LESSON 10-5 Tangents

Then

- You used the Pythagorean Theorem to find side lengths of right triangles.

Now

1. Use properties of tangents.
2. Solve problems involving circumscribed polygons.

Why?

- The first bicycles were moved by pushing your feet on the ground. Modern bicycles use pedals, a chain, and gears. The chain loops around circular gears. The length of the chain between these gears is measured from the points of tangency.

NewVocabulary
tangent
point of tangency
common tangent

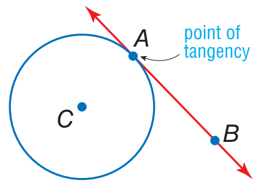
Common Core State Standards

Content Standards
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
G.C.4 Construct a tangent line from a point outside a given circle to the circle.

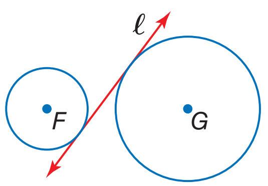
Mathematical Practices
1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.

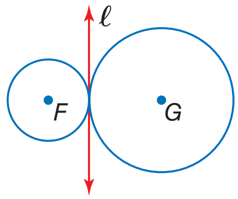
1 Tangents

A **tangent** is a line in the same plane as a circle that intersects the circle in exactly one point, called the **point of tangency**. $\overleftrightarrow{AB}$ is tangent to $\odot C$ at point A. $\overline{AB}$ and $\overrightarrow{AB}$ are also called tangents.



A **common tangent** is a line, ray, or segment that is tangent to two circles in the same plane. In each figure below, line ℓ is a common tangent of circles F and G.

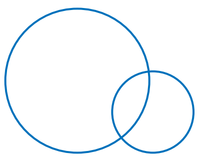




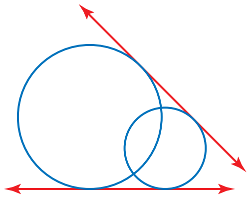
Example 1 Identify Common Tangents

Copy each figure and draw the common tangents. If no common tangent exists, state *no common tangent*.

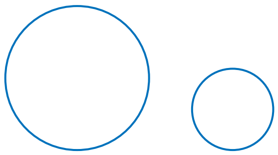
a.



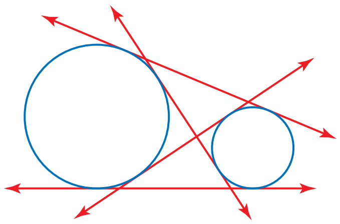
These circles have two common tangents.



b.

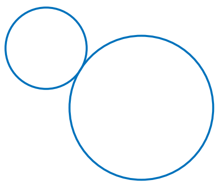


These circles have 4 common tangents.

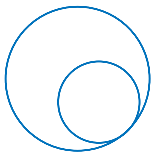


GuidedPractice

1A.



1B.

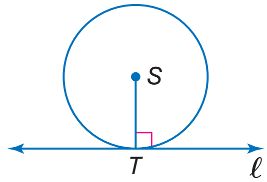


The shortest distance from a tangent to the center of a circle is the radius drawn to the point of tangency.

Theorem 10.10

Words In a plane, a line is tangent to a circle if and only if it is perpendicular to a radius drawn to the point of tangency.

Example Line ℓ is tangent to $\odot S$ if and only if $\ell \perp \overline{ST}$.



You will prove both parts of Theorem 10.10 in Exercises 32 and 33.

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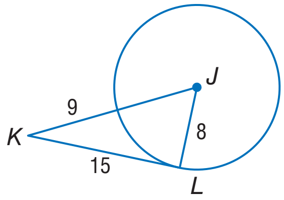
Example 2 Identify a Tangent

$\overline{JL}$ is a radius of $\odot J$. Determine whether $\overline{KL}$ is tangent to $\odot J$. Justify your answer.



Test to see if $\triangle JKL$ is a right triangle.

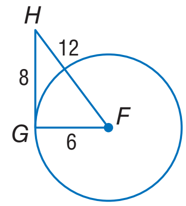
$8^2 + 15^2 \stackrel{?}{=} (8 + 9)^2$ Pythagorean Theorem

$289 = 289$ ✓ Simplify.

$\triangle JKL$ is a right triangle with right angle JLK. So $\overline{KL}$ is perpendicular to radius $\overline{JL}$ at point L. Therefore, by Theorem 10.10, $\overline{KL}$ is tangent to $\odot J$.

GuidedPractice

2. Determine whether $\overline{GH}$ is tangent to $\odot F$. Justify your answer.



You can also use Theorem 10.10 to identify missing values.

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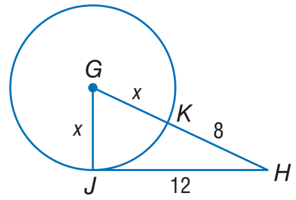
Example 3 Use a Tangent to Find Missing Values

$\overline{JH}$ is tangent to $\odot G$ at J. Find the value of x.



By Theorem 10.10, $\overline{JH} \perp \overline{GJ}$. So, $\triangle GHJ$ is a right triangle.

$GJ^2 + JH^2 = GH^2$ Pythagorean Theorem

$x^2 + 12^2 = (x + 8)^2$ $GJ = x$, $JH = 12$, and $GH = x + 8$

$x^2 + 144 = x^2 + 16x + 64$ Multiply.

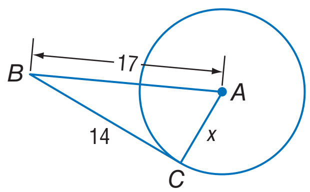
$80 = 16x$ Simplify.

$5 = x$ Divide each side by 16.

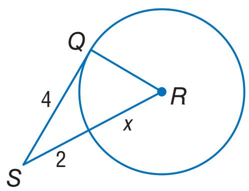
GuidedPractice

Find the value of x. Assume that segments that appear to be tangent are tangent.

3A.

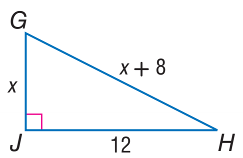


3B.



Problem-Solving Tip

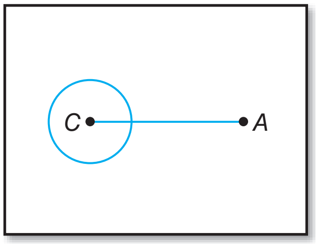
CCSS **Sense-Making** You can use the *solve a simpler problem* strategy by sketching and labeling the right triangles without the circles. A drawing of the triangle in Example 3 is shown below.



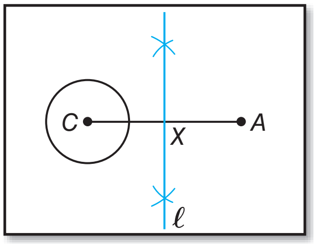
You can use Theorems 10.8 and 10.10 to construct a line tangent to a circle.

Construction Line Tangent to a Circle Through an External Point

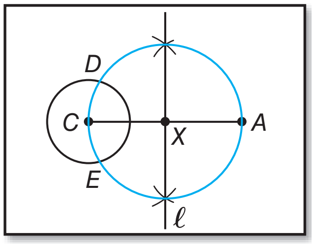
Step 1 Use a compass to draw circle C and a point A outside of circle C. Then draw $\overline{CA}$.



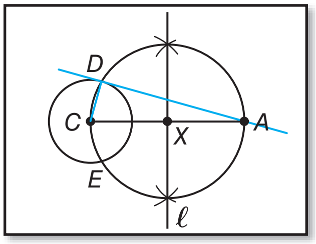
Step 2 Construct line ℓ, the perpendicular bisector of $\overline{CA}$. Label the point of intersection X.



Step 3 Construct circle X with radius $\overline{XC}$. Label the points of intersection of the two circles D and E.



Step 4 Draw $\overleftrightarrow{AD}$ and $\overline{DC}$. $\triangle ADC$ is inscribed in a semicircle. So, $\angle ADC$ is a right angle and $\overleftrightarrow{AD}$ is tangent to $\odot C$.



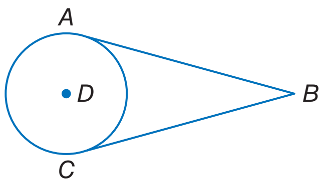
You will justify this construction in Exercise 36 and construct a line tangent to a circle through a point on the circle in Exercise 34.

More than one line can be tangent to the same circle.

Theorem 10.11

Words If two segments from the same exterior point are tangent to a circle, then they are congruent.

Example If $\overline{AB}$ and $\overline{CB}$ are tangent to $\odot D$, then $\overline{AB} \cong \overline{CB}$.



You will prove Theorem 10.11 in Exercise 28.

Example 4 Use Congruent Tangents to Find Measures

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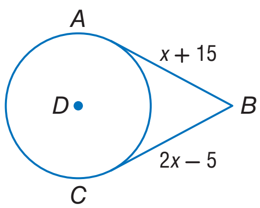
ALGEBRA $\overline{AB}$ **and** $\overline{CB}$ **are tangent to** $\odot D$. **Find the value of** x.

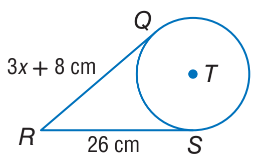


$AB = CB$	Tangents from the same exterior point are congruent.
$x + 15 = 2x - 5$	Substitution
$15 = x - 5$	Subtract x from each side.
$20 = x$	Add 5 to each side.

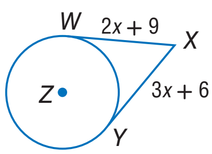
GuidedPractice

ALGEBRA **Find the value of** x. **Assume that segments that appear to be tangent are tangent.**

4A.



4B.

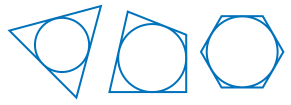
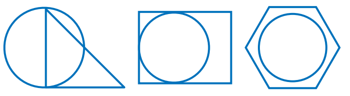


2 Circumscribed Polygons

A polygon is circumscribed about a circle if every side of the polygon is tangent to the circle.

WatchOut!

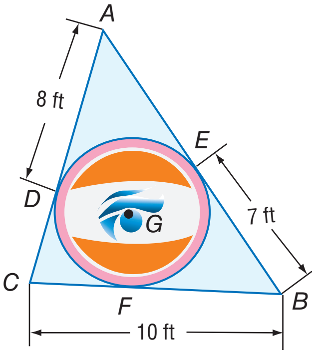
Identifying Circumscribed Polygons Just because the circle is tangent to one or more of the sides of a polygon does not mean that the polygon is circumscribed about the circle, as shown in the second set of figures.

Circumscribed Polygons	Polygons Not Circumscribed
	

You can use Theorem 10.11 to find missing measures in circumscribed polygons.

Real-World Example 5 Find Measures in Circumscribed Polygons

GRAPHIC DESIGN A graphic designer is giving directions to create a larger version of the triangular logo shown. If $\triangle ABC$ is circumscribed about $\odot G$, find the perimeter of $\triangle ABC$.



Step 1 Find the missing measures.

Since $\triangle ABC$ is circumscribed about $\odot G$, $\overline{AE}$ and $\overline{AD}$ are tangent to $\odot G$, as are $\overline{BE}$, $\overline{BF}$, $\overline{CF}$, and $\overline{CD}$. Therefore, $\overline{AE} \cong \overline{AD}$, $\overline{BF} \cong \overline{BE}$, and $\overline{CF} \cong \overline{CD}$.

So, $AE = AD = 8$ feet, $BF = BE = 7$ feet.

By Segment Addition, $CF + FB = CB$, so $CF = CB - FB = 10 - 7$ or 3 feet. So, $CD = CF = 3$ feet.

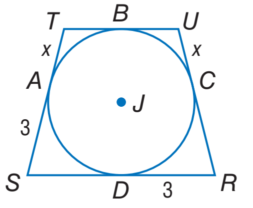
Step 2 Find the perimeter of $\triangle ABC$.

$$\text{perimeter} = AE + EB + BC + CD + DA$$
$$= 8 + 7 + 10 + 3 + 8 \text{ or } 36$$

So, the perimeter of $\triangle ABC$ is 36 feet.

GuidedPractice

5. Quadrilateral $RSTU$ is circumscribed about $\odot J$. If the perimeter is 18 units, find x.

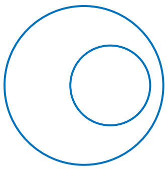


Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1

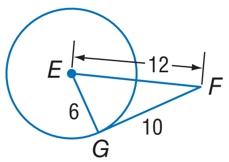
1. Copy the figure shown, and draw the common tangents. If no common tangent exists, state *no common tangent*.



Example 2

Determine whether $\overline{FG}$ is tangent to $\odot E$. Justify your answer.

2.



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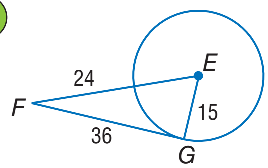
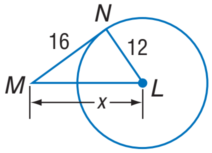
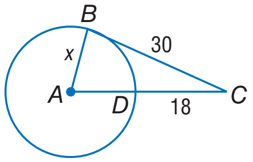


Examples 3–4 **Find x. Assume that segments that appear to be tangent are tangent.**

4.



5.



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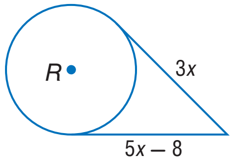
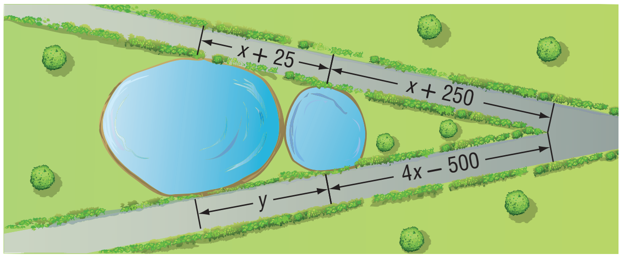


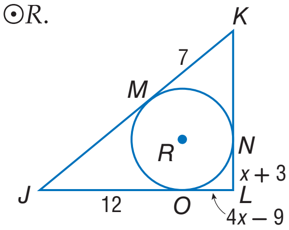
7. **LANDSCAPE ARCHITECT** A landscape architect is paving the two walking paths that are tangent to two approximately circular ponds as shown. The lengths given are in feet. Find the values of x and y.



Example 5 **8.** CCSS **SENSE-MAKING** Triangle JKL is circumscribed about $\odot R$.

a. Find x.

b. Find the perimeter of $\triangle JKL$.

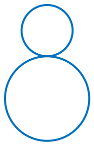


Practice and Problem Solving

Extra Practice is on page R10.

Example 1 **Copy each figure and draw the common tangents. If no common tangent exists, state *no common tangent*.**

9.



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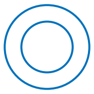

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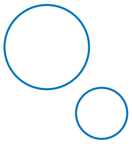

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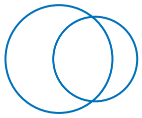
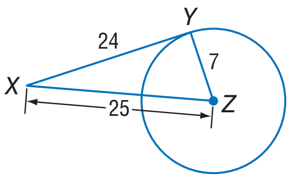


Example 2 **Determine whether each $\overline{XY}$ is tangent to the given circle. Justify your answer.**

13.



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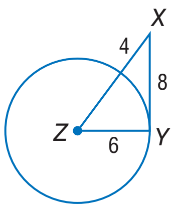

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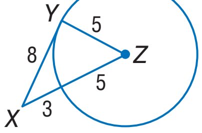

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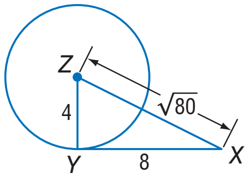

Examples 3–4 **Find x. Assume that segments that appear to be tangent are tangent. Round to the nearest tenth if necessary.**

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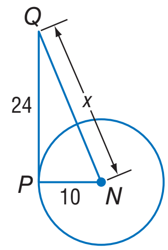

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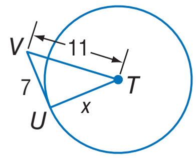

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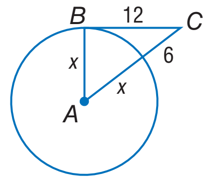

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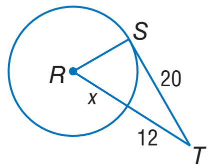

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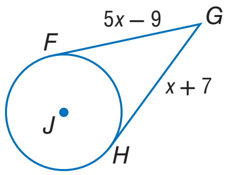

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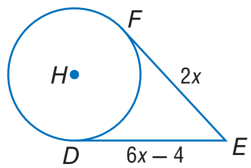
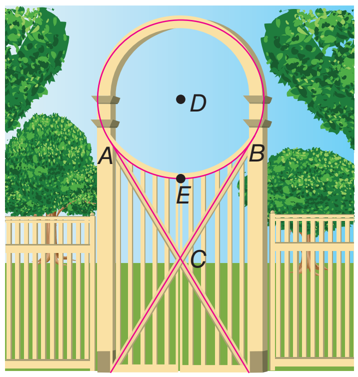


23. **ARBORS** In the arbor shown, $\overline{AC}$ and $\overline{BC}$ are tangents to $\odot D$. The radius of the circle is 26 inches and $EC = 20$ inches. Find each measure to the nearest hundredth.

a. AC **b.** BC



Example 5 **CCSS SENSE-MAKING** **Find the value of x. Then find the perimeter.**

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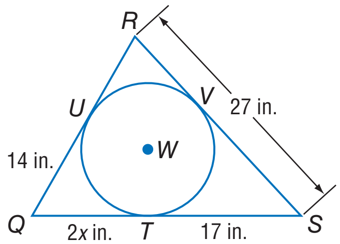

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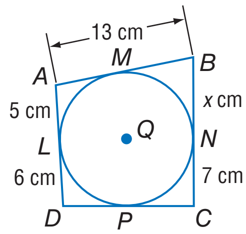
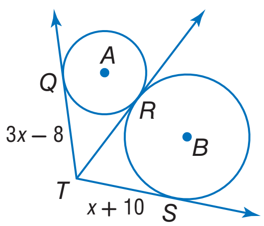


Find x to the nearest hundredth. Assume that segments that appear to be tangent are tangent.

26.


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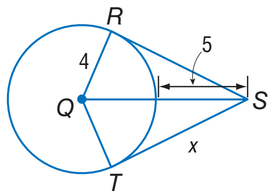
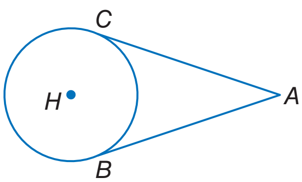


Write the specified type of proof.

28. two-column proof of Theorem 10.11

Given: $\overline{AC}$ is tangent to $\odot H$ at C.
$\overline{AB}$ is tangent to $\odot H$ at B.

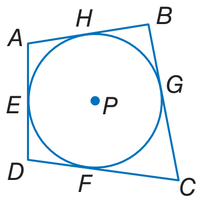
Prove: $\overline{AC} \cong \overline{AB}$



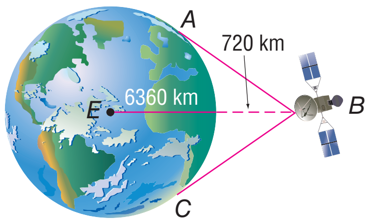
29. two-column proof

Given: Quadrilateral $ABCD$ is circumscribed about $\odot P$.

Prove: $AB + CD = AD + BC$



30. **SATELLITES** A satellite is 720 kilometers above Earth, which has a radius of 6360 kilometers. The region of Earth that is visible from the satellite is between the tangent lines $\overline{BA}$ and $\overline{BC}$. What is BA? Round to the nearest hundredth.



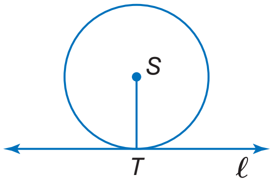
31 **SPACE TRASH** *Orbital debris* refers to materials from space missions that still orbit Earth. In 2007, a 1400-pound ammonia tank was discarded from a space mission. Suppose the tank has an altitude of 435 miles. What is the distance from the tank to the farthest point on Earth's surface from which the tank is visible? Assume that the radius of Earth is 4000 miles. Round to the nearest mile, and include a diagram of this situation with your answer.

32. **PROOF** Write an indirect proof to show that if a line is tangent to a circle, then it is perpendicular to a radius of the circle. (Part 1 of Theorem 10.10)

Given: ℓ is tangent to $\odot S$ at T; $\overline{ST}$ is a radius of $\odot S$.

Prove: $\ell \perp \overline{ST}$

(*Hint:* Assume ℓ is *not* $\perp$ to $\overline{ST}$.)

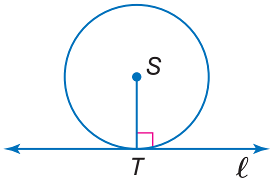


33. **PROOF** Write an indirect proof to show that if a line is perpendicular to the radius of a circle at its endpoint, then the line is a tangent of the circle. (Part 2 of Theorem 10.10)

Given: $\ell \perp \overline{ST}$; $\overline{ST}$ is a radius of $\odot S$.

Prove: ℓ is tangent to $\odot S$.

(*Hint:* Assume ℓ is *not* tangent to $\odot S$.)

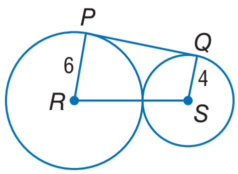


34. CCSS **TOOLS** Construct a line tangent to a circle through a point on the circle.

Use a compass to draw $\odot A$. Choose a point P on the circle and draw $\overleftrightarrow{AP}$. Then construct a segment through point P perpendicular to $\overleftrightarrow{AP}$. Label the tangent line t. Explain and justify each step.

H.O.T. Problems Use Higher-Order Thinking Skills

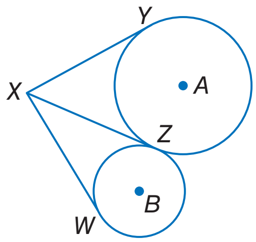
35. **CHALLENGE** $\overline{PQ}$ is tangent to circles R and S. Find PQ. Explain your reasoning.



36. **WRITING IN MATH** Explain and justify each step in the construction on page 734.

37. **OPEN ENDED** Draw a circumscribed triangle and an inscribed triangle.

38. **REASONING** In the figure, $\overline{XY}$ and $\overline{XZ}$ are tangent to $\odot A$. $\overline{XZ}$ and $\overline{XW}$ are tangent to $\odot B$. Explain how segments $\overline{XY}$, $\overline{XZ}$, and $\overline{XW}$ can all be congruent if the circles have different radii.



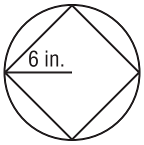
39. E? **WRITING IN MATH** Is it possible to draw a tangent from a point that is located anywhere outside, on, or inside a circle? Explain.

Standardized Test Practice

40. ⊙P has a radius of 10 centimeters, and $\overline{ED}$ is tangent to the circle at point D. F lies both on ⊙P and on segment $\overline{EP}$. If $ED = 24$ centimeters, what is the length of $\overline{EF}$?

A 10 cm **C** 21.8 cm
B 16 cm **D** 26 cm

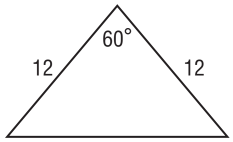
41. SHORT RESPONSE A square is inscribed in a circle having a radius of 6 inches. Find the length of each side of the square.



42. ALGEBRA Which of the following shows $25x^2 - 5x$ factored completely?

F $5x(x)$ **H** $x(x - 5)$
G $5x(5x - 1)$ **J** $x(5x - 1)$

43. SAT/ACT What is the perimeter of the triangle shown below?

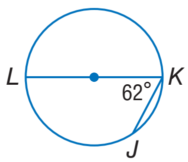


A 12 units **D** 36 units
B 24 units **E** 104 units
C 34.4 units

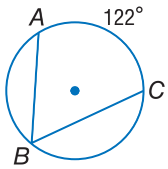
Spiral Review

Find each measure. (Lesson 10-4)

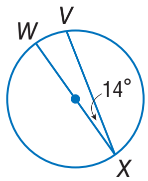
44. $m\widehat{JK}$



45. $m\angle B$

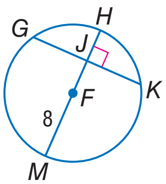


46. $m\widehat{VX}$

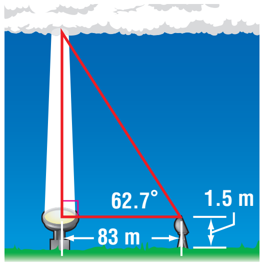


In ⊙F, $GK = 14$ and $m\widehat{GHK} = 142$. Find each measure. Round to the nearest hundredth. (Lesson 10-3)

47. $m\widehat{GH}$ **48.** JK **49.** $m\widehat{KM}$

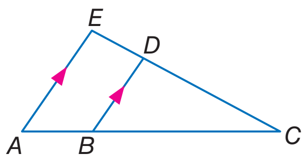


50. METEOROLOGY The altitude of the base of a cloud formation is called the *ceiling*. To find the ceiling one night, a meteorologist directed a spotlight vertically at the clouds. Using a theodolite, an optical instrument with a rotatable telescope, placed 83 meters from the spotlight and 1.5 meters above the ground, he found the angle of elevation to be 62.7°. How high was the ceiling? (Lesson 8-5)

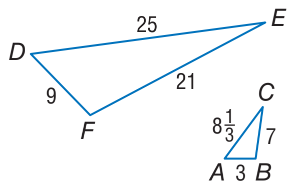


Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning. (Lesson 7-3)

51.



52.



Skills Review

Solve each equation.

53. $15 = \frac{1}{2}[(360 - x) - 2x]$ **54.** $x + 12 = \frac{1}{2}[(180 - 120)]$ **55.** $x = \frac{1}{2}[(180 - 64)]$