LESSON 1-6

Two-Dimensional Figures

Then

- You measured one-dimensional figures.

Now

1. Identify and name polygons.
2. Find perimeter, circumference, and area of two-dimensional figures.

Why?

- Mosaics are patterns or pictures created using small bits of colored glass or stone. They are usually set into a wall or floor and often make use of polygons.

NewVocabulary

polygon
vertex of a polygon
concave
convex
n-gon
equilateral polygon
equiangular polygon
regular polygon
perimeter
circumference
area

Common Core State Standards

Content Standards

G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Mathematical Practices

2 Reason abstractly and quantitatively.

6 Attend to precision.

1 Identify Polygons

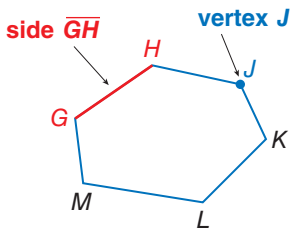
Most of the closed figures shown in the mosaic are polygons. The term *polygon* is derived from a Greek word meaning *many angles*.

KeyConcept Polygons

A **polygon** is a closed figure formed by a finite number of coplanar segments called *sides* such that

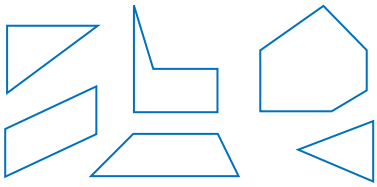
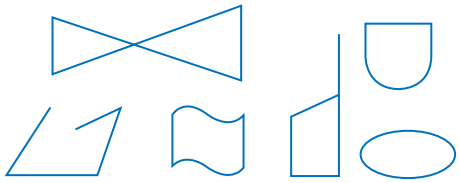
- the sides that have a common endpoint are noncollinear, and
- each side intersects exactly two other sides, but only at their endpoints.

The vertex of each angle is a **vertex of the polygon**. A polygon is named by the letters of its vertices, written in order of consecutive vertices.

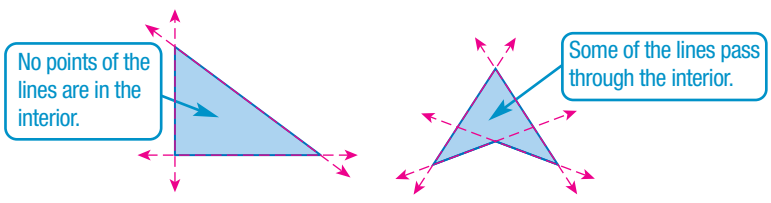


polygon *GHJKLM*

The table below shows some additional examples of polygons and some examples of figures that are not polygons.

Polygons	Not Polygons
	

Polygons can be **concave** or **convex**. Suppose the line containing each side is drawn. If any of the lines contain any point in the interior of the polygon, then it is concave. Otherwise it is convex.



convex polygon concave polygon

Pixoi Ltd/Alamy

StudyTip

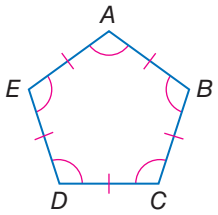
Naming Polygons The Greek prefixes used to name polygons are also used to denote number. For example a *bi*cycle has two wheels, and a *tri*pod has three legs.

In general, a polygon is classified by its number of sides. The table lists some common names for various categories of polygon. A polygon with n sides is an ***n*-gon**. For example, a polygon with 15 sides is a 15-gon.

Number of Sides	Polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
11	hendecagon
12	dodecagon
n	n-gon

An **equilateral polygon** is a polygon in which all sides are congruent. An **equiangular polygon** is a polygon in which all angles are congruent.

A convex polygon that is both equilateral and equiangular is called a **regular polygon**. An *irregular polygon* is a polygon that is *not* regular.



regular pentagon *ABCDE*

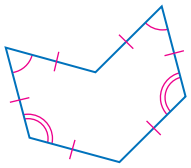
ReadingMath

Simple Closed Curves Polygons and circles are examples of *simple closed curves*. Such a curve begins and ends at the same point without crossing itself. The figures below are *not* simple closed curves.

Example 1 Name and Classify Polygons

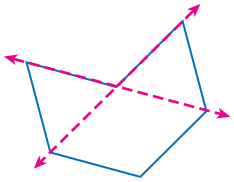
Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

a.



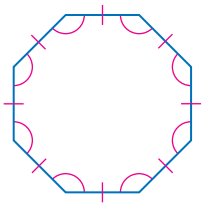
The polygon has 6 sides, so it is a hexagon.

Two of the lines containing the sides of the polygon will pass through the interior of the hexagon, so it is concave.



Only convex polygons can be regular, so this is an irregular hexagon.

b.



There are 8 sides, so this is an octagon.

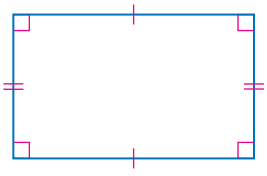
No line containing any of the sides will pass through the interior of the octagon, so it is convex.

All of the sides are congruent, so it is equilateral. All of the angles are congruent, so it is equiangular.

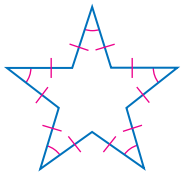
Since the polygon is convex, equilateral, and equiangular, it is regular. So this is a regular octagon.

GuidedPractice

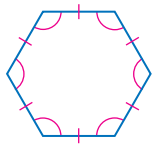
1A.



1B.



1C.

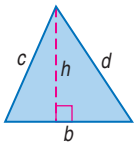
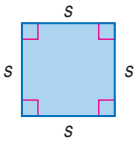
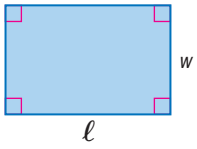
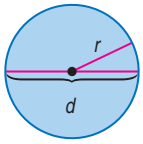


2 Perimeter, Circumference, and Area

The **perimeter** of a polygon is the sum of the lengths of the sides of the polygon. Some shapes have special formulas for perimeter, but all are derived from the basic definition of perimeter. You will derive these formulas in Chapter 11. The **circumference** of a circle is the distance around the circle.

The **area** of a figure is the number of square units needed to cover a surface. Review the formulas for the perimeter and area of three common polygons and circle given below.

KeyConcept Perimeter, Circumference, and Area

Triangle	Square	Rectangle	Circle
			
$P = b + c + d$	$P = s + s + s + s$ $= 4s$	$P = \ell + w + \ell + w$ $= 2\ell + 2w$	$C = 2\pi r$ or $C = \pi d$
$A = \frac{1}{2}bh$	$A = s^2$	$A = \ell w$	$A = \pi r^2$

P = perimeter of polygon; A = area of figure; C = circumference

b = base, h = height; ℓ = length, w = width; r = radius, d = diameter

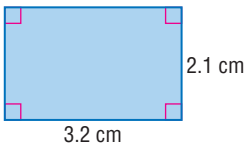
ReadingMath

Pi The symbol π is read *pi*. This is not a variable but an irrational number. The most accurate way to perform a calculation with π is to use a calculator. If no calculator is available, 3.14 is a good estimate for π.

Example 2 Find Perimeter and Area

Find the perimeter or circumference and area of each figure.

a.



$P = 2\ell + 2w$ Perimeter of rectangle

$= 2(3.2) + 2(2.1)$ $\ell = 3.2$, $w = 2.1$

$= 10.6$ Simplify.

The perimeter is 10.6 centimeters.

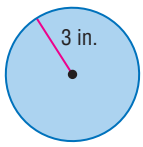
$A = \ell w$ Area of rectangle

$= (3.2)(2.1)$ $\ell = 3.2$, $w = 2.1$

$= 6.72$ Simplify.

The area is about 6.7 square centimeters.

b.



$C = 2\pi r$ Circumference

$= 2\pi(3)$ $r = 3$

≈ 18.85 Use a calculator.

The circumference is about 18.9 inches.

$A = \pi r^2$ Area of circle

$= \pi(3)^2$ $r = 3$

≈ 28.3 Use a calculator.

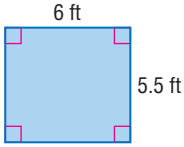
The area is about 28.3 square inches.

StudyTip

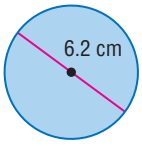
Perimeter vs. Area Since calculating the area of a figure involves multiplying two dimensions (unit × unit), *square units* are used. There is only one dimension used when finding the perimeter (the distance around), thus, it is given simply in *units*.

GuidedPractice

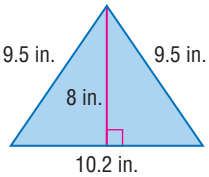
2A.



2B.



2C.



Standardized Test Example 3 Largest Area

Yolanda has 26 centimeters of cording to frame a photograph in her scrapbook. Which of these shapes would use *most* or all of the cording and enclose the *largest* area?

A right triangle with each leg about 7 centimeters long

B circle with a radius of about 4 centimeters

C rectangle with a length of 8 centimeters and a width of 4.5 centimeters

D square with a side length of 6 centimeters

Test-TakingTip

Mental Math When you are asked to compare measures for varying figures, it can be helpful to use mental math. Estimate the perimeter or area of each figure, and then check your calculations.

Read the Test Item

You are asked to compare the area and perimeter of four different shapes.

Solve the Test Item

Find the perimeter and area of each shape.

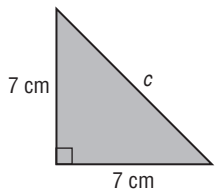
Right Triangle

Use the Pythagorean Theorem to find the length of the hypotenuse.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = 7^2 + 7^2$ or 98 $a = 7, b = 7$

$c = \sqrt{98}$ or about 9.9 Simplify.



StudyTip

Irrational Measures Notice that the triangle perimeter given in Example 3 is only an approximation. Because the length of the hypotenuse is an irrational number, the actual perimeter of the triangle is the irrational measure $(14 + \sqrt{98})$ centimeters.

$P = a + b + c$ Perimeter of a triangle

$\approx 7 + 7 + 9.9$ or about 23.9 cm Substitution

$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(7)(7)$ or 24.5 cm^2 Substitution

Circle	Rectangle	Square
$C = 2\pi r$	$P = 2\ell + 2w$	$P = 4s$
$= 2\pi(4)$	$= 2(8) + 2(4.5)$	$= 4(6)$
≈ 25.1 cm	$= 25$ cm	$= 24$ cm
$A = \pi r^2$	$A = \ell w$	$A = s^2$
$= \pi(4)^2$	$= (8)(4.5)$	$= 6^2$
≈ 50.3 cm^2	$= 36$ cm^2	$= 36$ cm^2

The shape that uses the most cording and encloses the largest area is the circle. The answer is B.

GuidedPractice

3. Dasan has 32 feet of fencing to fence in a play area for his dog. Which shape of play area uses *most* or all of the fencing and encloses the *largest* area?

F circle with radius of about 5 feet

G rectangle with length 5 feet and width 10 feet

H right triangle with legs of length 10 feet each

J square with side length 8 feet

You can use the Distance Formula to find the perimeter of a polygon graphed on a coordinate plane.

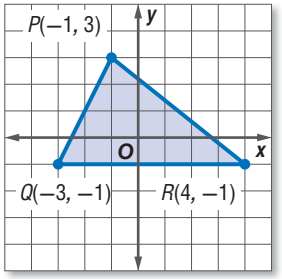
Example 4 Perimeter and Area on the Coordinate Plane

COORDINATE GEOMETRY **Find the perimeter and area of $\triangle PQR$ with vertices $P(-1, 3)$, $Q(-3, -1)$, and $R(4, -1)$.**

Step 1 Find the perimeter of $\triangle PQR$.

Graph $\triangle PQR$.

To find the perimeter of $\triangle PQR$, first find the lengths of each side. Counting the squares on the grid, we find that $QR = 7$ units. Use the Distance Formula to find the lengths of $\overline{PQ}$ and $\overline{PR}$.



$\overline{PQ}$ has endpoints at $P(-1, 3)$ and $Q(-3, -1)$.

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{[-1 - (-3)]^2 + [3 - (-1)]^2}$ Substitute.

$= \sqrt{2^2 + 4^2}$ Subtract.

$= \sqrt{20}$ or about 4.5 Simplify.

$\overline{PR}$ has endpoints at $P(-1, 3)$ and $R(4, -1)$.

$PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(-1 - 4)^2 + [3 - (-1)]^2}$ Substitute.

$= \sqrt{(-5)^2 + 4^2}$ Subtract.

$= \sqrt{41}$ or about 6.4 Simplify.

The perimeter of $\triangle PQR$ is $7 + \sqrt{20} + \sqrt{41}$ or about 17.9 units.

StudyTip
Linear and Square Units
Remember to use linear units with perimeter and square units with area.

Step 2 Find the area of $\triangle PQR$.

To find the area of the triangle, find the lengths of the height and base. The height is the perpendicular distance from P to $\overline{QR}$. Counting squares on the graph, the height is 4 units. The length of $\overline{QR}$ is 7 units.

$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(7)(4)$ or 14 Substitute and simplify.

The area of $\triangle PQR$ is 14 square units.

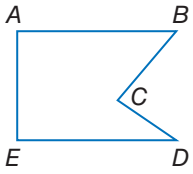
GuidedPractice

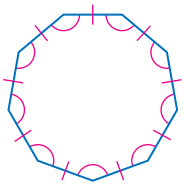
4. Find the perimeter and area of $\triangle ABC$ with vertices $A(-1, 4)$, $B(-1, -1)$, and $C(6, -1)$.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

1. 

2. 

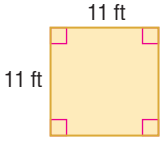
SIGNS **Identify the shape of each traffic sign and classify it as *regular* or *irregular*.**

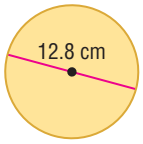
3. stop

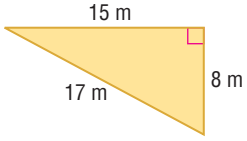

4. caution or warning


5. slow moving vehicle


Example 2 **Find the perimeter or circumference and area of each figure. Round to the nearest tenth.**

6. 

7. 

8. 

Example 3 9. **MULTIPLE CHOICE** Vanesa is making a banner for the game. She has 20 square feet of fabric. What shape will use *most* or all of the fabric?

A a square with a side length of 4 feet

B a rectangle with a length of 4 feet and a width of 3.5 feet

C a circle with a radius of about 2.5 feet

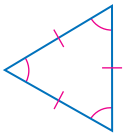
D a right triangle with legs of about 5 feet each

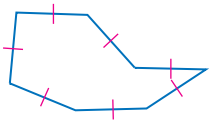
Example 4 10. CCSS **REASONING** Find the perimeter and area of $\triangle ABC$ with vertices $A(-1, 2)$, $B(3, 6)$, and $C(3, -2)$.

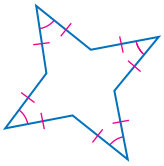
Practice and Problem Solving

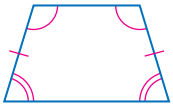
Extra Practice is on page R1.

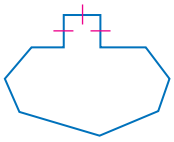
Example 1 **Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.**

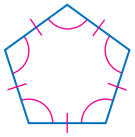
11. 

12. 

13. 

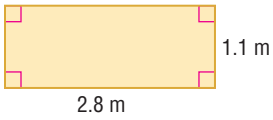
14. 

15. 

16. 

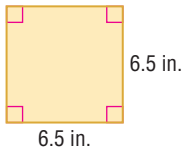
Examples 2–3 **Find the perimeter or circumference and area of each figure. Round to the nearest tenth.**

17.



18.

19.



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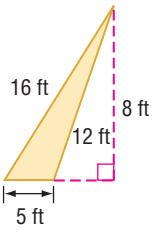

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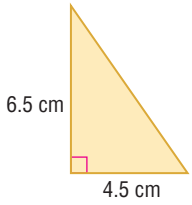


22.

23. **CRAFTS** Joy has a square picture that is 4 inches on each side. The picture is framed with a length of ribbon. She wants to use the same piece of ribbon to frame a circular picture. What is the maximum radius of the circular frame?

24. **LANDSCAPING** Mr. Jackson has a circular garden with a diameter of 10 feet surrounded by edging. Using the same length of edging, he is going to create a square garden. What is the maximum side length of the square?

Example 4 **CCSS REASONING** **Graph each figure with the given vertices and identify the figure. Then find the perimeter and area of the figure.**

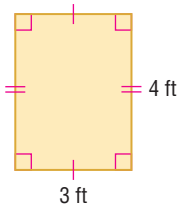
25. $D(-2, -2)$, $E(-2, 3)$, $F(2, -1)$

26. $J(-3, -3)$, $K(3, 2)$, $L(3, -3)$

27. $P(-1, 1)$, $Q(3, 4)$, $R(6, 0)$, $S(2, -3)$

28. $T(-2, 3)$, $U(1, 6)$, $V(5, 2)$, $W(2, -1)$

29. **CHANGING DIMENSIONS** Use the rectangle at the right.



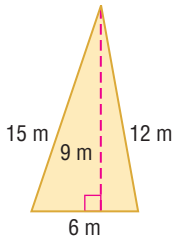
a. Find the perimeter of the rectangle.

b. Find the area of the rectangle.

c. Suppose the length and width of the rectangle are doubled. What effect would this have on the perimeter? the area? Justify your answer.

d. Suppose the length and width of the rectangle are halved. What effect does this have on the perimeter? the area? Justify your answer.

30. **CHANGING DIMENSIONS** Use the triangle at the right.



a. Find the perimeter of the triangle.

b. Find the area of the triangle.

c. Suppose the side lengths and height of the triangle were doubled. What effect would this have on the perimeter? the area? Justify your answer.

d. Suppose the side lengths and height of the triangle were divided by three. What effect would this have on the perimeter? the area? Justify your answer.

31. **ALGEBRA** A rectangle of area 360 square yards is 10 times as long as it is wide. Find its length and width.

32. **ALGEBRA** A rectangle of area 350 square feet is 14 times as wide as it is long. Find its length and width.

33 **DISC GOLF** The diameter of the most popular brand of flying disc used in disc golf measures between 8 and 10 inches. Find the range of possible circumferences and areas for these flying discs to the nearest tenth.

ALGEBRA Find the perimeter or circumference for each figure described.

34. The area of a square is 36 square units.

35. The length of a rectangle is half the width. The area is 25 square meters.

36. The area of a circle is 25π square units.

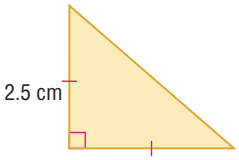
37. The area of a circle is 32π square units.

38. A rectangle's length is 3 times its width. The area is 27 square inches.

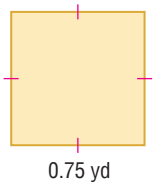
39. A rectangle's length is twice its width. The area is 48 square inches.

CCSS **PRECISION Find the perimeter and area of each figure in inches. Round to the nearest hundredth, if necessary.**

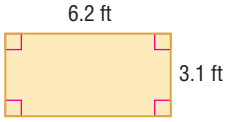
40.



41.



42.



43. **MULTIPLE REPRESENTATIONS** Collect and measure the diameter and circumference of ten round objects using a millimeter measuring tape.

Object	d	C	$\frac{C}{d}$
1			
2			
3			
⋮			
10			

a. Tabular Record the measures in a table as shown.

b. Algebraic Compute the value of $\frac{C}{d}$ to the nearest hundredth for each object and record the result.

c. Graphical Make a scatter plot of the data with d-values on the horizontal axis and C-values on the vertical axis.

d. Verbal Find an equation for a line of best fit for the data. What does this equation represent? What does the slope of the line represent?

H.O.T. Problems Use Higher-Order Thinking Skills

44. **WHICH ONE DOESN'T BELONG?** Identify the term that does not belong with the other three. Explain your reasoning.

square	circle	triangle	pentagon

45. **CHALLENGE** The vertices of a rectangle with side lengths of 10 and 24 units are on a circle of radius 13 units. Find the area between the figures.

46. **REASONING** Name a polygon that is always regular and a polygon that is sometimes regular. Explain your reasoning.

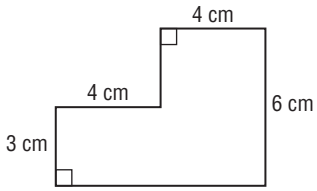
47. **OPEN ENDED** Draw a pentagon. Is your pentagon *convex* or *concave*? Is your pentagon *regular* or *irregular*? Justify your answers.

48. **CHALLENGE** A rectangular room measures 20 feet by 12.5 feet. How many 5-inch square tiles will it take to cover the floor of this room? Explain.

49. **WRITING IN MATH** Describe two possible ways that a polygon can be equiangular but not a regular polygon.

Standardized Test Practice

50. Find the perimeter of the figure.



A 17 cm
B 25 cm
C 28 cm
D 31 cm

51. PROBABILITY In three successive rolls of a fair number cube, Matt rolls a 6. What is the probability of Matt rolling a 6 if the number cube is rolled a fourth time?

F $\frac{1}{6}$
G $\frac{1}{4}$
H $\frac{1}{3}$
J 1

52. SHORT RESPONSE Miguel is planning a party for 80 guests. According to the pattern in the table, how many gallons of ice cream should Miguel buy?

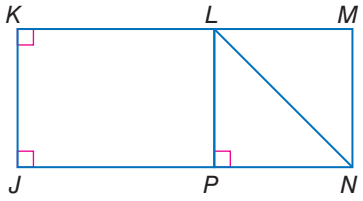
Number of Guests	Gallons of Ice Cream
8	2
16	4
24	6
32	8

53. SAT/ACT A frame 2 inches wide surrounds a painting that is 18 inches wide and 14 inches tall. What is the area of the frame?

A 68 in^2
B 84 in^2
C 144 in^2
D 252 in^2
E 396 in^2

Spiral Review

Determine whether each statement can be assumed from the figure. Explain. (Lesson 1-5)



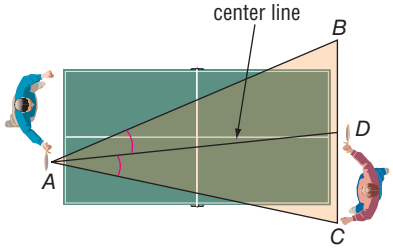
54. $\angle KJN$ is a right angle.

55. $\angle PLN \cong \angle NLM$

56. $\angle PNL$ and $\angle MNL$ are complementary.

57. $\angle KLN$ and $\angle MLN$ are supplementary.

58. TABLE TENNIS The diagram shows the angle of play for a table tennis player. If a right-handed player has a strong forehand, he should stand to the left of the center line of his opponent's angle of play. (Lesson 1-4)



a. What geometric term describes the center line?

b. If the angle of play shown in the diagram measures 43°, what is $m\angle BAD$?

Name an appropriate method to solve each system of equations. Then solve the system. (Lesson 0-8)

59. $-5x + 2y = 13$
$2x + 3y = -9$

60. $y = -5x + 7$
$y = 3x - 17$

61. $x - 8y = 16$
$7x - 4y = -18$

Skills Review

Evaluate each expression if $P = 10$, $B = 12$, $h = 6$, $r = 3$, and $\ell = 5$. Round to the nearest tenth, if necessary.

62. $\frac{1}{2}P\ell + B$

63. $\frac{1}{3}Bh$

64. $\frac{1}{3}\pi r^2 h$

65. $2\pi rh + 2\pi r^2$

EXTEND

1-6 Geometry Software Lab
Two-Dimensional Figures

You can use The Geometer's Sketchpad® to draw and investigate polygons.

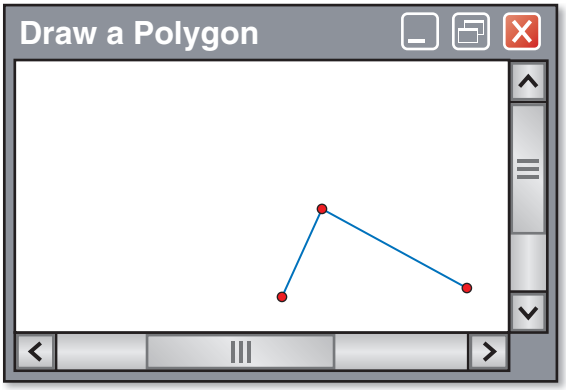
Common Core State Standards
Content Standards
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
Mathematical Practices 5

Activity 1 Draw a Polygon

Draw $\triangle XYZ$.

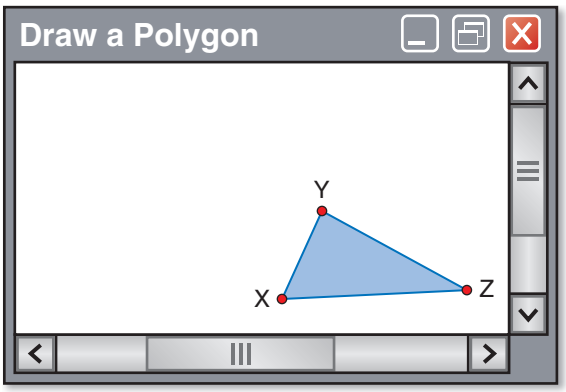
Step 1 Select the segment tool from the toolbar, and click to set the first endpoint X of side $\overline{XY}$. Then drag the cursor, and click again to set the other endpoint Y.

Step 2 Click on point Y to set the endpoint of $\overline{YZ}$. Drag the cursor and click to set point Z.



Step 3 Click on point Z to set the endpoint of $\overline{ZX}$. Then move the cursor to highlight point X. Click on X to draw $\overline{ZX}$.

Step 4 Use the pointer tool to click on points X, Y, and Z. Under the **Display** menu, select **Show Labels** to label the vertices of your triangle.



Activity 2 Measure Sides

Find XY, YZ, and ZX.

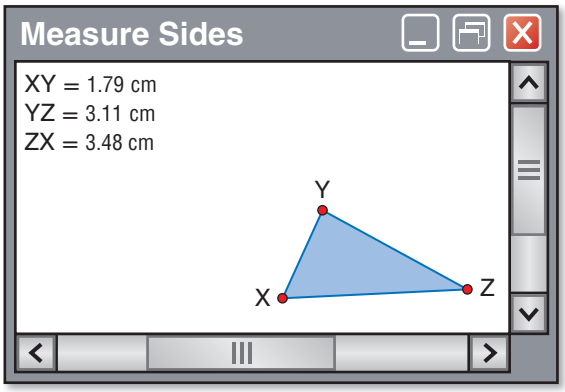
Step 1 Use the pointer tool to select $\overline{XY}$, $\overline{YZ}$, and $\overline{ZX}$.

Step 2 Select the **Length** command under the **Measure** menu to display the lengths of $\overline{XY}$, $\overline{YZ}$, and $\overline{ZX}$.

$XY = 1.79$ cm

$YZ = 3.11$ cm

$ZX = 3.48$ cm



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