LESSON 10-3 Arcs and Chords

Then

- You used the relationships between arcs and angles to find measures.

Now

1. Recognize and use relationships between arcs and chords.
2. Recognize and use relationships between arcs, chords, and diameters.

Why?

- Embroidery hoops are used in sewing, quilting, and cross-stitching, as well as for embroidering. The endpoints of the snowflake shown are both the endpoints of a chord and the endpoints of an arc.

CCSS Common Core State Standards

Content Standards

G.C.2 Identify and describe relationships among inscribed angles, radii, and chords.

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

Mathematical Practices

4 Model with mathematics.

3 Construct viable arguments and critique the reasoning of others.

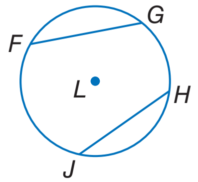
1 Arcs and Chords

A *chord* is a segment with endpoints on a circle. If a chord is not a diameter, then its endpoints divide the circle into a major and a minor arc.

Theorem 10.2

Words In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

Example $\widehat{FG} \cong \widehat{HJ}$ if and only if $\overline{FG} \cong \overline{HJ}$.

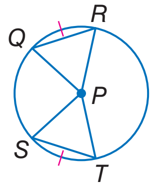


Proof Theorem 10.2 (part 1)

Given: $\odot P$; $\widehat{QR} \cong \widehat{ST}$

Prove: $\overline{QR} \cong \overline{ST}$

Proof:



Statements	Reasons
1. $\odot P$, $\widehat{QR} \cong \widehat{ST}$	**1.** Given
2. $\angle QPR \cong \angle SPT$	**2.** If arcs are ≅, their corresponding central ⦞ are ≅.
3. $\overline{QP} \cong \overline{PR} \cong \overline{SP} \cong \overline{PT}$	**3.** All radii of a circle are ≅.
4. $\triangle PQR \cong \triangle PST$	**4.** SAS
5. $\overline{QR} \cong \overline{ST}$	**5.** CPCTC

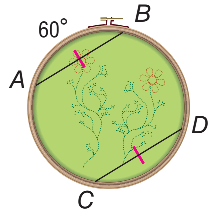
You will prove part 2 of Theorem 10.2 in Exercise 25.

Real-World Example 1 Use Congruent Chords to Find Arc Measure

PT

CRAFTS In the embroidery hoop, $\overline{AB} \cong \overline{CD}$ and $m\widehat{AB} = 60$. Find $m\widehat{CD}$.

$\overline{AB}$ and $\overline{CD}$ are congruent chords, so the corresponding arcs $\widehat{AB}$ and $\widehat{CD}$ are congruent. $m\widehat{AB} = m\widehat{CD} = 60$

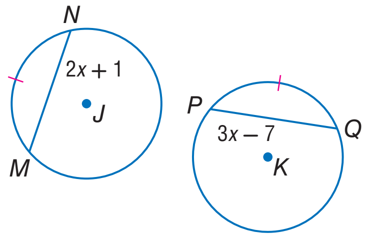


Guided Practice

1. If $m\widehat{AB} = 78$ in the embroidery hoop, find $m\widehat{CD}$.

Example 2 Use Congruent Arcs to Find Chord Lengths

ALGEBRA In the figures, $\odot J \cong \odot K$ and $\overset{\frown}{MN} \cong \overset{\frown}{PQ}$. Find PQ.



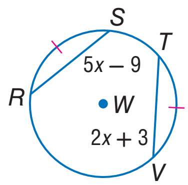
$\overset{\frown}{MN}$ and $\overset{\frown}{PQ}$ are congruent arcs in congruent circles, so the corresponding chords $\overline{MN}$ and $\overline{PQ}$ are congruent.

$MN = PQ$	Definition of congruent segments
$2x + 1 = 3x - 7$	Substitution
$8 = x$	Simplify.

So, $PQ = 3(8) - 7$ or 17.

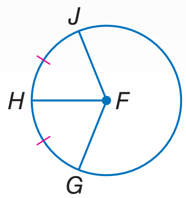
GuidedPractice

2. In $\odot W$, $\overset{\frown}{RS} \cong \overset{\frown}{TV}$. Find RS.



StudyTip

Arc Bisectors In the figure below, $\overline{FH}$ is an arc bisector of $\overset{\frown}{JG}$.



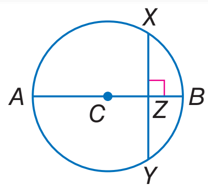
2 Bisecting Arcs and Chords

If a line, segment, or ray divides an arc into two congruent arcs, then it *bisects* the arc.

Theorems

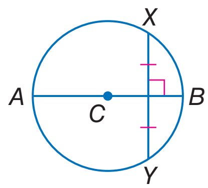
10.3 If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc.

Example If diameter $\overline{AB}$ is perpendicular to chord $\overline{XY}$, then $\overline{XZ} \cong \overline{ZY}$ and $\overset{\frown}{XB} \cong \overset{\frown}{BY}$.



10.4 The perpendicular bisector of a chord is a diameter (or radius) of the circle.

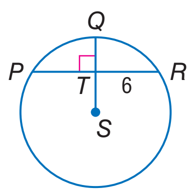
Example If $\overline{AB}$ is a perpendicular bisector of chord $\overline{XY}$, then $\overline{AB}$ is a diameter of $\odot C$.



You will prove Theorems 10.3 and 10.4 in Exercises 26 and 28, respectively.

Example 3 Use a Radius Perpendicular to a Chord

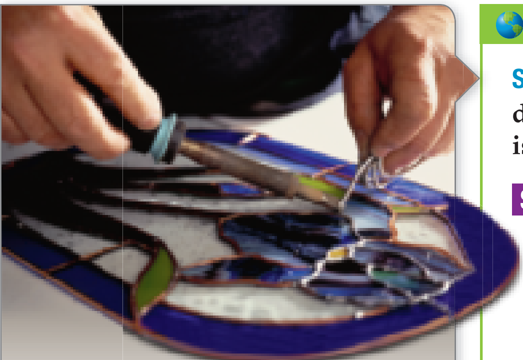
In $\odot S$, $m\overset{\frown}{PQR} = 98$. Find $m\overset{\frown}{PQ}$.



Radius $\overline{SQ}$ is perpendicular to chord $\overline{PR}$. So by Theorem 10.3, $\overline{SQ}$ bisects $\overset{\frown}{PQR}$. Therefore, $m\overset{\frown}{PQ} = m\overset{\frown}{QR}$. By substitution, $m\overset{\frown}{PQ} = \frac{98}{2}$ or 49.

GuidedPractice

3. In $\odot S$, find PR.



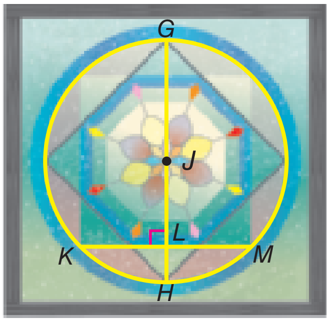
Real-WorldLink

To make stained glass windows, glass is heated to a temperature of 2000 degrees, until it is the consistency of taffy. The colors are caused by the addition of metallic oxides.

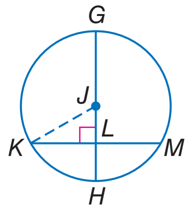
Source: *Artistic Stained Glass by Regg*

Real-World Example 4 Use a Diameter Perpendicular to a Chord

STAINED GLASS **In the stained glass window, diameter $\overline{GH}$ is 30 inches long and chord $\overline{KM}$ is 22 inches long. Find JL.**



Step 1 Draw radius $\overline{JK}$.



This forms right $\triangle JKL$.

Step 2 Find JK and KL.

Since $GH = 30$ inches, $JH = 15$ inches. All radii of a circle are congruent, so $JK = 15$ inches.

Since diameter $\overline{GH}$ is perpendicular to $\overline{KM}$, $\overline{GH}$ bisects chord $\overline{KM}$ by Theorem 10.3. So, $KL = \frac{1}{2}(22)$ or 11 inches.

Step 3 Use the Pythagorean Theorem to find JL.

$KL^2 + JL^2 = JK^2$	Pythagorean Theorem
$11^2 + JL^2 = 15^2$	$KL = 11$ and $JK = 15$
$121 + JL^2 = 225$	Simplify.
$JL^2 = 104$	Subtract 121 from each side.
$JL = \sqrt{104}$	Take the positive square root of each side.

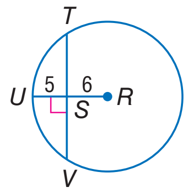
So, JL is $\sqrt{104}$ or about 10.20 inches long.

StudyTip

Drawing Segments You can add any known information to a figure to help you solve the problem. In Example 4, radius $\overline{JK}$ was drawn.

GuidedPractice

4. In $\odot R$, find TV. Round to the nearest hundredth.

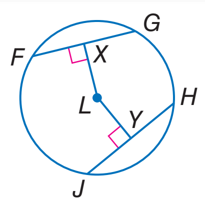


In addition to Theorem 10.2, you can use the following theorem to determine whether two chords in a circle are congruent.

Theorem 10.5

Words In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

Example $\overline{FG} \cong \overline{JH}$ if and only if $LX = LY$.

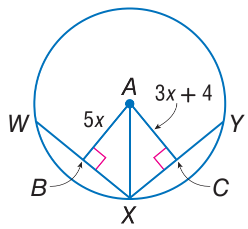


You will prove Theorem 10.5 in Exercises 29 and 30.

Dan Lim/Masterfile

Example 5 Chords Equidistant from Center

ALGEBRA **In ⊙A, $WX = XY = 22$. Find AB.**



Since chords $\overline{WX}$ and $\overline{XY}$ are congruent, they are equidistant from A. So, $AB = AC$.

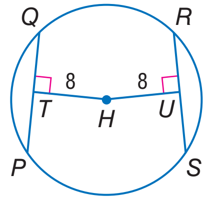
$AB = AC$

$5x = 3x + 4$ Substitution

$x = 2$ Simplify.

So, $AB = 5(2)$ or 10.

GuidedPractice

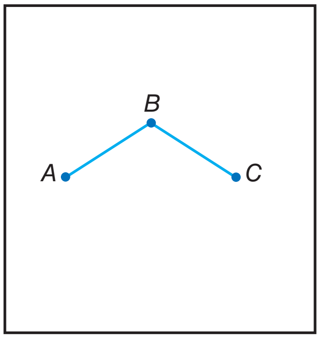


5. In ⊙H, $PQ = 3x - 4$ and $RS = 14$. Find x.

You can use Theorem 10.5 to find the point equidistant from three noncollinear points.

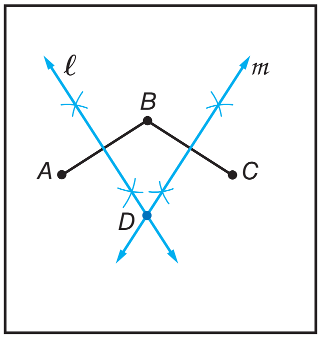
Construction Circle Through Three Noncollinear Points

Step 1



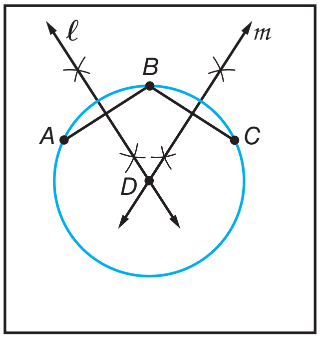
Draw three noncollinear points A, B, and C. Then draw segments $\overline{AB}$ and $\overline{BC}$.

Step 2



Construct the perpendicular bisectors ℓ and m of $\overline{AB}$ and $\overline{BC}$. Label the point of intersection D.

Step 3



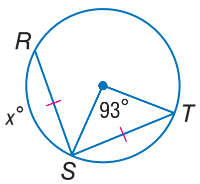
By Theorem 10.4, lines ℓ and m contain diameters of ⊙D. With the compass at point D, draw a circle through points A, B, and C.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Examples 1–2 **ALGEBRA** **Find the value of x.**

1



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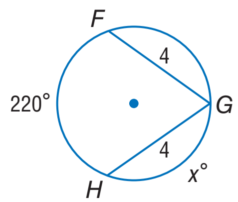
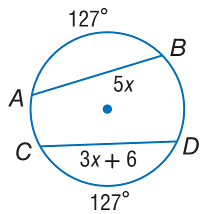
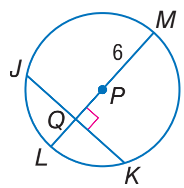


3.



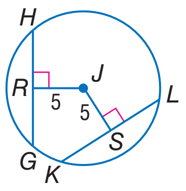
Examples 3–4 **In ⊙P, $JK = 10$ and $m\widehat{JLK} = 134$. Find each measure. Round to the nearest hundredth.**



4. $m\widehat{JL}$

5. PQ

Example 5 **6.** In $\odot J$, $GH = 9$, $KL = 4x + 1$. Find x.

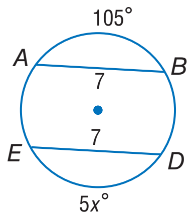


Practice and Problem Solving

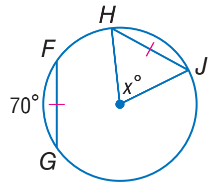
Extra Practice is on page R10.

Examples 1–2 **ALGEBRA** **Find the value of x.**

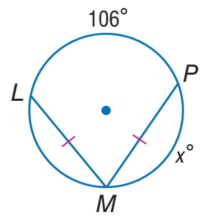
7.



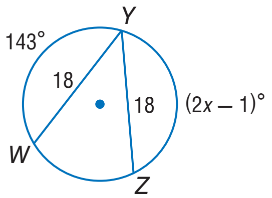
8.



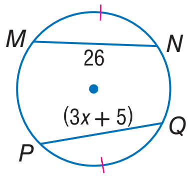
9.



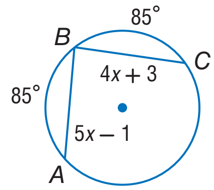
10.



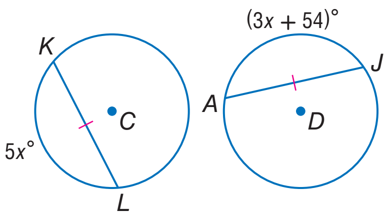
11.



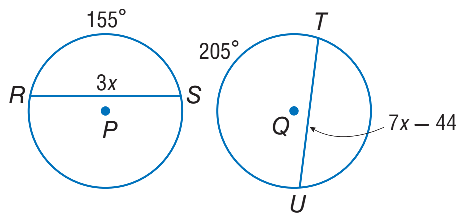
12.



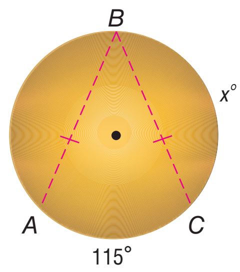
13 $\odot C \cong \odot D$



14. $\odot P \cong \odot Q$



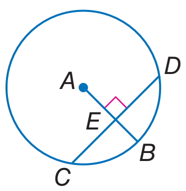
15. CCSS **MODELING** Angie is in a jewelry making class at her local arts center. She wants to make a pair of triangular earrings from a metal circle. She knows that $\widehat{AC}$ is 115°. If she wants to cut two equal parts off so that $\widehat{AB} = \widehat{BC}$, what is x?



Examples 3–4 **In $\odot A$, the radius is 14 and $CD = 22$. Find each measure. Round to the nearest hundredth, if necessary.**

16. CE

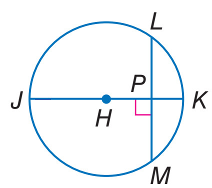
17. EB



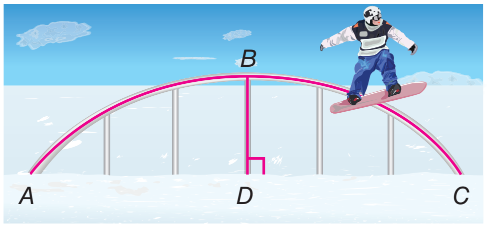
In $\odot H$, the diameter is 18, $LM = 12$, and $m\widehat{LM} = 84$. Find each measure. Round to the nearest hundredth, if necessary.

18. $m\widehat{LK}$

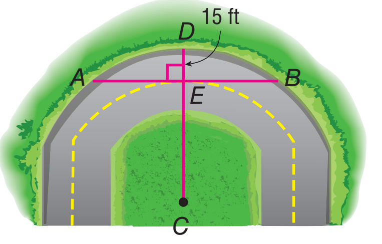
19. HP



20. **SNOWBOARDING** The snowboarding rail shown is an arc of a circle in which $\overline{BD}$ is part of the diameter. If $\widehat{ABC}$ is about 32% of a complete circle, what is $m\widehat{AB}$?

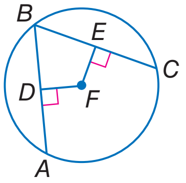


21 **ROADS** The curved road at the right is part of $\odot C$, which has a radius of 88 feet. What is AB? Round to the nearest tenth.

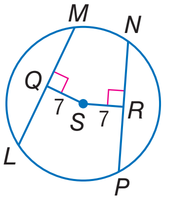


Example 5

22. **ALGEBRA** In $\odot F$, $\overline{AB} \cong \overline{BC}$, $DF = 3x - 7$, and $FE = x + 9$. What is x?



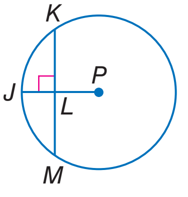
23. **ALGEBRA** In $\odot S$, $LM = 16$ and $PN = 4x$. What is x?



PROOF **Write a two-column proof.**

24. **Given:** $\odot P$, $\overline{KM} \perp \overline{JP}$

Prove: $\overline{JP}$ bisects $\overline{KM}$ and $\widehat{KM}$.

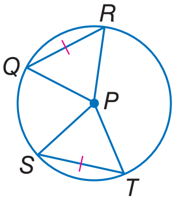


PROOF **Write the specified type of proof.**

25. paragraph proof of Theorem 10.2, part 2

Given: $\odot P$, $\overline{QR} \cong \overline{ST}$

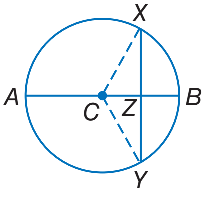
Prove: $\widehat{QR} \cong \widehat{ST}$



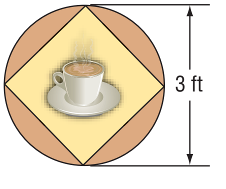
26. two-column proof of Theorem 10.3

Given: $\odot C$, $\overline{AB} \perp \overline{XY}$

Prove: $\overline{XZ} \cong \overline{YZ}$, $\widehat{XB} \cong \widehat{YB}$



27. **DESIGN** Roberto is designing a logo for a friend's coffee shop according to the design at the right, where each chord is equal in length. What is the measure of each arc and the length of each chord?



28. CCSS **ARGUMENTS** Write a two-column proof of Theorem 10.4.

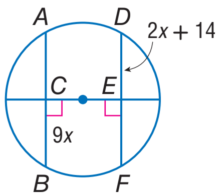
CCSS ARGUMENTS **Write a two-column proof of the indicated part of Theorem 10.5.**

29. In a circle, if two chords are equidistant from the center, then they are congruent.

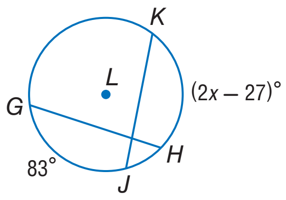
30. In a circle, if two chords are congruent, then they are equidistant from the center.

ALGEBRA **Find the value of x.**

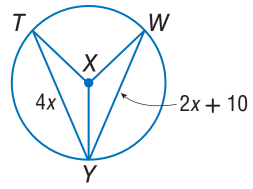
31 $\overline{AB} \cong \overline{DF}$



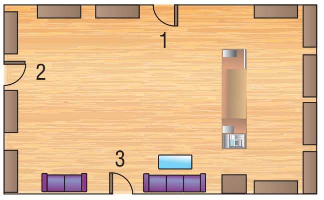
32. $\overline{GH} \cong \overline{KJ}$



33. $\widehat{WTY} \cong \widehat{TWY}$

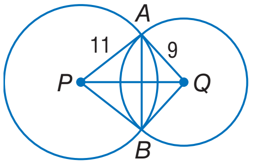


34. ADVERTISING A bookstore clerk wants to set up a display of new books. If there are three entrances into the store as shown in the figure at the right, where should the display be to get maximum exposure?



H.O.T. Problems Use Higher-Order Thinking Skills

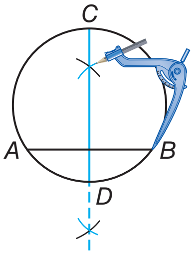
35. CHALLENGE The common chord $\overline{AB}$ between $\odot P$ and $\odot Q$ is perpendicular to the segment connecting the centers of the circles. If $AB = 10$, what is the length of $\overline{PQ}$? Explain your reasoning.



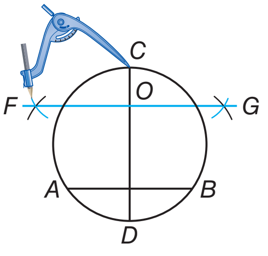
36. REASONING In a circle, $\overline{AB}$ is a diameter and $\overline{HG}$ is a chord that intersects $\overline{AB}$ at point X. Is it *sometimes*, *always*, or *never* true that $HX = GX$? Explain.

37. CHALLENGE Use a compass to draw a circle with chord $\overline{AB}$. Refer to this construction for the following problem.

Step 1 Construct $\overline{CD}$, the perpendicular bisector of $\overline{AB}$.



Step 2 Construct $\overline{FG}$, the perpendicular bisector of $\overline{CD}$. Label the point of intersection O.



a. Use an indirect proof to show that $\overline{CD}$ passes through the center of the circle by assuming that the center of the circle is *not* on $\overline{CD}$.

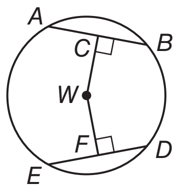
b. Prove that O is the center of the circle.

38. OPEN ENDED Construct a circle and draw a chord. Measure the chord and the distance that the chord is from the center. Find the length of the radius.

39. WRITING IN MATH If the measure of an arc in a circle is tripled, will the chord of the new arc be three times as long as the chord of the original arc? Explain your reasoning.

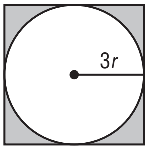
Standardized Test Practice

40. If $CW = WF$ and $ED = 30$, what is DF?



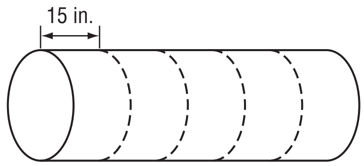
A 60
B 45
C 30
D 15

41. ALGEBRA Write the ratio of the area of the circle to the area of the square in simplest form.

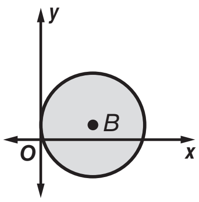


F $\frac{\pi}{4}$ **H** $\frac{3\pi}{4}$
G $\frac{\pi}{2}$ **J** π

42. SHORT RESPONSE The pipe shown is divided into five equal sections. How long is the pipe in feet (ft) and inches (in.)?



43. SAT/ACT Point B is the center of a circle, tangent to the y-axis, and the coordinates of Point B are (3, 1). What is the area of the circle?

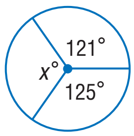


A π units^2
B 3π units^2
C 4π units^2
D 6π units^2
E 9π units^2

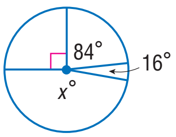
Spiral Review

Find x. (Lesson 10-2)

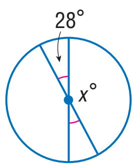
44.



45.



46.



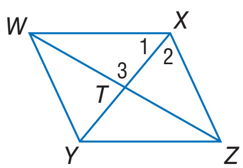
47. CRAFTS Ruby created a pattern to sew flowers onto a quilt by first drawing a regular pentagon that was 3.5 inches long on each side. Then she added a semicircle onto each side of the pentagon to create the appearance of five petals. How many inches of gold trim does she need to edge 10 flowers? Round to the nearest inch. (Lesson 10-1)

Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as *acute*, *obtuse*, or *right*. Justify your answer. (Lesson 8-2)

48. 8, 15, 17 **49.** 20, 21, 31 **50.** 10, 16, 18

Skills Review

ALGEBRA Quadrilateral $WXZY$ is a rhombus. Find each value or measure.



51. If $m\angle 3 = y^2 - 31$, find y.

52. If $m\angle XZY = 56$, find $m\angle YWZ$.