LESSON 10-7

Special Segments in a Circle

Then

- You found measures of diagonals that intersect in the interior of a parallelogram.

Now

- 1 Find measures of segments that intersect in the interior of a circle.
- 2 Find measures of segments that intersect in the exterior of a circle.

Why?

- A large circular cake is cut lengthwise instead of into wedges to serve more people for a party. Only a small portion of the original cake remains. Using the geometry of circles, you can determine the diameter of the original cake.

NewVocabulary

chord segment
secant segment
external secant segment
tangent segment

Common Core State Standards

Content Standards

Reinforcement of G.C.4 Construct a tangent line from a point outside a given circle to the circle.

Mathematical Practices

1 Make sense of problems and persevere in solving them.
7 Look for and make use of structure.

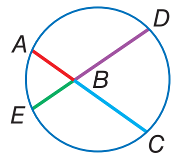
1 Segments Intersecting Inside a Circle

When two chords intersect inside a circle, each chord is divided into two segments, called **chord segments**.

Theorem 10.15 Segments of Chords Theorem

Words If two chords intersect in a circle, then the products of the lengths of the chord segments are equal.

Example $AB \cdot BC = DB \cdot BE$

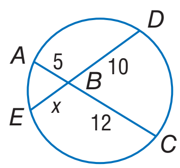


You will prove Theorem 10.15 in Exercise 23.

Example 1 Use the Intersection of Two Chords

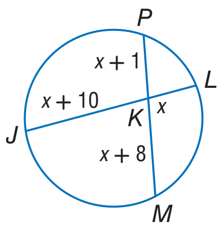
Find x.

a.



$AB \cdot BC = EB \cdot BD$	Theorem 10.15
$5 \cdot 12 = x \cdot 10$	Substitution
$60 = 10x$	Multiply.
$6 = x$	Divide each side by 10.

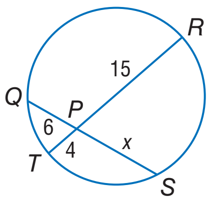
b.



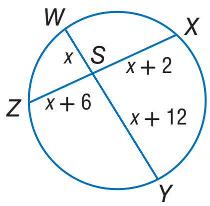
$JK \cdot KL = PK \cdot KM$	Theorem 10.15
$(x + 10) \cdot x = (x + 1)(x + 8)$	Substitution
$x^2 + 10x = x^2 + 9x + 8$	Multiply.
$10x = 9x + 8$	Subtract x^2 from each side.
$x = 8$	Subtract $9x$ from each side.

GuidedPractice

1A.



1B.



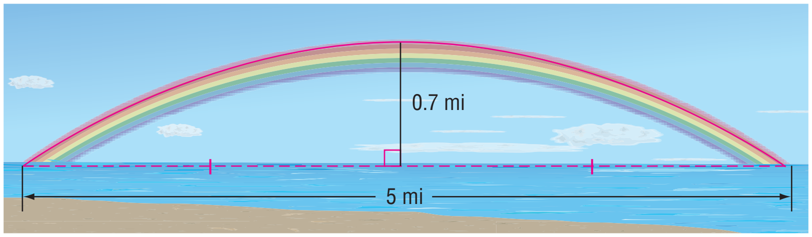
Real-WorldLink

The lower the Sun is to the horizon, the more of a rainbow you can see. At sunset, you could see a full semicircle of a rainbow with the top of the arch 42 degrees above the horizon.

Source: The National Center for Atmospheric Research

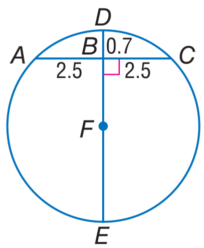
Real-World Example 2 Find Measures of Segments in Circles

SCIENCE The true shape of a rainbow is a complete circle. However, we see only the arc of the circle that appears above Earth's horizon. What is the radius of the circle containing the arc of the rainbow shown?



Understand You know that the rainbow's arc is part of a whole circle. $\overline{AC}$ is a chord of this circle, and $\overline{DB}$ is a perpendicular bisector of $\overline{AC}$.

Plan Draw a model. Since it bisects chord $\overline{AC}$, $\overline{DE}$ is a diameter of the circle. Use the products of the lengths of the intersecting chords to find the length of the diameter.

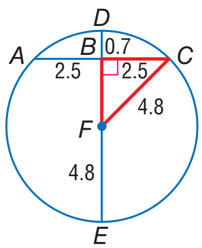


Solve

$AB \cdot BC = DB \cdot BE$	Theorem 10.15
$2.5 \cdot 2.5 = 0.7 \cdot BE$	Substitution
$6.25 = 0.7BE$	Multiply.
$8.9 \approx BE$	Divide each side by 0.7.
$DE = DB + BE$	Segment Addition Postulate
$= 0.7 + 8.9$	Substitution
$= 9.6$	Add.

Since the diameter of the circle is about 9.6 miles, the radius is about 9.6 ÷ 2 or 4.8 miles.

Check Use the Pythagorean Theorem to check the triangle in the circle formed by the radius, the chord, and part of the diameter.

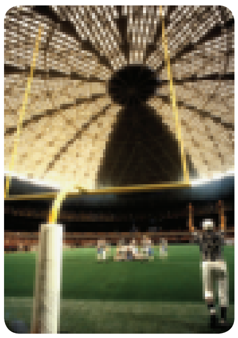


$DB + BF = DF$	Segment Addition Postulate
$0.7 + BF = 4.8$	Substitution
$BF = 4.1$	Subtract 0.7 from each side.
$BF^2 + BC^2 = CF^2$	Pythagorean Theorem
$4.1^2 + 2.5^2 \stackrel{?}{=} 4.8^2$	Substitution
$23.06 \approx 23.04$ ✓	Simplify.

Problem-SolvingTip

Make a Drawing When solving word problems involving circles, it is helpful to make a drawing and label all parts of the circle that are known. Use a variable to label the unknown measure.

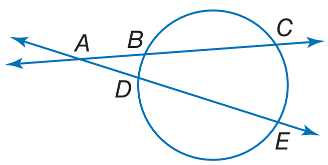
GuidedPractice



2. ASTRODOME The highest point, or apex, of the Astrodome is 208 feet high, and the diameter of the circle containing the arc is 710 feet. How long is the stadium from one side to the other?

2 Segments Intersecting Outside a Circle

A **secant segment** is a segment of a secant line that has exactly one endpoint on the circle. In the figure, $\overline{AC}$, $\overline{AB}$, $\overline{AE}$ and $\overline{AD}$ are secant segments.



A secant segment that lies in the exterior of the circle is called an **external secant segment**. In the figure, $\overline{AB}$ and $\overline{AD}$ are external secant segments.

A special relationship exists among secants and external secant segments.

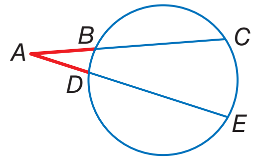
StudyTip

Simplify the Theorem Each side of the equation in Theorem 10.16 is the product of the lengths of the exterior part and the whole segment.

Theorem 10.16 Secant Segments Theorem

Words If two secants intersect in the exterior of a circle, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant and its external secant segment.

Example $AC \cdot AB = AE \cdot AD$



You will prove Theorem 10.16 in Exercise 24.

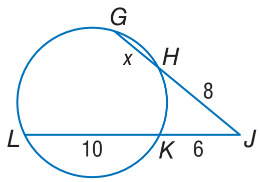
WatchOut!

Use the Correct Equation Be sure to multiply the length of the secant segment by the length of the external secant segment. Do not multiply the length of the internal secant segment, or chord, by the length of the external secant segment.

Example 3 Use the Intersection of Two Chords

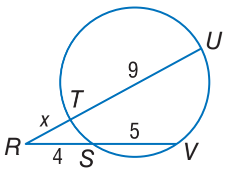
Find *x*.

$JG \cdot JH = JL \cdot JK$	Theorem 10.16
$(x + 8)8 = (10 + 6)6$	Substitution
$8x + 64 = 96$	Multiply.
$8x = 32$	Subtract 64 from each side.
$x = 4$	Divide each side by 8.

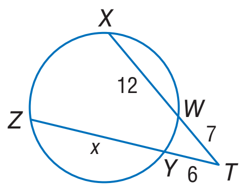


GuidedPractice

3A.



3B.

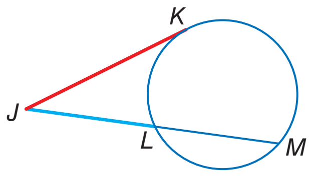


An equation similar to the one in Theorem 10.16 can be used when a secant and a tangent intersect outside a circle. In this case, the **tangent segment**, or segment of a tangent with one endpoint on the circle, is both the exterior and whole segment.

Theorem 10.17

Words If a tangent and a secant intersect in the exterior of a circle, then the square of the measure of the tangent is equal to the product of the measures of the secant and its external secant segment.

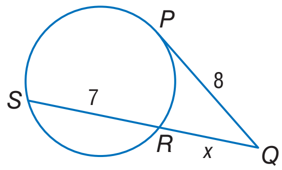
Example $JK^2 = JL \cdot JM$



You will prove Theorem 10.17 in Exercise 25.

Example 4 Use the Intersection of a Secant and a Tangent

$\overline{PQ}$ is tangent to the circle. Find x. Round to the nearest tenth.



$PQ^2 = QR \cdot QS$	Theorem 10.17
$8^2 = x(x + 7)$	Substitution
$64 = x^2 + 7x$	Multiply.
$0 = x^2 + 7x - 64$	Subtract 64 from each side.

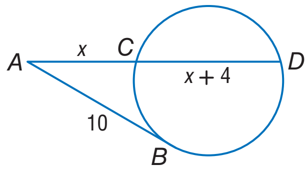
Since the expression is not factorable, use the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$= \frac{-7 \pm \sqrt{7^2 - 4(1)(-64)}}{2(1)}$	$a = 1$, $b = 7$, and $c = -64$
$= \frac{-7 \pm \sqrt{305}}{2}$	Simplify.
≈ 5.2 or -12.2	Use a calculator.

Since lengths cannot be negative, the value of x is about 5.2.

GuidedPractice

4. $\overline{AB}$ is tangent to the circle. Find x. Round to the nearest tenth.



Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Examples 1, 3 and 4

Find x. Assume that segments that appear to be tangent are tangent.

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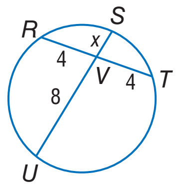

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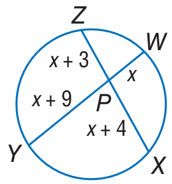

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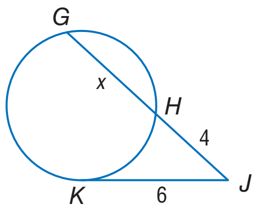

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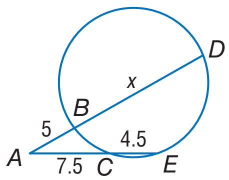
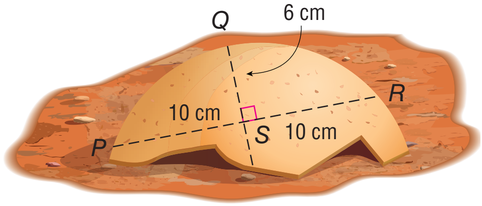


Example 2

5. **SCIENCE** A piece of broken pottery found at an archaeological site is shown. $\overline{QS}$ lies on a diameter of the circle. What was the circumference of the original pottery? Round to the nearest hundredth.

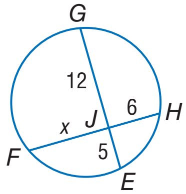


Practice and Problem Solving

Extra Practice is on page R10.

Examples 1, 3 and 4 **Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.**

6.



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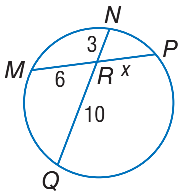

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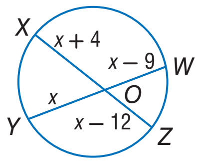

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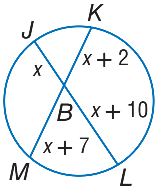

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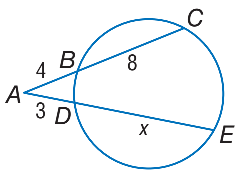

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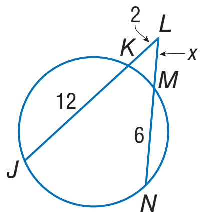

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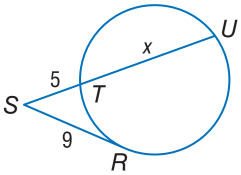

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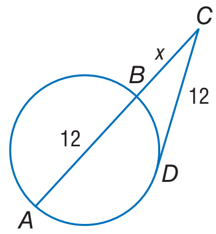
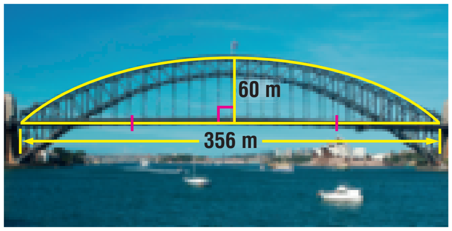
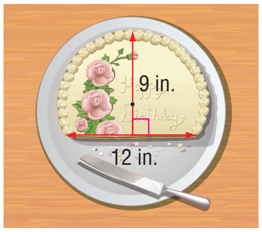


Example 2

14. BRIDGES What is the diameter of the circle containing the arc of the Sydney Harbour Bridge shown? Round to the nearest tenth.

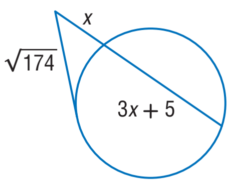


15. CAKES Sierra is serving cake at a party. If the dimensions of the remaining cake are shown below, what was the original diameter of the cake?



CCSS STRUCTURE **Find each variable to the nearest tenth. Assume that segments that appear to be tangent are tangent.**

16.



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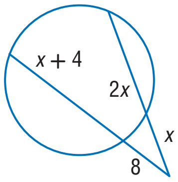

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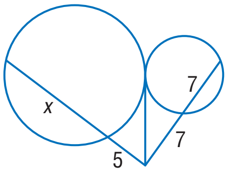

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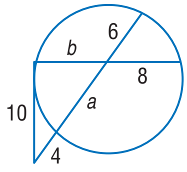

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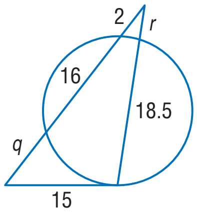

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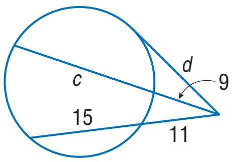


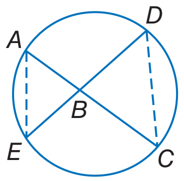
22. **INDIRECT MEASUREMENT** Gwendolyn is standing 16 feet from a giant sequoia tree and Chet is standing next to the tree, as shown. The distance between Gwendolyn and Chet is 27 feet. Draw a diagram of this situation, and then find the diameter of the tree.

PROOF **Prove each theorem.**

23. two-column proof of Theorem 10.15

Given: $\overline{AC}$ and $\overline{DE}$ intersect at B.

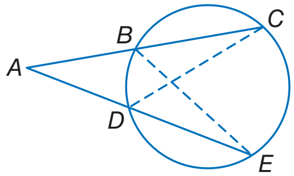
Prove: $AB \cdot BC = EB \cdot BD$



24. paragraph proof of Theorem 10.16

Given: Secants $\overline{AC}$ and $\overline{AE}$

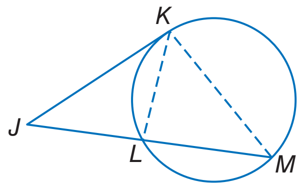
Prove: $AB \cdot AC = AD \cdot AE$



25. two-column proof of Theorem 10.17

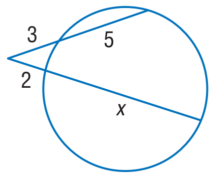
Given: tangent $\overline{JK}$,
secant $\overline{JM}$

Prove: $JK^2 = JL \cdot JM$



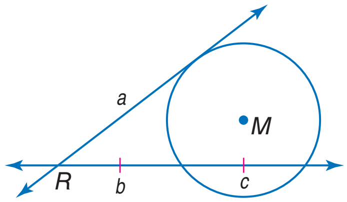
H.O.T. Problems Use Higher-Order Thinking Skills

26. CCSS **CRITIQUE** Tiffany and Jun are finding the value of x in the figure at the right. Tiffany wrote $3(5) = 2x$, and Jun wrote $3(8) = 2(2 + x)$. Is either of them correct? Explain your reasoning.



27. **WRITING IN MATH** Compare and contrast the methods for finding measures of segments when two secants intersect in the exterior of a circle and when a secant and a tangent intersect in the exterior of a circle.

28. **CHALLENGE** In the figure, a line tangent to circle M and a secant line intersect at R. Find a. Show the steps that you used.



29. **REASONING** When two chords intersect at the center of a circle, are the measures of the intercepting arcs *sometimes, always,* or *never* equal to each other?

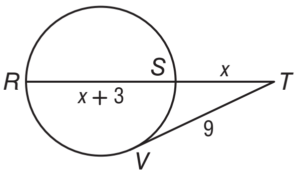
30. **OPEN ENDED** Investigate Theorem 10.17 by drawing and labeling a circle that has a secant and a tangent intersecting outside the circle. Measure and label the two parts of the secant segment to the nearest tenth of a centimeter. Use an equation to find the measure of the tangent segment. Verify your answer by measuring the segment.

31. **WRITING IN MATH** Describe the relationship among segments in a circle when two secants intersect inside a circle.

Standardized Test Practice

32. $\overline{TV}$ is tangent to the circle, and R and S are points on the circle. What is the value of x to the nearest tenth?

A 7.6 **C** 5.7
B 6.4 **D** 4.8



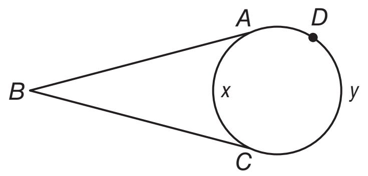
33. ALGEBRA A department store has all of its jewelry discounted 40%. It is having a sale that says you receive an additional 20% off the already discounted price. How much will you pay for a ring with an original price of $200?

F $80 **H** $120
G $96 **J** $140

34. EXTENDED RESPONSE The degree measures of minor arc $\widehat{AC}$ and major arc $\widehat{ADC}$ are x and y, respectively.

a. If $m\angle ABC = 70°$, write two equations relating x and y.

b. Find x and y.

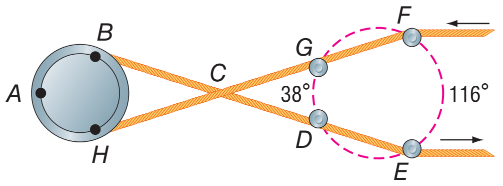


35. SAT/ACT During the first two weeks of summer vacation, Antonia earned $100 per week. During the next six weeks, she earned $150 per week. What was her average weekly pay?

A $50 **D** $135
B $112.50 **E** $137.50
C $125

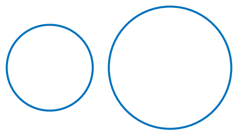
Spiral Review

36. WEAVING Once yarn is woven from wool fibers, it is often dyed and then threaded along a path of pulleys to dry. One set of pulleys is shown. Note that the yarn appears to intersect itself at C, but in reality it does not. Use the information from the diagram to find $m\widehat{BH}$. (Lesson 10-6)

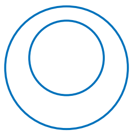


Copy the figure shown and draw the common tangents. If no common tangent exists, state *no common tangent*. (Lesson 10-5)

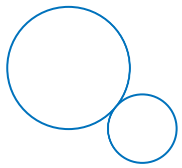
37.



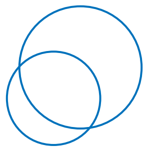
38.



39.



40.



COORDINATE GEOMETRY **Graph each figure and its image along the given vector.** (Lesson 9-2)

41. $\triangle KLM$ with vertices $K(5, -2)$, $L(-3, -1)$, and $M(0, 5)$; $\langle -3, -4 \rangle$

42. quadrilateral $PQRS$ with vertices $P(1, 4)$, $Q(-1, 4)$, $R(-2, -4)$, and $S(2, -4)$; $\langle -5, 3 \rangle$

43. $\triangle EFG$ with vertices $E(0, -4)$, $F(-4, -4)$, and $G(0, 2)$; $\langle 2, -1 \rangle$

Skills Review

Write an equation in slope-intercept form of the line having the given slope and y-intercept.

44. m: 3, y-intercept: -4

45. m: 2, $(0, 8)$

46. m: $\frac{5}{8}$, $(0, -6)$

47. m: $\frac{2}{9}$, y-intercept: $\frac{1}{3}$

48. m: -1, b: -3

49. m: $-\frac{1}{12}$, b: 1