LESSON 10-1

Circles and Circumference

Then

- You identified and used parts of parallelograms.

Now

1. Identify and use parts of circles.
2. Solve problems involving the circumference of a circle.

Why?

- The maxAir ride shown speeds back and forth and rotates counterclockwise. At times, the riders are upside down 140 feet above the ground experiencing "airtime"—a feeling of weightlessness. The ride's width, or *diameter,* is 44 feet. You can find the distance that a rider travels in one rotation by using this measure.

Ilene MacDonald/Alamy

NewVocabulary

circle
center
radius
chord
diameter
concentric circles
circumference
pi (π)
inscribed
circumscribed

Common Core State Standards

Content Standards

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G.C.1 Prove that all circles are similar.

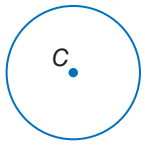
Mathematical Practices

4 Model with mathematics.

1 Make sense of problems and persevere in solving them.

1 Segments in Circles

A **circle** is the locus or set of all points in a plane equidistant from a given point called the **center** of the circle.



Circle C or $\odot C$

Segments that intersect a circle have special names.

KeyConcept Special Segments in a Circle

A **radius** (plural radii) is a segment with endpoints at the center and on the circle.

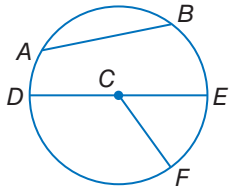
Examples $\overline{CD}$, $\overline{CE}$, and $\overline{CF}$ are radii of $\odot C$.

A **chord** is a segment with endpoints on the circle.

Examples $\overline{AB}$ and $\overline{DE}$ are chords of $\odot C$.

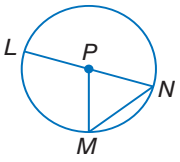
A **diameter** of a circle is a chord that passes through the center and is made up of collinear radii.

Example $\overline{DE}$ is a diameter of $\odot C$. Diameter $\overline{DE}$ is made up of collinear radii $\overline{CD}$ and $\overline{CE}$.



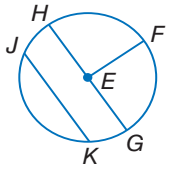
Example 1 Identify Segments in a Circle

a. Name the circle and identify a radius.



The circle has a center at P, so it is named circle P, or $\odot P$. Three radii are shown: $\overline{PL}$, $\overline{PN}$, and $\overline{PM}$.

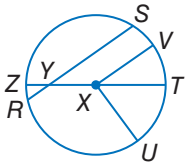
b. Identify a chord and a diameter of the circle.



Two chords are shown: $\overline{JK}$ and $\overline{HG}$. $\overline{HG}$ goes through the center, so $\overline{HG}$ is a diameter.

GuidedPractice

1. Name the circle, a radius, a chord, and a diameter of the circle.



ReadingMath

CCSS **Precision** The words *radius* and *diameter* are used to describe lengths as well as segments. Since a circle has many different radii and diameters, the phrases *the radius* and *the diameter* refer to lengths rather than segments.

By definition, the distance from the center of a circle to any point on the circle is always the same. Therefore, all radii r of a circle are congruent. Since a diameter d is composed of two radii, all diameters of a circle are also congruent.

KeyConcept Radius and Diameter Relationships

If a circle has radius r and diameter d, the following relationships are true.

Radius Formula $r = \frac{d}{2}$ or $r = \frac{1}{2}d$ **Diameter Formula** $d = 2r$

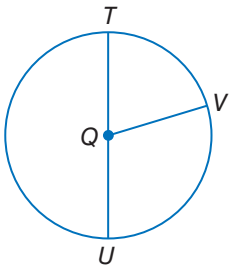
Example 2 Find Radius and Diameter

If $QV = 8$ inches, what is the diameter of $\odot Q$?

$d = 2r$ Diameter Formula

$= 2(8)$ or 16 Substitute and simplify.

The diameter of $\odot Q$ is 16 inches.



GuidedPractice

2A. If $TU = 14$ feet, what is the radius of $\odot Q$?

2B. If $QT = 11$ meters, what is QU?

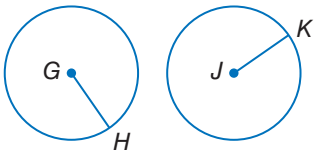
As with other figures, pairs of circles can be congruent, similar, or share other special relationships.

ReviewVocabulary

coplanar points that lie in the same plane

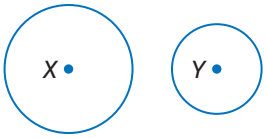
KeyConcept Circle Pairs

Two circles are congruent if and only if they have congruent radii.



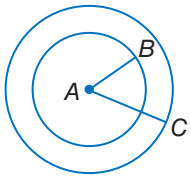
Example $\overline{GH} \cong \overline{JK}$, so $\odot G \cong \odot J$.

All circles are similar.



Example $\odot X \sim \odot Y$

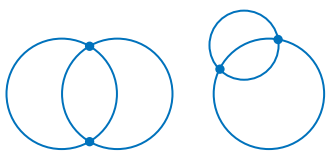
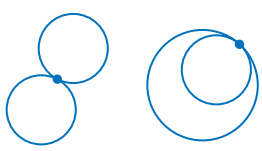
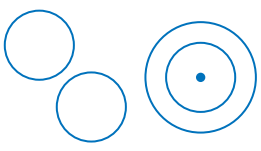
Concentric circles are coplanar circles that have the same center.



Example $\odot A$ with radius $\overline{AB}$ and $\odot A$ with radius $\overline{AC}$ are concentric.

You will prove that all circles are similar in Exercise 52.

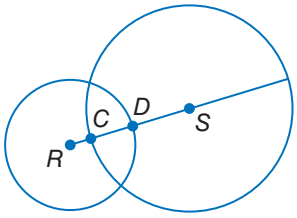
Two circles can intersect in two different ways.

2 Points of Intersection	1 Point of Intersection	No Points of Intersection
		

The segment connecting the centers of the two intersecting circles contains the radii of the two circles.

Example 3 Find Measures in Intersecting Circles

The diameter of ⊙S is 30 units, the diameter of ⊙R is 20 units, and $DS = 9$ units. Find CD.



Since the diameter of ⊙S is 30, $CS = 15$.
$\overline{CD}$ is part of radius $\overline{CS}$.

$CD + DS = CS$	Segment Addition Postulate
$CD + 9 = 15$	Substitution
$CD = 6$	Subtract 9 from each side.

GuidedPractice

3. Use the diagram above to find RC.

2 Circumference

The **circumference** of a circle is the distance around the circle. By definition, the ratio $\frac{C}{d}$ is an irrational number called **pi (π)**. Two formulas for circumference can be derived by using this definition.

$\frac{C}{d} = \pi$	Definition of pi
$C = \pi d$	Multiply each side by d.
$C = \pi(2r)$	$d = 2r$
$C = 2\pi r$	Simplify.

KeyConcept Circumference

Words	If a circle has diameter d or radius r, the circumference C equals the diameter times pi or twice the radius times pi.
Symbols	$C = \pi d$ or $C = 2\pi r$

David Cannon/Getty Images Sport/Getty Images

Real-WorldLink

In 2005, Roger Federer and Andre Agassi played tennis on the helipad of the Burj Al Arab hotel in the United Arab Emirates. The helipad has a diameter of 79 feet and is nearly 700 feet high.

Source: Burj Al Arab, Emporis Buildings

Real-World Example 4 Find Circumference

TENNIS Find the circumference of the helipad described at the left.

$C = \pi d$	Circumference formula
$= \pi(79)$	Substitution
$= 79\pi$	Simplify.
≈ 248.19	Use a calculator.

The circumference of the helipad is 79π feet or about 248.19 feet.

GuidedPractice

Find the circumference of each circle described. Round to the nearest hundredth.

4A. radius = 2.5 centimeters

4B. diameter = 16 feet

These circumference formulas can also be used to determine the diameter and radius of a circle when the circumference is given.

Example 5 Find Diameter and Radius

Find the diameter and radius of a circle to the nearest hundredth if the circumference of the circle is 106.4 millimeters.

$C = \pi d$	Circumference Formula	$r = \frac{1}{2}d$	Radius Formula
$106.4 = \pi d$	Substitution	$\approx \frac{1}{2}(33.87)$	$d \approx 33.87$
$\frac{106.4}{\pi} = d$	Divide each side by π.	≈ 16.94 mm	Use a calculator.
$33.87 \text{ mm} \approx d$	Use a calculator.		

StudyTip

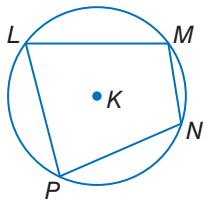
Levels of Accuracy Since π is irrational, its value cannot be given as a terminating decimal. Using a value of 3 for π provides a quick estimate in calculations. Using a value of 3.14 or $\frac{22}{7}$ provides a closer approximation. For the most accurate approximation, use the π key on a calculator. Unless stated otherwise, assume that in this text, a calculator with a π key was used to generate answers.

GuidedPractice

5. Find the diameter and radius of a circle to the nearest hundredth if the circumference of the circle is 77.8 centimeters.

A polygon is **inscribed** in a circle if all of its vertices lie on the circle. A circle is **circumscribed** about a polygon if it contains all the vertices of the polygon.

- Quadrilateral *LMNP* is *inscribed in* ⊙*K*.
- Circle *K* is *circumscribed about* quadrilateral *LMNP*.



Standardized Test Example 6 Circumference of Circumscribed Polygon

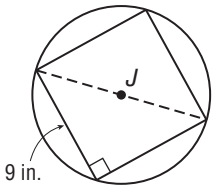
SHORT RESPONSE A square with side length of 9 inches is inscribed in ⊙*J*. Find the exact circumference of ⊙*J*.

Read the Test Item

You need to find the diameter of the circle and use it to calculate the circumference.

Solve the Test Item

First, draw a diagram. The diagonal of the square is the diameter of the circle and the hypotenuse of a right triangle.



$a^2 + b^2 = c^2$	Pythagorean Theorem
$9^2 + 9^2 = c^2$	Substitution
$162 = c^2$	Simplify.
$9\sqrt{2} = c$	Take the positive square root of each side.

The diameter of the circle is $9\sqrt{2}$ inches.

Find the circumference in terms of π by substituting $9\sqrt{2}$ for d in $C = \pi d$. The exact circumference is $9\pi\sqrt{2}$ inches.

StudyTip

Circumcircle A *circumcircle* is a circle that passes through all of the vertices of a polygon.

GuidedPractice

Find the exact circumference of each circle by using the given polygon.

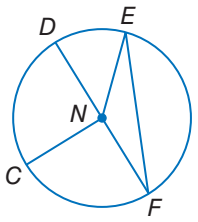
6A. inscribed right triangle with legs 7 meters and 3 meters long

6B. circumscribed square with side 10 feet long

Check Your Understanding

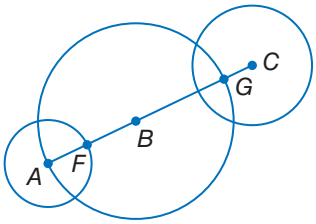
= Step-by-Step Solutions begin on page R14.

Examples 1–2 **For Exercises 1–4, refer to ⊙N.**



1. Name the circle.
2. Identify each.
   a. a chord **b.** a diameter **c.** a radius
3. If $CN = 8$ centimeters, find DN.
4. If $EN = 13$ feet, what is the diameter of the circle?

Example 3 **The diameters of ⊙A, ⊙B, and ⊙C are 8 inches, 18 inches, and 11 inches, respectively. Find each measure.**



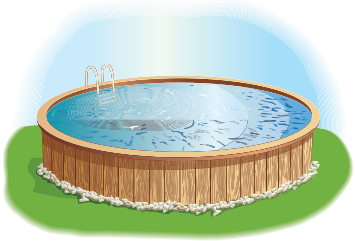
5. FG
6. FB

Example 4

7. **RIDES** The circular ride described at the beginning of the lesson has a diameter of 44 feet. What are the radius and circumference of the ride? Round to the nearest hundredth, if necessary.

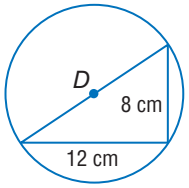
Example 5

8. **MODELING** The circumference of the circular swimming pool shown is about 56.5 feet. What are the diameter and radius of the pool? Round to the nearest hundredth.



Example 6

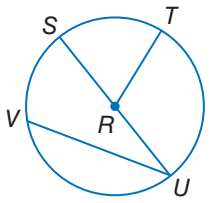
9. **SHORT RESPONSE** The right triangle shown is inscribed in ⊙D. Find the exact circumference of ⊙D.



Practice and Problem Solving

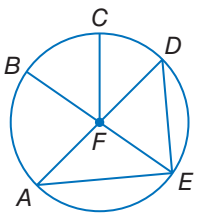
Extra Practice is on page R10.

Examples 1–2 **For Exercises 10–13, refer to ⊙R.**



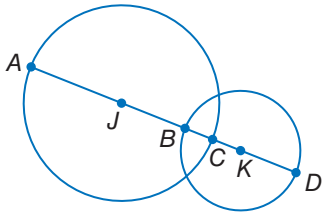
10. Name the center of the circle.
11. Identify a chord that is also a diameter.
12. Is $\overline{VU}$ a radius? Explain.
13. If $SU = 16.2$ centimeters, what is RT?

For Exercises 14–17, refer to ⊙F.



14. Identify a chord that is not a diameter.
15. If $CF = 14$ inches, what is the diameter of the circle?
16. Is $\overline{AF} \cong \overline{EF}$? Explain.
17. If $DA = 7.4$ centimeters, what is EF?

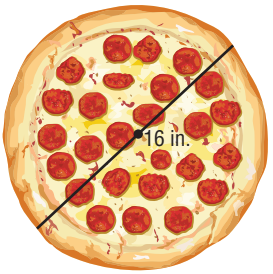
Example 3 **Circle J has a radius of 10 units, $\odot K$ has a radius of 8 units, and $BC = 5.4$ units. Find each measure.**



18. CK **19.** AB

20. JK **21.** AD

Example 4 **22. PIZZA** Find the radius and circumference of the pizza shown. Round to the nearest hundredth, if necessary.



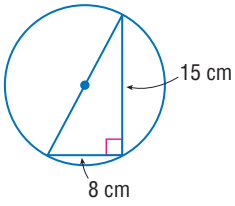
23. BICYCLES A bicycle has tires with a diameter of 26 inches. Find the radius and circumference of a tire. Round to the nearest hundredth, if necessary.

Example 5 **Find the diameter and radius of a circle with the given circumference. Round to the nearest hundredth.**

24. $C = 18$ in. **25.** $C = 124$ ft **26.** $C = 375.3$ cm **27.** $C = 2608.25$ m

Example 6 CCSS **SENSE-MAKING** **Find the exact circumference of each circle by using the given inscribed or circumscribed polygon.**

28.



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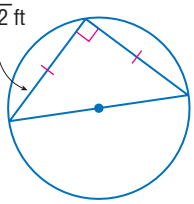

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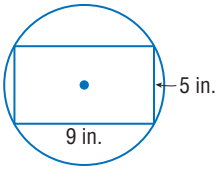

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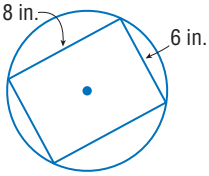

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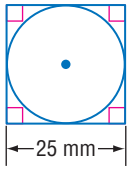

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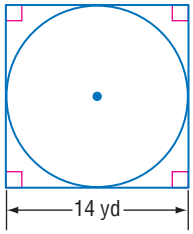
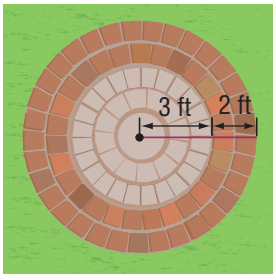


34. DISC GOLF Disc golf is similar to regular golf, except that a flying disc is used instead of a ball and clubs. For professional competitions, the maximum weight of a disc in grams is 8.3 times the diameter in centimeters. What is the maximum allowable weight for a disc with circumference 66.92 centimeters? Round to the nearest tenth.

35. PATIOS Mr. Martinez is going to build the patio shown.

a. What is the patio's approximate circumference?

b. If Mr. Martinez changes the plans so that the inner circle has a circumference of approximately 25 feet, what should the radius of the circle be to the nearest foot?



The radius, diameter, or circumference of a circle is given. Find each missing measure to the nearest hundredth.

36. $d = 8\frac{1}{2}$ in., $r =$ ___?___, $C =$ ___?___

37. $r = 11\frac{2}{5}$ ft, $d =$ ___?___, $C =$ ___?___

38. $C = 35x$ cm, $d =$ ___?___, $r =$ ___?___

39. $r = \frac{x}{8}$, $d =$ _____, $C =$ ___?___

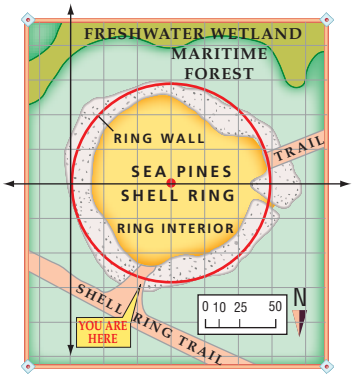
Determine whether the circles in the figures below appear to be *congruent*, *concentric*, or *neither*.

40.

41.

42.

43 **HISTORY** The *Indian Shell Ring* on Hilton Head Island approximates a circle. If each unit on the coordinate grid represents 25 feet, how far would someone have to walk to go completely around the ring? Round to the nearest tenth.



44. CCSS **MODELING** A brick path is being installed around a circular pond. The pond has a circumference of 68 feet. The outer edge of the path is going to be 4 feet from the pond all the way around. What is the approximate circumference of the path? Round to the nearest hundredth.

45. **MULTIPLE REPRESENTATIONS** In this problem, you will explore changing dimensions in circles.

a. Geometric Use a compass to draw three circles in which the scale factor from each circle to the next larger circle is 1:2.

b. Tabular Calculate the radius (to the nearest tenth) and circumference (to the nearest hundredth) of each circle. Record your results in a table.

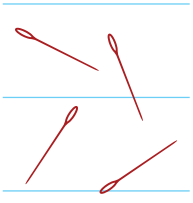
c. Verbal Explain why these three circles are geometrically similar.

d. Verbal Make a conjecture about the ratio between the circumferences of two circles when the ratio between their radii is 2.

e. Analytical The scale factor from $\odot A$ to $\odot B$ is $\frac{b}{a}$. Write an equation relating the circumference (C_A) of $\odot A$ to the circumference (C_B) of $\odot B$.

f. Numerical If the scale factor from $\odot A$ to $\odot B$ is $\frac{1}{3}$, and the circumference of $\odot A$ is 12 inches, what is the circumference of $\odot B$?

46. **BUFFON'S NEEDLE** Measure the length ℓ of a needle (or toothpick) in centimeters. Next, draw a set of horizontal lines that are ℓ centimeters apart on a sheet of plain white paper.

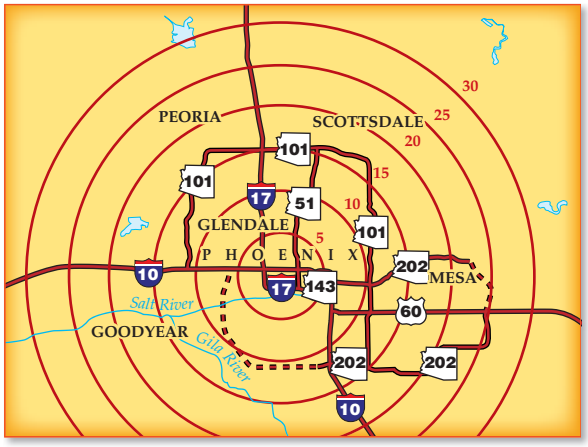


a. Drop the needle onto the paper. When the needle lands, record whether it touches one of the lines as a hit. Record the number of hits after 25, 50, and 100 drops.

b. Calculate the ratio of two times the total number of drops to the number of hits after 25, 50, and 100 drops.

c. How are the values you found in part **b** related to π?

47. MAPS The concentric circles on the map below show the areas that are 5, 10, 15, 20, 25, and 30 miles from downtown Phoenix.



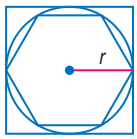
a. How much greater is the circumference of the outermost circle than the circumference of the center circle?

b. As the radii of the circles increase by 5 miles, by how much does the circumference increase?

H.O.T. Problems Use Higher-Order Thinking Skills

48. WRITING IN MATH How can we describe the relationships that exist between circles and lines?

49. REASONING In the figure, a circle with radius r is inscribed in a regular polygon and circumscribed about another.



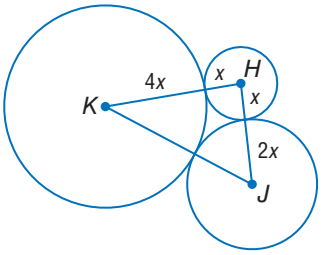
a. What are the perimeters of the circumscribed and inscribed polygons in terms of r? Explain.

b. Is the circumference C of the circle greater or less than the perimeter of the circumscribed polygon? the inscribed polygon? Write a compound inequality comparing C to these perimeters.

c. Rewrite the inequality from part **b** in terms of the diameter d of the circle and interpret its meaning.

d. As the number of sides of both the circumscribed and inscribed polygons increase, what will happen to the upper and lower limits of the inequality from part **c**, and what does this imply?

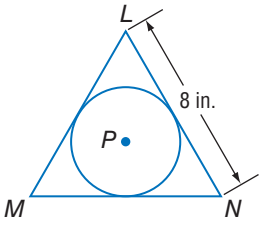
50. CHALLENGE The sum of the circumferences of circles H, J, and K shown at the right is 56π units. Find KJ.



51. REASONING Is the distance from the center of a circle to a point in the interior of a circle *sometimes*, *always*, or *never* less than the radius of the circle? Explain.

52. ARGUMENTS Use the locus definition of a circle and dilations to prove that all circles are similar.

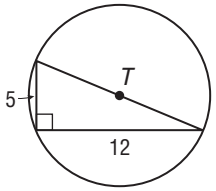
53. CHALLENGE In the figure, $\odot P$ is inscribed in equilateral triangle LMN. What is the circumference of $\odot P$?



54. WRITING IN MATH Research and write about the history of pi and its importance to the study of geometry.

Standardized Test Practice

55. GRIDDED RESPONSE What is the circumference of $\odot T$? Round to the nearest tenth.



56. What is the radius of a table with a circumference of 10 feet?

A 1.6 ft **C** 3.2 ft
B 2.5 ft **D** 5 ft

57. ALGEBRA Bill is planning a circular vegetable garden with a fence around the border. If he can use up to 50 feet of fence, what radius can he use for the garden?

F 10 **G** 9 **H** 8 **J** 7

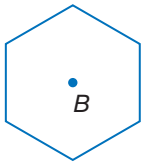
58. SAT/ACT What is the radius of a circle with an area of $\frac{\pi}{4}$ square units?

A 0.4 units **D** 4 units
B 0.5 units **E** 16 units
C 2 units

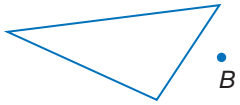
Spiral Review

Copy each figure and point B. Then use a ruler to draw the image of the figure under a dilation with center B and the scale factor r indicated. (Lesson 9-6)

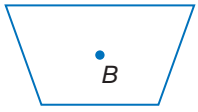
59. $r = \frac{1}{5}$



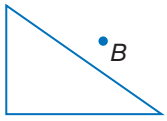
60. $r = \frac{2}{5}$



61. $r = 2$



62. $r = 3$

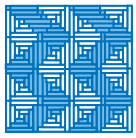


State whether each figure has rotational symmetry. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry. (Lesson 9-5)

63.

64.

65.



66.

Determine the truth value of the following statement for each set of conditions. Explain your reasoning. (Lesson 2-2)

If you are over 18 years old, then you vote in all elections.

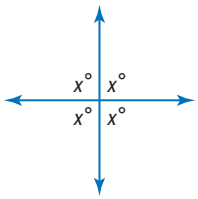
67. You are 19 years old and you vote.

68. You are 21 years old and do not vote.

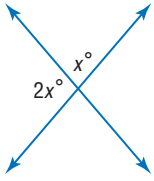
Skills Review

Find x.

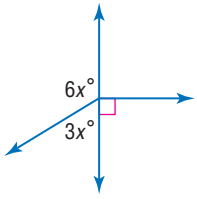
69.



70.



71.



72.