LESSON 10-4

Inscribed Angles

Then

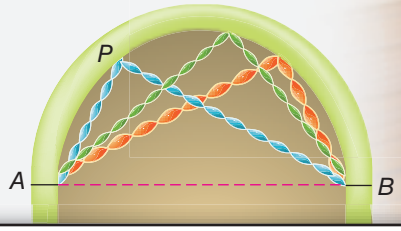
- You found measures of interior angles of polygons.

Now

1. Find measures of inscribed angles.
2. Find measures of angles of inscribed polygons.

Why?

- The entrance to a school prom has a semicircular arch. Streamers are attached with one end at point *A* and the other end at point *B*. The middle of each streamer can then be attached to a different point *P* along the arch.



New Vocabulary

inscribed angle
intercepted arc

Common Core State Standards

Content Standards

G.C.2 Identify and describe relationships among inscribed angles, radii, and chords.

G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

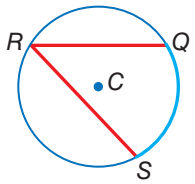
Mathematical Practices

7 Look for and make use of structure.

3 Construct viable arguments and critique the reasoning of others.

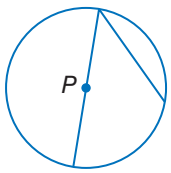
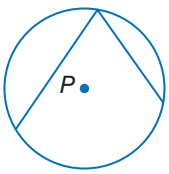
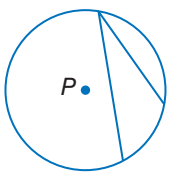
1 Inscribed Angles

Notice that the angle formed by each streamer appears to be congruent, no matter where point *P* is placed along the arch. An **inscribed angle** has a vertex on a circle and sides that contain chords of the circle. In $\odot C$, $\angle QRS$ is an inscribed angle.



An **intercepted arc** has endpoints on the sides of an inscribed angle and lies in the interior of the inscribed angle. In $\odot C$, minor arc $\overset{\frown}{QS}$ is intercepted by $\angle QRS$.

There are three ways that an angle can be inscribed in a circle.

Case 1	Case 2	Case 3
		
Center *P* is on a side of the inscribed angle.	Center *P* is inside the inscribed angle.	The center *P* is in the exterior of the inscribed angle.

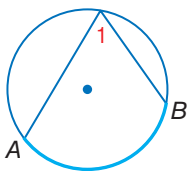
In Case 1, the side of the angle is a diameter of the circle.

For each of these cases, the following theorem holds true.

Theorem 10.6 Inscribed Angle Theorem

Words If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

Example $m\angle 1 = \frac{1}{2}m\overset{\frown}{AB}$ and $m\overset{\frown}{AB} = 2m\angle 1$



You will prove Cases 2 and 3 of the Inscribed Angle Theorem in Exercises 37 and 38.

VocabularyLink

Inscribed

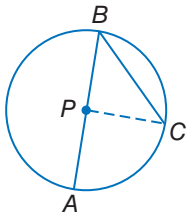
Everyday Use: written on or in a surface, such as inscribing the inside of a ring with an inscription

Math Use: touching only the sides (or interior) of another figure

Proof Inscribed Angle Theorem (Case 1)

Given: $\angle B$ is inscribed in $\odot P$.

Prove: $m\angle B = \frac{1}{2}m\overset{\frown}{AC}$



Proof:

Statements	Reasons
1. Draw an auxiliary radius $\overline{PC}$.	1. Two points determine a line.
2. $\overline{PB} \cong \overline{PC}$	2. All radii of a circle are $\cong$.
3. $\triangle PBC$ is isosceles.	3. Definition of isosceles triangle
4. $m\angle B = m\angle C$	4. Isosceles Triangle Theorem
5. $m\angle APC = m\angle B + m\angle C$	5. Exterior Angle Theorem
6. $m\angle APC = 2m\angle B$	6. Substitution (Steps 4, 5)
7. $m\overset{\frown}{AC} = m\angle APC$	7. Definition of arc measure
8. $m\overset{\frown}{AC} = 2m\angle B$	8. Substitution (Steps 6, 7)
9. $2m\angle B = m\overset{\frown}{AC}$	9. Symmetric Property of Equality
10. $m\angle B = \frac{1}{2}m\overset{\frown}{AC}$	10. Division Property of Equality

PT

Example 1 Use Inscribed Angles to Find Measures

Find each measure.

a. $m\angle P$

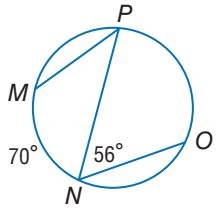
$m\angle P = \frac{1}{2}m\overset{\frown}{MN}$

$= \frac{1}{2}(70)$ or 35

b. $m\overset{\frown}{PO}$

$m\overset{\frown}{PO} = 2m\angle N$

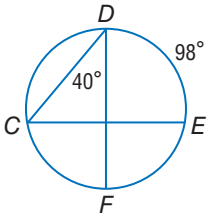
$= 2(56)$ or 112



GuidedPractice

1A. $m\overset{\frown}{CF}$

1B. $m\angle C$

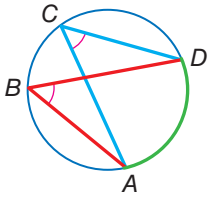


Two inscribed angles that intercept the same arc of a circle are related.

Theorem 10.7

Words If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent.

Example $\angle B$ and $\angle C$ both intercept $\overset{\frown}{AD}$. So, $\angle B \cong \angle C$.



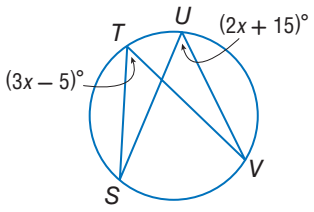
You will prove Theorem 10.7 in Exercise 39.

StudyTip

Inscribed Polygons
Remember that for a polygon to be an inscribed polygon, *all* of its vertices must lie on the circle.

Example 2 Use Inscribed Angles to Find Measures

ALGEBRA Find $m\angle T$.



$\angle T \cong \angle U$	$\angle T$ and $\angle U$ both intercept $\overset{\frown}{SV}$.
$m\angle T = m\angle U$	Definition of congruent angles
$3x - 5 = 2x + 15$	Substitution
$x = 20$	Simplify.

So, $m\angle T = 3(20) - 5$ or 55.

GuidedPractice

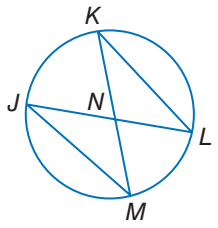
2. If $m\angle S = 3x$ and $m\angle V = (x + 16)$, find $m\angle S$.

Example 3 Use Inscribed Angles in Proofs

Write a two-column proof.

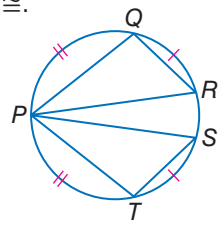
Given: $\overset{\frown}{JM} \cong \overset{\frown}{KL}$

Prove: $\triangle JMN \cong \triangle KLN$



Proof:

Statements	Reasons
1. $\overset{\frown}{JM} \cong \overset{\frown}{KL}$	**1.** Given
2. $\overline{JM} \cong \overline{KL}$	**2.** If minor arcs are $\cong$, their corresponding chords are $\cong$.
3. $\angle M$ intercepts $\overset{\frown}{JK}$. $\angle L$ intercepts $\overset{\frown}{JK}$.	**3.** Definition of intercepted arc
4. $\angle M \cong \angle L$	**4.** Inscribed $\measuredangle$s of same arc are $\cong$.
5. $\angle JNM \cong \angle KNL$	**5.** Vertical $\measuredangle$s are $\cong$.
6. $\triangle JMN \cong \triangle KLN$	**6.** AAS



GuidedPractice

3. Given: $\overset{\frown}{QR} \cong \overset{\frown}{ST}$, $\overset{\frown}{PQ} \cong \overset{\frown}{PT}$

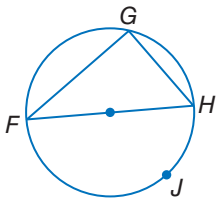
Prove: $\triangle PQR \cong \triangle PTS$

2 Angles of Inscribed Polygons

Triangles and quadrilaterals that are inscribed in circles have special properties.

Theorem 10.8

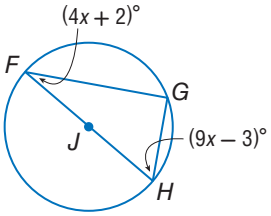
Words	An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle.
Example	If $\overset{\frown}{FJH}$ is a semicircle, then $m\angle G = 90$. If $m\angle G = 90$, then $\overset{\frown}{FJH}$ is a semicircle and $\overline{FH}$ is a diameter.



You will prove Theorem 10.8 in Exercise 40.

Example 4 Find Angle Measures in Inscribed Triangles

ALGEBRA Find $m\angle F$.



$\triangle FGH$ is a right triangle because $\angle G$ inscribes a semicircle.

$m\angle F + m\angle G + m\angle H = 180$	Angle Sum Theorem
$(4x + 2) + 90 + (9x - 3) = 180$	Substitution
$13x + 89 = 180$	Simplify.
$13x = 91$	Subtract 89 from each side.
$x = 7$	Divide each side by 13.

So, $m\angle F = 4(7) + 2$ or 30.

GuidedPractice

4. If $m\angle F = 7x + 2$ and $m\angle H = 17x - 8$, find x.

While many different types of triangles, including right triangles, can be inscribed in a circle, only certain quadrilaterals can be inscribed in a circle.

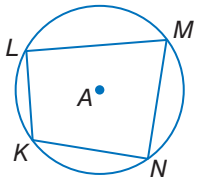
StudyTip

CCSS Arguments Theorem 10.9 can be verified by considering that the arcs intercepted by opposite angles of an inscribed quadrilateral form a circle.

Theorem 10.9

Words If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

Example If quadrilateral $KLMN$ is inscribed in $\odot A$, then $\angle L$ and $\angle N$ are supplementary and $\angle K$ and $\angle M$ are supplementary.



You will prove Theorem 10.9 in Exercise 31.

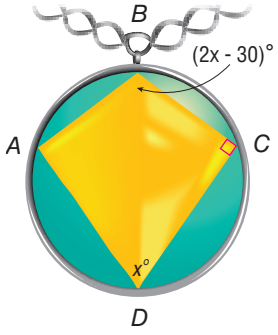
Real-WorldLink

Charms for jewelry first became popular during the age of the Egyptian Pharaohs. They were repopularized by Queen Victoria in the early 20th century and by Louis Vuitton in 2001.

Source: *My Mother's Charms*

Real-World Example 5 Find Angle Measures

JEWELRY **The necklace charm shown uses a quadrilateral inscribed in a circle. Find $m\angle A$ and $m\angle B$.**



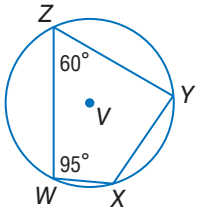
Since $ABCD$ is inscribed in a circle, opposite angles are supplementary.

$m\angle A + m\angle C = 180$	$m\angle B + m\angle D = 180$
$m\angle A + 90 = 180$	$(2x - 30) + x = 180$
$m\angle A = 90$	$3x - 30 = 180$
	$3x = 210$
	$x = 70$

So, $m\angle A = 90$ and $m\angle B = 2(70) - 30$ or 110.

GuidedPractice

5. Quadrilateral $WXYZ$ is inscribed in $\odot V$. Find $m\angle X$ and $m\angle Y$.

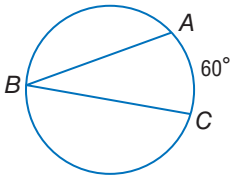


Check Your Understanding

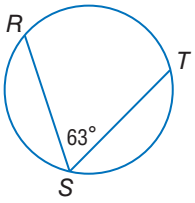
= Step-by-Step Solutions begin on page R14.

Example 1 **Find each measure.**

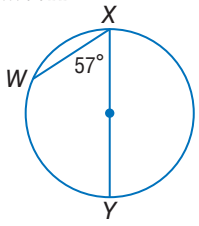
1. $m\angle B$



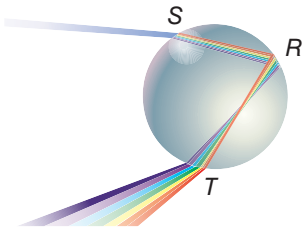
2. $m\overset{\frown}{RT}$



3. $m\overset{\frown}{WX}$

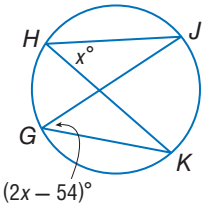


4. SCIENCE The diagram shows how light bends in a raindrop to make the colors of the rainbow. If $m\overset{\frown}{ST} = 144$, what is $m\angle R$?

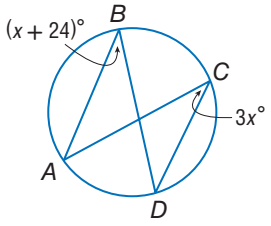


Example 2 **ALGEBRA Find each measure.**

5. $m\angle H$



6. $m\angle B$

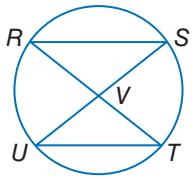


Example 3

7. PROOF Write a two-column proof.

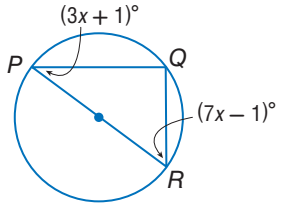
Given: $\overline{RT}$ bisects $\overline{SU}$.

Prove: $\triangle RVS \cong \triangle UVT$

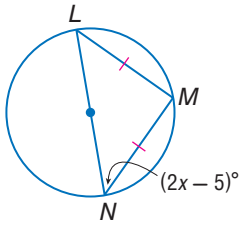


Examples 4–5 **STRUCTURE Find each value.**

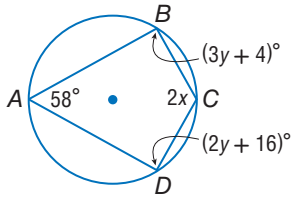
8. $m\angle R$



9. x



10. $m\angle C$ and $m\angle D$

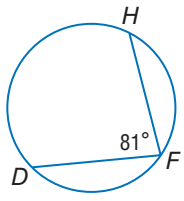


Practice and Problem Solving

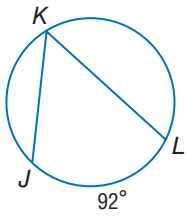
Extra Practice is on page R10.

Example 1 **Find each measure.**

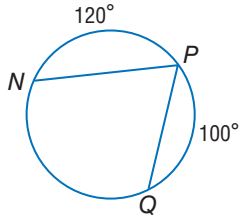
11. $m\overset{\frown}{DH}$



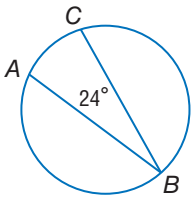
12. $m\angle K$



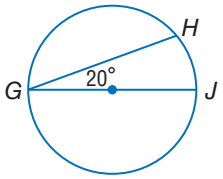
13 $m\angle P$



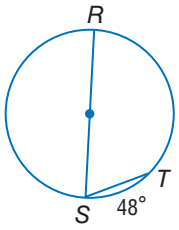
14. $m\overset{\frown}{AC}$



15. $m\overset{\frown}{GH}$



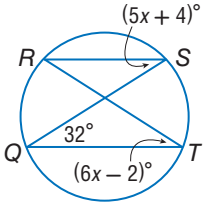
16. $m\angle S$



Example 2 **ALGEBRA** **Find each measure.**

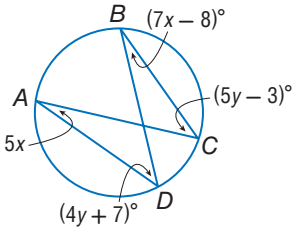
17. $m\angle R$

18. $m\angle S$



19. $m\angle A$

20. $m\angle C$

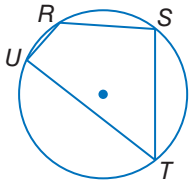


Example 3 **PROOF** **Write the specified type of proof.**

21. paragraph proof

Given: $m\angle T = \frac{1}{2}m\angle S$

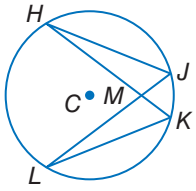
Prove: $m\overset{\frown}{TUR} = 2m\overset{\frown}{URS}$



22. two-column proof

Given: $\odot C$

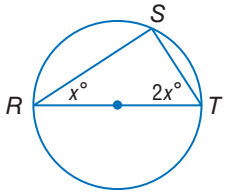
Prove: $\triangle KML \sim \triangle JMH$



Example 4 **ALGEBRA** **Find each value.**

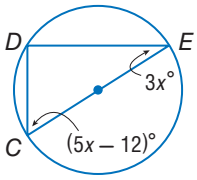
23. x

24. $m\angle T$



25. x

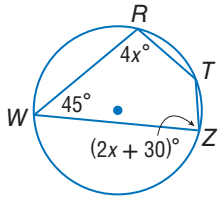
26. $m\angle C$



Example 5 CCSS **STRUCTURE** **Find each measure.**

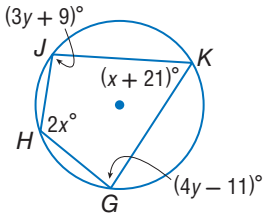
27. $m\angle T$

28. $m\angle Z$



29. $m\angle H$

30. $m\angle G$



31. **PROOF** Write a paragraph proof for Theorem 10.9.

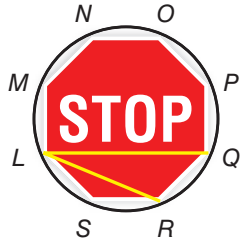
SIGNS **A stop sign in the shape of a regular octagon is inscribed in a circle. Find each measure.**

32. $m\overset{\frown}{NQ}$

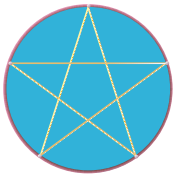
33 $m\angle RLQ$

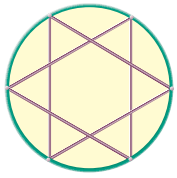
34. $m\angle LRQ$

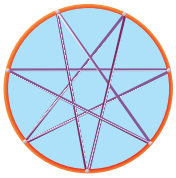
35. $m\angle LSR$

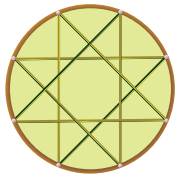


36. ART Four different string art star patterns are shown. If all of the inscribed angles of each star shown are congruent, find the measure of each inscribed angle.

a. 

b. 

c. 

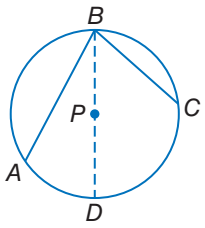
d. 

PROOF **Write a two-column proof for each case of Theorem 10.6.**

37. Case 2

Given: P lies inside $\angle ABC$.
$\overline{BD}$ is a diameter.

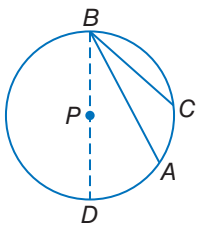
Prove: $m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$



38. Case 3

Given: P lies outside $\angle ABC$.
$\overline{BD}$ is a diameter.

Prove: $m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$



PROOF **Write the specified proof for each theorem.**

39 Theorem 10.7, two-column proof

40. Theorem 10.8, paragraph proof

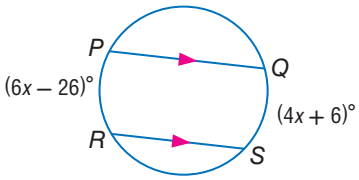
41. MULTIPLE REPRESENTATIONS In this problem, you will investigate the relationship between the arcs of a circle that are cut by two parallel chords.

a. Geometric Use a compass to draw a circle with parallel chords $\overline{AB}$ and $\overline{CD}$. Connect points A and D by drawing segment $\overline{AD}$.

b. Numerical Use a protractor to find $m\angle A$ and $m\angle D$. Then determine $m\overset{\frown}{AC}$ and $m\overset{\frown}{BD}$. What is true about these arcs? Explain.

c. Verbal Draw another circle and repeat parts **a** and **b**. Make a conjecture about arcs of a circle that are cut by two parallel chords.

d. Analytical Use your conjecture to find $m\overset{\frown}{PR}$ and $m\overset{\frown}{QS}$ in the figure at the right. Verify by using inscribed angles to find the measures of the arcs.



H.O.T. Problems Use Higher-Order Thinking Skills

CCSS ARGUMENTS **Determine whether the quadrilateral can *always*, *sometimes*, or *never* be inscribed in a circle. Explain your reasoning.**

42. square **43.** rectangle **44.** parallelogram **45.** rhombus **46.** kite

47. CHALLENGE A square is inscribed in a circle. What is the ratio of the area of the circle to the area of the square?

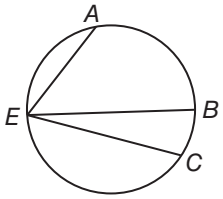
48. WRITING IN MATH A 45°-45°-90° right triangle is inscribed in a circle. If the radius of the circle is given, explain how to find the lengths of the right triangle's legs.

49. OPEN ENDED Find and sketch a real-world logo with an inscribed polygon.

50. WRITING IN MATH Compare and contrast inscribed angles and central angles of a circle. If they intercept the same arc, how are they related?

Standardized Test Practice

51. In the circle below, $m\overset{\frown}{AC} = 160$ and $m\angle BEC = 38$. What is $m\angle AEB$?

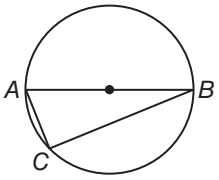


A 42
B 61
C 80
D 84

52. ALGEBRA Simplify $4(3x - 2)(2x + 4) + 3x^2 + 5x - 6$.

F $9x^2 + 3x - 14$
G $9x^2 + 13x - 14$
H $27x^2 + 37x - 38$
J $27x^2 + 27x - 26$

53. SHORT RESPONSE In the circle below, $\overline{AB}$ is a diameter, $AC = 8$ inches, and $BC = 15$ inches. Find the diameter, the radius, and the circumference of the circle.

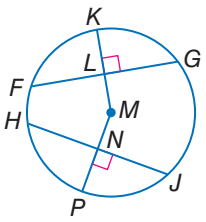


54. SAT/ACT The sum of three consecutive integers is −48. What is the least of the three integers?

A −15
B −16
C −17
D −18
E −19

Spiral Review

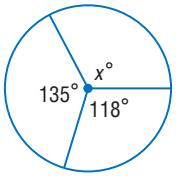
In $\odot M$, $FL = 24$, $HJ = 48$, and $m\overset{\frown}{HP} = 65$. Find each measure. (Lesson 10-3)



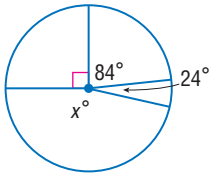
55. FG
56. $m\overset{\frown}{PJ}$
57. NJ
58. $m\overset{\frown}{HJ}$

Find x. (Lesson 10-2)

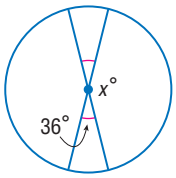
59.



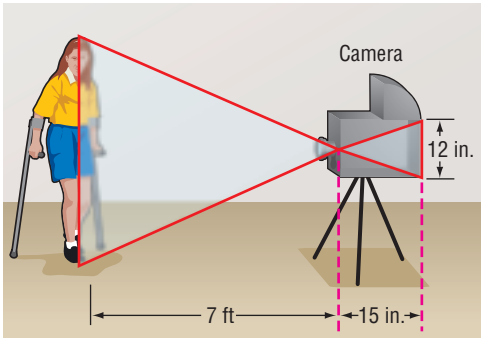
60.



61.



62. PHOTOGRAPHY In one of the first cameras invented, light entered an opening in the front. An image was reflected in the back of the camera, upside down, forming similar triangles. Suppose the image of the person on the back of the camera is 12 inches, the distance from the opening to the person is 7 feet, and the camera itself is 15 inches long. How tall is the person being photographed? (Lesson 7-3)



Skills Review

ALGEBRA Suppose B is the midpoint of $\overline{AC}$. Use the given information to find the missing measure.

63. $AB = 4x - 5$, $BC = 11 + 2x$, $AC = ?$
64. $AB = 6y - 14$, $BC = 10 - 2y$, $AC = ?$
65. $BC = 6 - 4m$, $AC = 8$, $m = ?$
66. $AB = 10s + 2$, $AC = 40$, $s = ?$