LESSON 1-7

Three-Dimensional Figures

Then

- You identified and named two-dimensional figures.

Now

1. Identify and name three-dimensional figures.
2. Find surface area and volume.

Why?

- Architects often provide three-dimensional models of their ideas to clients. These models give their clients a better idea of what the completed structure will look like than a two-dimensional drawing. Three-dimensional figures, or *solids*, are made up of flat or curved surfaces.

NewVocabulary

polyhedron
face
edge
vertex
prism
base
pyramid
cylinder
cone
sphere
regular polyhedron
Platonic solid
surface area
volume

Common Core State Standards

Content Standards

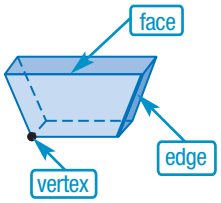
G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★

Mathematical Practices

2 Reason abstractly and quantitatively.
6 Attend to precision.

1 Identify Three-Dimensional Figures

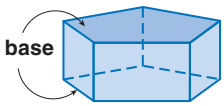
A solid with all flat surfaces that enclose a single region of space is called a **polyhedron**. Each flat surface or **face** is a polygon. The line segments where the faces intersect are called **edges**. The point where three or more edges intersect is called a **vertex**. Below are examples and definitions of polyhedrons and other types of solids.



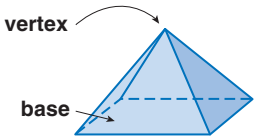
KeyConcept Types of Solids

Polyhedrons

A **prism** is a polyhedron with two parallel congruent faces called **bases** connected by parallelogram faces.

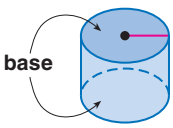


A **pyramid** is a polyhedron that has a polygonal base and three or more triangular faces that meet at a common vertex.

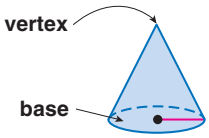


Not Polyhedrons

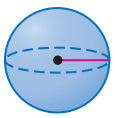
A **cylinder** is a solid with congruent parallel circular bases connected by a curved surface.



A **cone** is a solid with a circular base connected by a curved surface to a single vertex.



A **sphere** is a set of points in space that are the same distance from a given point. A sphere has no faces, edges, or vertices.



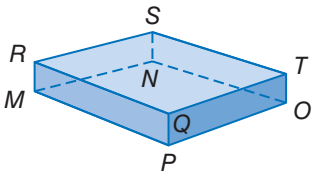
Polyhedrons or *polyhedra* are named by the shape of their bases.

hana/Datacraft/Getty Images

Example 1 Identify Solids

Determine whether each solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the bases, faces, edges, and vertices.

a.



The solid is formed by polygonal faces, so it is a polyhedron. There are two parallel congruent rectangular bases, so it is a rectangular prism.

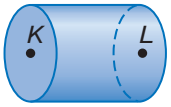
Bases: $\square MNOP$, $\square RSTQ$

Faces: $\square RQPM$, $\square RSNM$, $\square STON$, $\square QTOP$, $\square RSTQ$, $\square MNOP$

Edges: $\overline{MN}$, $\overline{NO}$, $\overline{OP}$, $\overline{PM}$, $\overline{RS}$, $\overline{ST}$, $\overline{TQ}$, $\overline{QR}$, $\overline{RM}$, $\overline{SN}$, $\overline{TO}$, $\overline{QP}$

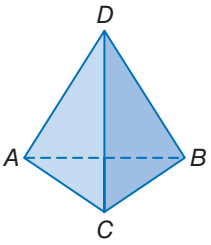
Vertices: M, N, O, P, Q, R, S, T

b.



The solid has a curved surface, so it is not a polyhedron. It has two congruent circular bases, so it is a cylinder.

c.



The solid is formed by polygonal faces, so it is a polyhedron. The base is a triangle, and the three faces meet in a vertex, so it is a triangular pyramid.

Bases: $\triangle ABC$

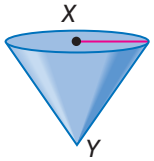
Faces: $\triangle ABC$, $\triangle ADC$, $\triangle CDB$, $\triangle BDA$

Edges: $\overline{AB}$, $\overline{BC}$, $\overline{CA}$, $\overline{DA}$, $\overline{DB}$, $\overline{DC}$

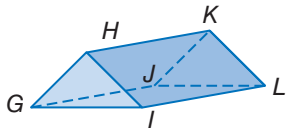
Vertices: A, B, C, D

GuidedPractice

1A.

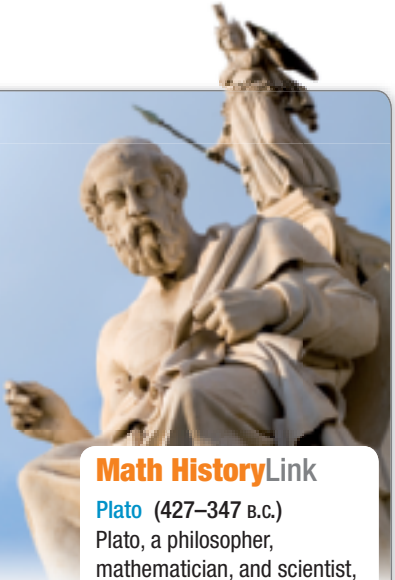


1B.



ReadingMath

Symbols Symbols can be used in naming the focus of polyhedra. The symbol $\square$ means rectangle. The symbol $\triangle$ means triangle. The symbol $\odot$ means circle.

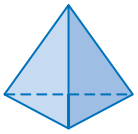
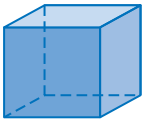
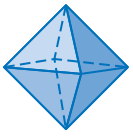


Math HistoryLink

Plato (427–347 B.C.) Plato, a philosopher, mathematician, and scientist, lived in Athens, Greece. He is best known for founding a school known as "The Academy." In mathematics, he was concerned with the idea of proofs, and he insisted that definitions must be accurate and hypotheses must be clear.

A polyhedron is a **regular polyhedron** if all of its faces are regular congruent polygons and all of the edges are congruent. There are exactly five types of regular polyhedrons, called **Platonic Solids** because Plato used them extensively.

KeyConcept Platonic Solids

Tetrahedron	Hexahedron or Cube	Octahedron	Dodecahedron	Icosahedron
				
4 equilateral triangle faces	6 square faces	8 equilateral triangular faces	12 regular pentagonal faces	20 equilateral triangular faces

RICHARD NOWITZ/National Geographic Image Collection

StudyTip

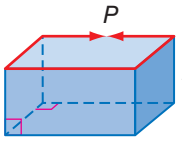
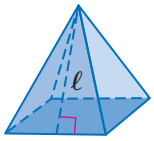
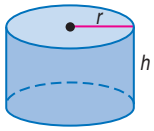
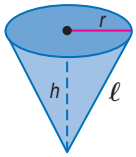
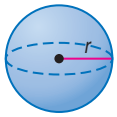
Euclidean Solids The *Euclidean solids* include the cube, the pyramid, the cylinder, the cone, and the sphere.

2 Surface Area and Volume

Surface area is a two-dimensional measurement of the surface of a solid figure. The surface area of a polyhedron is the sum of the areas of each face. **Volume** is the measure of the amount of space enclosed by a solid figure.

Review the formulas for the surface area and volume of five common solids given below. You will derive these formulas in Chapter 12.

KeyConcept Surface Area and Volume

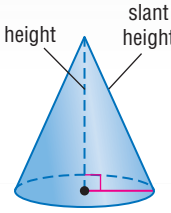
Prism	Regular Pyramid	Cylinder	Cone	Sphere
				
$S = Ph + 2B$	$S = \frac{1}{2}P\ell + B$	$S = 2\pi rh + 2\pi r^2$	$S = \pi r\ell + \pi r^2$	$S = 4\pi r^2$
$V = Bh$	$V = \frac{1}{3}Bh$	$V = \pi r^2 h$	$V = \frac{1}{3}\pi r^2 h$	$V = \frac{4}{3}\pi r^3$

S = total surface area; V = volume; h = height of a solid

P = perimeter of the base; B = area of base; ℓ = slant height, r = radius

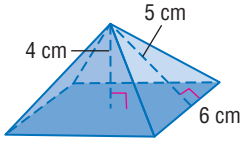
WatchOut!

Height vs. Slant Height The *height* of a pyramid or cone is not the same as its *slant height*.



Example 2 Find Surface Area and Volume

Find the surface area and volume of the square pyramid.



Surface Area

Since the base of the pyramid is a square, the perimeter P of the base is 4 • 6 or 24 centimeters. The area of the base B is 6 • 6 or 36 square centimeters. The slant height is 5 centimeters.

$S = \frac{1}{2}P\ell + B$ Surface area of pyramid

$= \frac{1}{2}(24)(5) + 36$ or 96 $P = 24$ cm, $\ell = 5$ cm, $B = 36$ cm^2

The surface area of the square pyramid is 96 square centimeters.

Volume

The height of the pyramid is 4 centimeters.

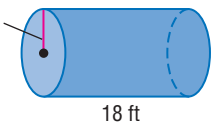
$V = \frac{1}{3}Bh$ Volume of pyramid

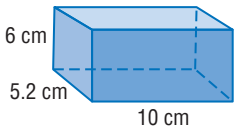
$= \frac{1}{3}(36)(4)$ or 48 $B = 36$ cm^2, $h = 4$ cm

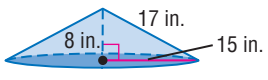
The volume is 48 cubic centimeters.

GuidedPractice

Find the surface area and volume of each solid to the nearest tenth.

2A. 

2B. 

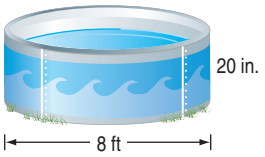
2C. 

StudyTip

CCSS **Precision** Be sure that you have converted all units of measure to be consistent before you begin volume or surface area calculations.

Real-World Example 3 Surface Area and Volume

POOLS **The diameter of the pool Mr. Sato purchased is 8 feet. The height of the pool is 20 inches. Find each measure to the nearest tenth.**



a. surface area of the pool

The pool is a cylinder.

$A = 2\pi rh + \pi r^2$ Surface area of cylinder with one base

$= 2\pi(4)\left(1\frac{2}{3}\right) + \pi(4)^2$ $r = 4$ ft, $h = 20$ in. or $1\frac{2}{3}$ ft

≈ 92.2 Use a calculator.

The surface area of the pool is about 92.2 square feet.

b. the volume of water needed to fill the pool to a depth of 16 inches

$V = \pi r^2 h$ Volume of cylinder

$= \pi(4)^2\left(1\frac{1}{3}\right)$ $r = 4$ ft, $h = 16$ in. or $1\frac{1}{3}$ ft

≈ 67.0 Use a calculator.

The volume of water needed is approximately 67.0 cubic feet.

GuidedPractice

3. CRAFTS Jessica is making spherical candles using a mold that is 10 centimeters in diameter. Find each measure to the nearest tenth.

A. the volume of wax needed to fill the mold

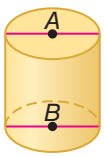
B. the surface area of the finished candle

Check Your Understanding

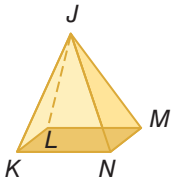
= Step-by-Step Solutions begin on page R14.

Example 1 **Determine whether the solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the bases, faces, edges, and vertices.**

1.

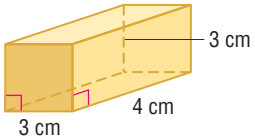


2.



Example 2 **Find the surface area and volume of each solid to the nearest tenth.**

3



4.

Example 3 **5. PARTY FAVORS** Lawana is making cone-shaped hats 4 inches in diameter, 6.5 inches tall, with a slant height of 6.8 inches for party favors. Find each measure to the nearest tenth.

a. the volume of candy that will fill each cone

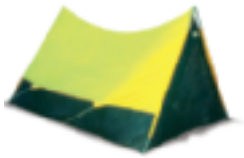
b. the area of material needed to make each hat assuming there is no overlap of material

Nichola Evans/Iconica/Getty Images

Example 1 Identify the solid modeled by each object. State whether the solid modeled is a polyhedron.

6. 

7. 

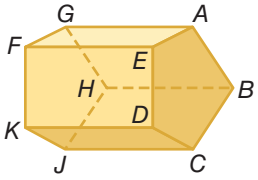
8. 

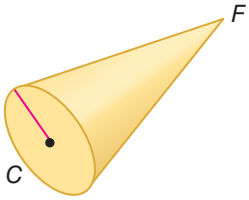
9. 

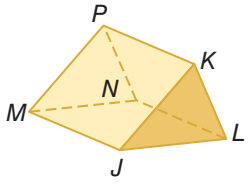
10. 

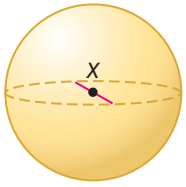
11. 

CCSS **STRUCTURE** Determine whether the solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the bases, faces, edges, and vertices.

12. 

13. 

14. 

15. 

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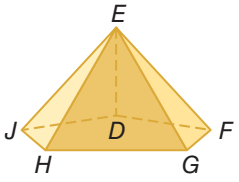
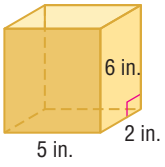
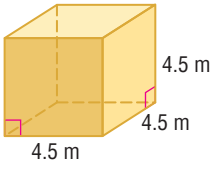
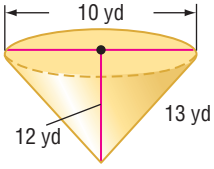
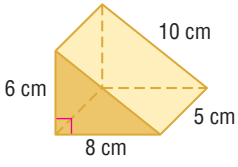
17. 

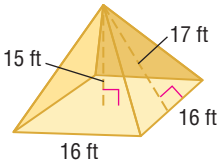
Example 2 Find the surface area and volume of each solid to the nearest tenth.

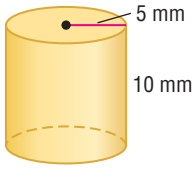
18. 

19. 

20. 

21. 

22. 

23. 

Example 3

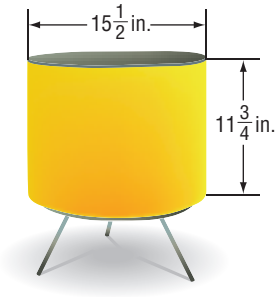
24. **SANDBOX** A rectangular sandbox is 3 feet by 4 feet. The depth of the box is 8 inches, but the depth of the sand is $\frac{3}{4}$ of the depth of the box. Find each measure to the nearest tenth.

a. the surface area of the sandbox assuming there is no lid

b. the volume of sand in the sandbox

(tc)Image Source Black/Alamy, (tr)David Wasserman/Jupiterimages, (bl)Lauri Rotko/Gorilla Creative Images/Getty Images, (bc)Cone 6 Productions/Photodisc/Getty Images, (br)Tetra Images/Getty Images

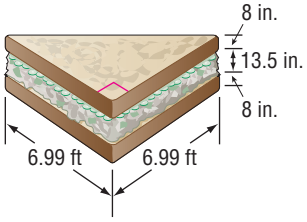
25 **ART** Fernando and Humberto Campana designed the Inflating Table shown. The diameter of the table is $15\frac{1}{2}$ inches. Suppose the height of the cylinder is $11\frac{3}{4}$ inches. Find each measure to the nearest tenth. Assume that the sides of the table are perpendicular to the bases of the table.



a. the volume of air that will fully inflate the table

b. the surface area of the table when fully inflated

26. CCSS **SENSE-MAKING** In 1999, Marks & Spencer, a British department store, created the biggest sandwich ever made. The tuna and cucumber sandwich was in the form of a triangular prism. Suppose each slice of bread was 8 inches thick. Find each measure to the nearest tenth.



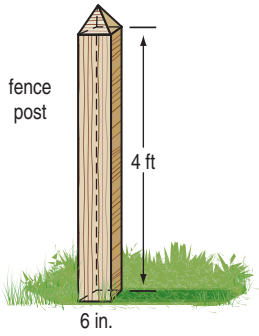
a. the surface area in square feet of the sandwich when filled

b. the volume of filling in cubic feet to the nearest tenth

27. **ALGEBRA** The surface area of a cube is 54 square inches. Find the length of each edge.

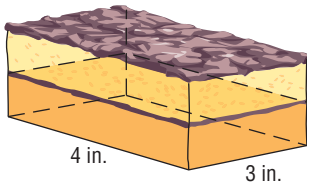
28. **ALGEBRA** The volume of a cube is 729 cubic centimeters. Find the length of each edge.

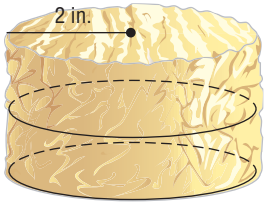
29. **PAINTING** Tara is painting her family's fence. Each post is composed of a square prism and a square pyramid. The height of the pyramid is 4 inches. Determine the surface area and volume of each post.



30. **COLLECT DATA** Use a ruler or tape measure and what you have learned in this lesson to find the surface area and volume of a soup can.

31. **CAKES** Cakes come in many shapes and sizes. Often they are stacked in two or more layers, like those in the diagrams shown below.





a. If each layer of the rectangular prism cake is 3 inches high, calculate the area of the cake that will be frosted assuming there is no frosting between layers.

b. Calculate the area of the cylindrical cake that will be frosted, if each layer is 4 inches in height.

c. If one can of frosting will cover 50 square inches of cake, how many cans of frosting will be needed for each cake?

d. If the height of each layer of cake is 5 inches, what does the radius of the cylindrical cake need to be, so the same amount of frosting is used for both cakes? Explain your reasoning.

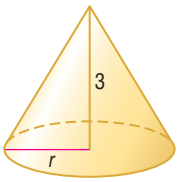
32. CHANGING UNITS A gift box has a surface area of 6.25 square feet. What is the surface area of the box in square inches?

33. CHANGING UNITS A square pyramid has a volume of 4320 cubic inches. What is the volume of this pyramid in cubic feet?

34. EULER'S FORMULA The number of faces F, vertices V, and edges E of a polyhedron are related by Euler's (OY luhrz) Formula: $F + V = E + 2$. Determine whether Euler's Formula is true for each of the figures in Exercises 18–23.

35. CHANGING DIMENSIONS A rectangular prism has a length of 12 centimeters, width of 18 centimeters, and height of 22 centimeters. Describe the effect on the volume of a rectangular prism when each dimension is doubled.

36. MULTIPLE REPRESENTATIONS In this problem, you will investigate how changing the length of the radius of a cone affects the cone's volume.



a. Tabular Create a table showing the volume of a cone when doubling the radius. Use radius values between 1 and 8.

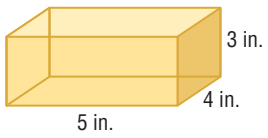
b. Graphical Use the values from your table to create a graph of radius versus volume.

c. Verbal Make a conjecture about the effect of doubling the radius of a cone on the volume. Explain your reasoning.

d. Algebraic If r is the radius of a cone, write an expression showing the effect doubling the radius has on the cone's volume.

H.O.T. Problems Use Higher-Order Thinking Skills

37. CCSS CRITIQUE Alex and Emily are calculating the surface area of the rectangular prism shown. Is either of them correct? Explain your reasoning.



Alex	Emily
$(5 \cdot 3) \cdot 6$ faces	$2(5 \cdot 4 \cdot 3)$
$= 90$ in^2	$= 120$ in^2

38. REASONING Is a cube a regular polyhedron? Explain.

39. CHALLENGE Describe the solid that results if the number of sides of each base increases infinitely. The bases of each solid are regular polygons inscribed in a circle.

a. pyramid **b.** prism

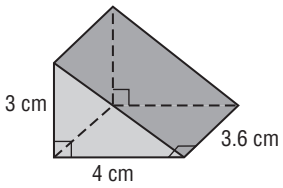
40. OPEN ENDED Draw an irregular 14-sided polyhedron which has two congruent bases.

41. CHALLENGE Find the volume of a cube that has a total surface area of 54 square millimeters.

42. WRITING IN MATH A reference sheet listed the formula for the surface area of a prism as $SA = Bh + 2B$. Use units of measure to explain why there must be a typographical error in this formula.

Standardized Test Practice

43. GRIDDED RESPONSE What is the surface area of the triangular prism in square centimeters?



44. ALGEBRA What is the value of $(-0.8)^2 + (-0.3)^3$?

A 0.627 **C** 0.370
B 0.613 **D** 0.327

45. The length of each side of a cube is multiplied by 5. What is the change in the volume of the cube?

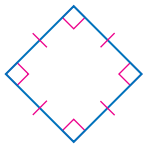
F The volume is 125 times the original volume.
G The volume is 25 times the original volume.
H The volume is 10 times the original volume.
J The volume is 5 times the original volume.

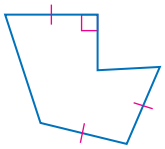
46. SAT/ACT What is the difference in surface area between a cube with an edge length of 7 inches and a cube with edge length of 4 inches?

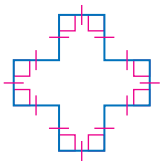
A 18 in^2 **D** 99 in^2
B 33 in^2 **E** 198 in^2
C 66 in^2

Spiral Review

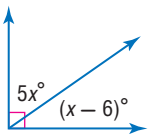
Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*. (Lesson 1-6)

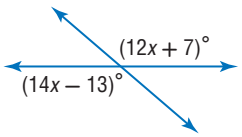
47. 

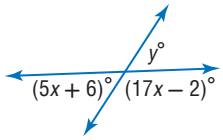
48. 

49. 

Find the value of each variable. (Lesson 1-5)

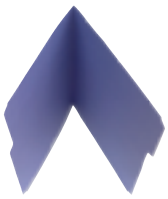
50. 

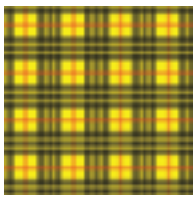
51. 

52. 

GAMES What type of geometric intersection is modeled in each photograph? (Lesson 1-1)

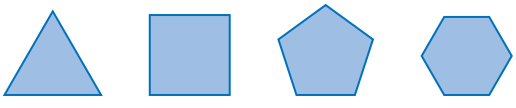
53. 

54. 

55. 

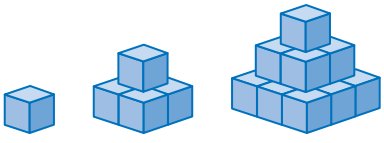
Skills Review

Sketch the next two figures in each pattern.

56. 

57. 

58. 

59. 

(l)PhotoDisc/Getty Images, (c)Mark Ransom Photography, (r)Oxana Zubov/age fotostock