 + m\angle 5 + m\angle 6$. Then use substitution to write an equation that relates $m\angle 4 + m\angle 5 + m\angle 6$ to an integer.
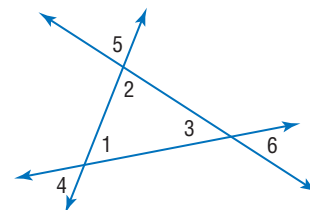
H.O.T. Problems   Use Higher-Order Thinking Skills

49. CCSS **REASONING** Are there angles that do not have a complement? Explain.

50. **OPEN ENDED** Draw a pair of intersecting lines that forms a pair of complementary angles. Explain your reasoning.

51. **CHALLENGE** If a line, line segment, or ray is perpendicular to a plane, it is perpendicular to every line, line segment, or ray in the plane that intersects it.

a. If a line is perpendicular to each of two intersecting lines at their point of intersection, then the line is perpendicular to the plane determined by them. If line a is perpendicular to line ℓ and line m at point X, what must also be true?

b. If a line is perpendicular to a plane, then any line perpendicular to the given line at the point of intersection with the given plane is in the given plane. If line a is perpendicular to plane $\mathcal{P}$ and line m at point X, what must also be true?

c. If a line is perpendicular to a plane, then every plane containing the line is perpendicular to the given plane. If line a is perpendicular to plane $\mathcal{P}$, what must also be true?
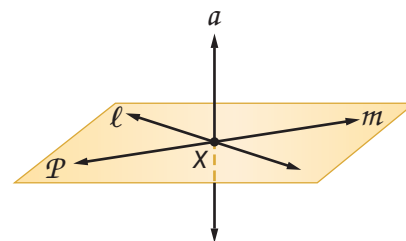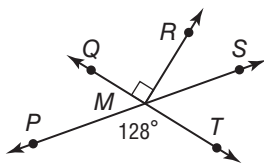
52. **WRITING IN MATH** Describe three different ways you can determine that an angle is a right angle.

53. What is $m\angle RMS$ in the figure below?

A 26

B 38

C 52

D 128

54. EXTENDED RESPONSE For a fundraiser, a theater club is making 400 cookies. They want to make twice as many chocolate chip as peanut butter cookies and three times as many peanut butter as oatmeal raisin cookies. Determine how many of each type of cookie the theater club will make. Show your work.

55. ALGEBRA Which inequality is graphed below?
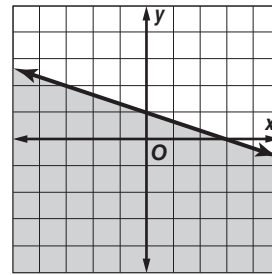
F $y > -\frac{1}{3}x + 1$ **H** $y \geq -\frac{1}{3}x + 1$

G $y < -\frac{1}{3}x + 1$ **J** $y \leq -\frac{1}{3}x + 1$

56. SAT/ACT One third of a number is three more than one fourth the same number. What is the number?

A 3   **D** 42

B 12   **E** 48

C 36

Spiral Review

Copy the diagram shown and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree. (Lesson 1-4)
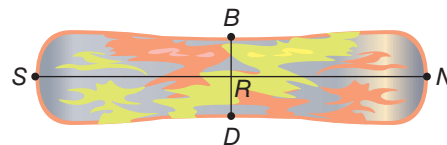
57. $\angle ABC$

58. $\angle DBC$

59. $\angle ABD$
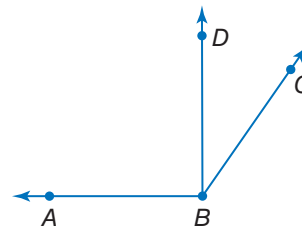
Find the coordinates of the midpoint of a segment with the given endpoints. (Lesson 1-3)

60. $P(3, -7), Q(9, 6)$ **61.** $A(-8, -5), B(1, 7)$ **62.** $J(-7, 4), K(3, 1)$

63. SNOWBOARDING In the design on the snowboard shown, $\overline{BD}$ bisects $\overline{SN}$ at R. If $SN = 163$ centimeters, find RN. (Lesson 1-2)

Skills Review

Name the congruent sides and angles in each figure.

64.

65.

66.

67.