LESSON 11-1

Areas of Parallelograms and Triangles

Then

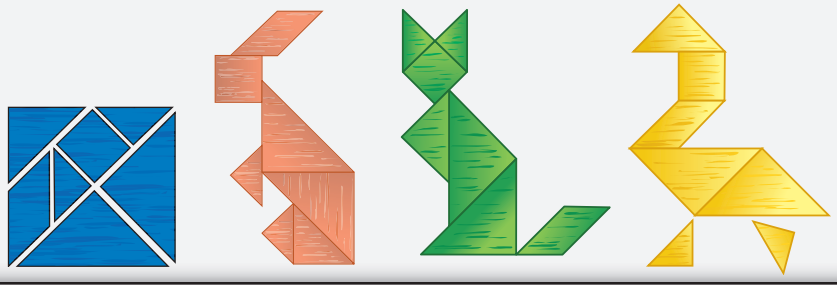
- You found areas of rectangles and squares.

Now

1. Find perimeters and areas of parallelograms.
2. Find perimeters and areas of triangles.

Why?

- A tangram is an ancient Chinese puzzle that can be rearranged to form different images, such as the animals shown. The area of the puzzle, before and after being rearranged, remains the same. It is the sum of all the areas of its pieces.



New Vocabulary

base of a parallelogram
height of a parallelogram
base of a triangle
height of a triangle

Common Core State Standards

Content Standards

G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

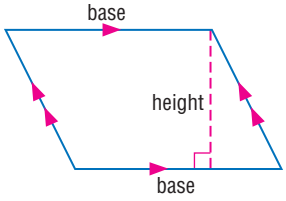
Mathematical Practices

1 Make sense of problems and persevere in solving them.

7 Look for and make use of structure.

1 Areas of Parallelograms

In Lesson 6-2, you learned that a *parallelogram* is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called the **base of a parallelogram**. The **height of a parallelogram** is the perpendicular distance between any two parallel bases.

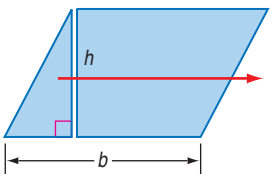


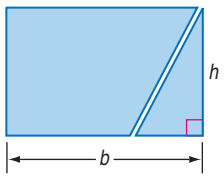
You can use the following postulate to develop the formula for the area of a parallelogram.

Postulate 11.1 Area Addition Postulate

The area of a region is the sum of the areas of its nonoverlapping parts.

In the figures below, a right triangle is cut off from one side of a parallelogram and translated to the other side as shown to form a rectangle with the same base and height.

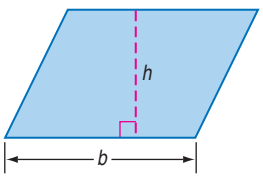




Recall from Lesson 1-6 that the area of a rectangle is the product of its base and height. By the Area Addition Postulate, a parallelogram with base b and height h has the same area as a rectangle with base b and height h.

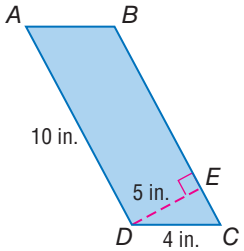
KeyConcept Area of a Parallelogram

Words	The area A of a parallelogram is the product of a base b and its corresponding height h.
Symbols	$A = bh$



Example 1 Perimeter and Area of a Parallelogram

Find the perimeter and area of $\square ABCD$.



Perimeter

Since opposite sides of a parallelogram are congruent, $\overline{AB} \cong \overline{DC}$ and $\overline{BC} \cong \overline{AD}$. So $AB = 4$ inches and $BC = 10$ inches.

Perimeter of $\square ABCD = AB + BC + DC + AD$
$= 4 + 10 + 4 + 10$ or 28 in.

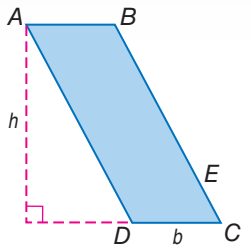
Area

The height given, DE, is 5 inches. $\overline{BC}$ is the base, which measures 10 inches.

$A = bh$ Area of a parallelogram
$= (10)(5)$ or 50 in^2 $b = 10$ and $h = 5$

StudyTip

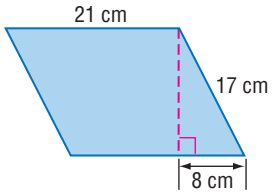
Heights of Figures
The height of a figure can be measured by extending a base. In Example 1, the height of $\square ABCD$ that corresponds to base $\overline{DC}$ can be measured by extending $\overline{DC}$.



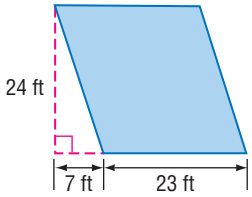
GuidedPractice

Find the perimeter and area of each parallelogram.

1A.



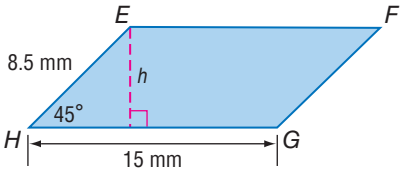
1B.



You may need to use trigonometry to find the area of a parallelogram.

Example 2 Area of a Parallelogram

Find the area of $\square EFGH$.



Step 1 Use a 45°-45°-90° triangle to find the height h of the parallelogram.

Recall that if the measure of the leg opposite the 45° angle is h, then the measure of the hypotenuse is $h\sqrt{2}$.

$h\sqrt{2} = 8.5$ Substitute 8.5 for the measure of the hypotenuse.
$h = \frac{8.5}{\sqrt{2}}$ or about 6 mm Divide each side by $\sqrt{2}$.

Step 2 Find the area.

$A = bh$ Area of a parallelogram
$\approx (15)(6)$ or 90 mm^2 $b = 15$ and $h \approx 6$

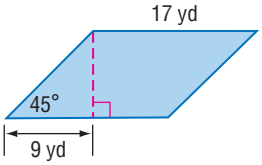
WatchOut!

CCSS Precision Remember that perimeter is measured in linear units such as inches and centimeters. Area is measured in square units such as square feet and square millimeters.

GuidedPractice

Find the area of each parallelogram. Round to the nearest tenth if necessary.

2A.



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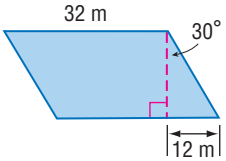
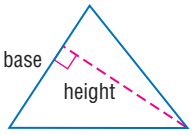


ReviewVocabulary

altitude of a triangle
a segment from a vertex of a triangle to the line containing the opposite side and perpendicular to the line containing that side

2 Areas of Triangles

Like the base of a parallelogram, the **base of a triangle** can be any side. The **height of a triangle** is the length of an altitude drawn to a given base.

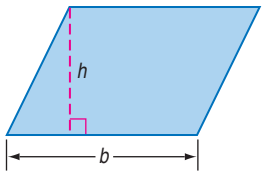


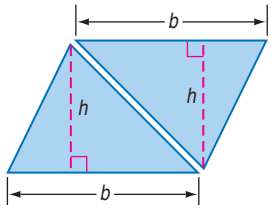
You can use the following postulate to develop the formula for the area of a triangle.

Postulate 11.2 Area Congruence Postulate

If two figures are congruent, then they have the same area.

In the figures below, a parallelogram is cut in half along a diagonal to form two congruent triangles with the same base and height.



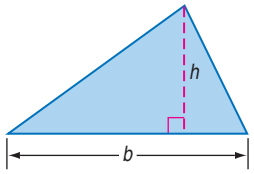


By the Area Congruence Postulate, the two congruent triangles have the same area. So, one triangle with base b and height h has half the area of a parallelogram with base b and height h.

KeyConcept Area of a Triangle

Words The area A of a triangle is one half the product of a base b and its corresponding height h.

Symbols $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

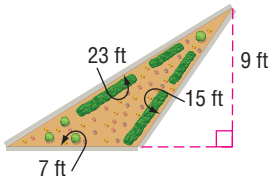


Real-WorldLink
Triangular gardens can serve as focal points in landscaping or simply result from intersecting walkways.

Janet Johnson/Gap Photos/age fotostock

Real-World Example 3 Perimeter and Area of a Triangle

GARDENING **D'Andre needs enough mulch to cover the triangular garden shown and enough paving stones to border it. If one bag of mulch covers 12 square feet and one paving stone provides a 4-inch border, how many bags of mulch and how many stones does he need to buy?**



Step 1 Find the perimeter of the garden.

Perimeter of garden = 23 + 15 + 7 or 45 ft

Step 2 Find the area of the garden.

$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(7)(9)$ or 31.5 ft^2 $b = 7$ and $h = 9$

Step 3 Use unit analysis to determine how many of each item are needed.

Bags of Mulch

$$31.5 \text{ ft}^2 \cdot \frac{1 \text{ bag}}{12 \text{ ft}^2} = 2.625 \text{ bags}$$

Paving Stones

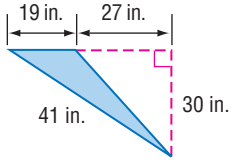
$$45 \text{ ft} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} \cdot \frac{1 \text{ stone}}{4 \text{ in.}} = 135 \text{ stones}$$

Round the number of bags up so there is enough mulch. He will need 3 bags of mulch and 135 paving stones.

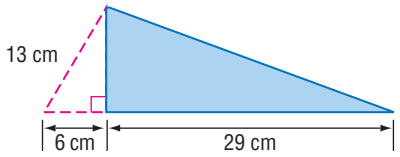
GuidedPractice

Find the perimeter and area of each triangle.

3A.



3B.



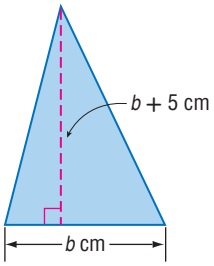
You can use algebra to solve for unknown measures in parallelograms and triangles.

Example 4 Use Area to Find Missing Measures

ALGEBRA The height of a triangle is 5 centimeters more than its base. The area of the triangle is 52 square centimeters. Find the base and height.

Step 1 Write expressions to represent each measure.

Let b represent the base of the triangle. Then the height is $b + 5$.



Step 2 Use the formula for the area of a triangle to find b.

$A = \frac{1}{2}bh$	Area of a triangle
$52 = \frac{1}{2}b(b + 5)$	Replace A with 52 and h with $b + 5$.
$104 = b(b + 5)$	Multiply each side by 2.
$104 = b^2 + 5b$	Distributive Property
$0 = b^2 + 5b - 104$	Subtract 104 from each side.
$0 = (b + 13)(b - 8)$	Factor.
$b + 13 = 0$ and $b - 8 = 0$	Zero Product Property
$b = -13$ $\quad b = 8$	Solve for b.

StudyTip

Zero Product Property
If the product of two factors is 0, then at least one of the factors must be 0.

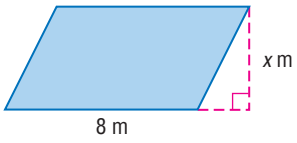
Step 3 Use the expressions from Step 1 to find each measure.

Since a length cannot be negative, the base measures 8 centimeters and the height measures 8 + 5 or 13 centimeters.

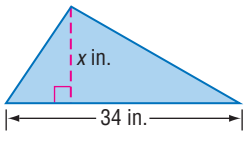
GuidedPractice

ALGEBRA Find x.

4A. $A = 148 \text{ m}^2$



4B. $A = 357 \text{ in}^2$



4C. ALGEBRA The base of a parallelogram is twice its height. If the area of the parallelogram is 72 square feet, find its base and height.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Examples 1–3 Find the perimeter and area of each parallelogram or triangle. Round to the nearest tenth if necessary.

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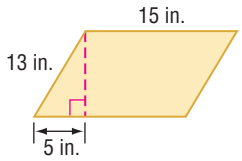

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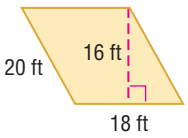

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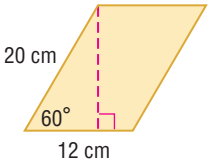

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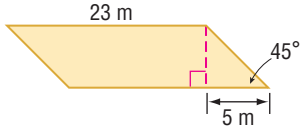

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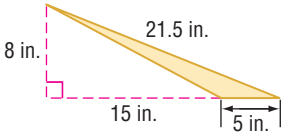

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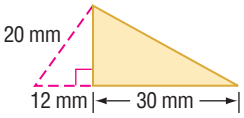
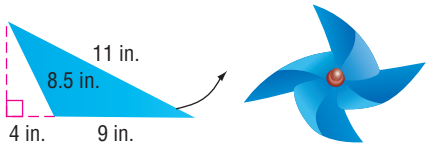
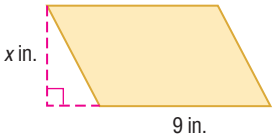


7. CRAFTS Marquez and Victoria are making pinwheels. Each pinwheel is composed of 4 triangles with the dimensions shown. Find the perimeter and area of one triangle.

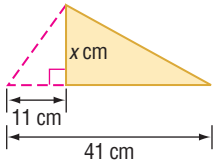


Example 4 Find x.

8. $A = 153$ in^2



9. $A = 165$ cm^2



Practice and Problem Solving

Extra Practice is on page R11.

Examples 1–3 CCSS **STRUCTURE** Find the perimeter and area of each parallelogram or triangle. Round to the nearest tenth if necessary.

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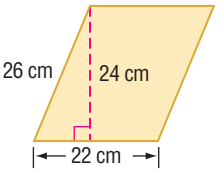

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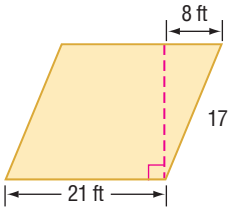

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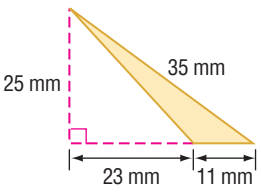

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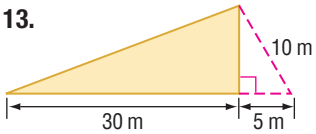

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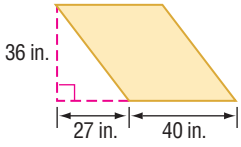

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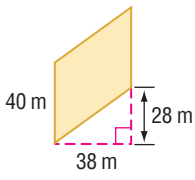
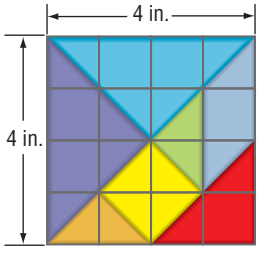


16. TANGRAMS The tangram shown is a 4-inch square.

a. Find the perimeter and area of the purple triangle. Round to the nearest tenth.

b. Find the perimeter and area of the blue parallelogram. Round to the nearest tenth.



Example 2 **STRUCTURE** Find the area of each parallelogram. Round to the nearest tenth if necessary.

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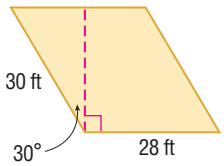

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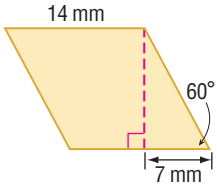

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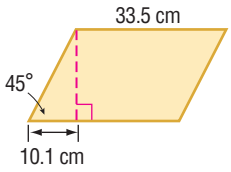

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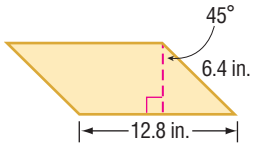

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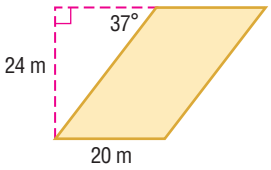

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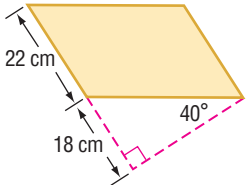
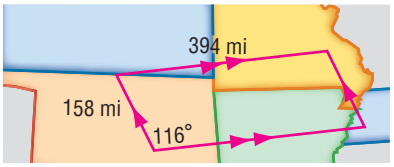


23 **WEATHER** Tornado watch areas are often shown on weather maps using parallelograms. What is the area of the region affected by the tornado watch shown? Round to the nearest square mile.



Example 4

24. The height of a parallelogram is 4 millimeters more than its base. If the area of the parallelogram is 221 square millimeters, find its base and height.

25. The height of a parallelogram is one fourth of its base. If the area of the parallelogram is 36 square centimeters, find its base and height.

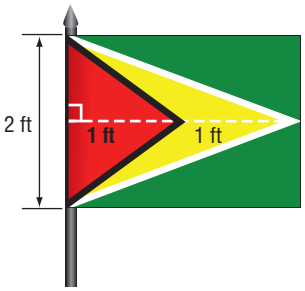
26. The base of a triangle is twice its height. If the area of the triangle is 49 square feet, find its base and height.

27. The height of a triangle is 3 meters less than its base. If the area of the triangle is 44 square meters, find its base and height.

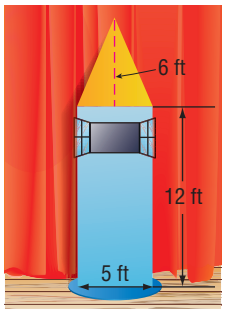
28. **FLAGS** Omar wants to make a replica of Guyana's national flag.

a. What is the area of the piece of fabric he will need for the red region? for the yellow region?

b. If the fabric costs $3.99 per square yard for each color and he buys exactly the amount of fabric he needs, how much will it cost to make the flag?

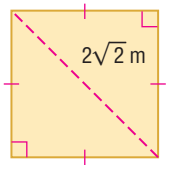


29. **DRAMA** Madison is in charge of the set design for her high school's rendition of *Romeo and Juliet*. One pint of paint covers 80 square feet. How many pints will she need of each color if the roof and tower each need 3 coats of paint?



Find the perimeter and area of each figure. Round to the nearest hundredth, if necessary.

30.



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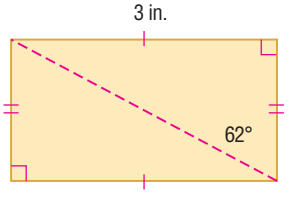

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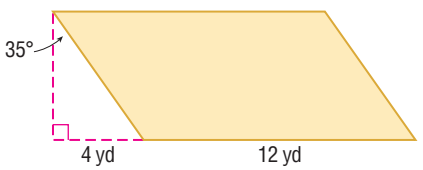


COORDINATE GEOMETRY **Find the area of each figure. Explain the method that you used.**

33 $\square ABCD$ with $A(4, 7)$, $B(2, 1)$, $C(8, 1)$, and $D(10, 7)$

34. $\triangle RST$ with $R(-8, -2)$, $S(-2, -2)$, and $T(-3, -7)$

35. HERON'S FORMULA Heron's Formula relates the lengths of the sides of a triangle to the area of the triangle. The formula is $A = \sqrt{s(s-a)(s-b)(s-c)}$, where s is the *semiperimeter*, or one half the perimeter, of the triangle and a, b, and c are the side lengths.

a. Use Heron's Formula to find the area of a triangle with side lengths 7, 10, and 4.

b. Show that the areas found for a 5-12-13 right triangle are the same using Heron's Formula and using the triangle area formula you learned earlier in this lesson.

36. MULTIPLE REPRESENTATIONS In this problem, you will investigate the relationship between the area and perimeter of a rectangle.

a. Algebraic A rectangle has a perimeter of 12 units. If the length of the rectangle is x and the width of the rectangle is y, write equations for the perimeter and area of the rectangle.

b. Tabular Tabulate all possible whole-number values for the length and width of the rectangle, and find the area for each pair.

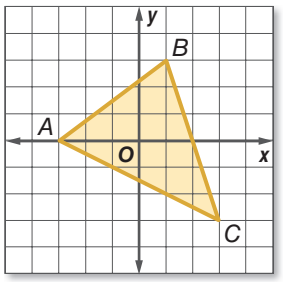
c. Graphical Graph the area of the rectangle with respect to its length.

d. Verbal Describe how the area of the rectangle changes as its length changes.

e. Analytical For what whole-number values of length and width will the area be greatest? least? Explain your reasoning.

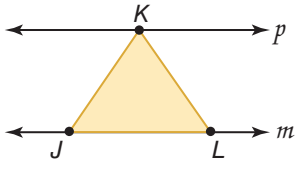
H.O.T. Problems Use Higher-Order Thinking Skills

37. CHALLENGE Find the area of $\triangle ABC$ graphed at the right. Explain your method.



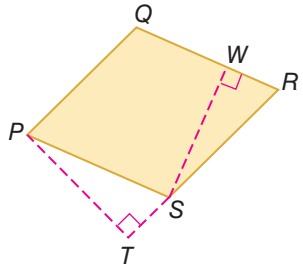
38. CCSS ARGUMENTS Will the perimeter of a nonrectangular parallelogram *always*, *sometimes*, or *never* be greater than the perimeter of a rectangle with the same area and the same height? Explain.

39. WRITING IN MATH Points J and L lie on line m. Point K lies on line p. If lines m and p are parallel, describe how the area of $\triangle JKL$ will change as K moves along line p.



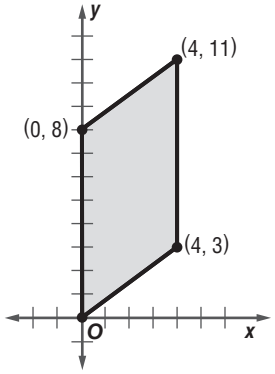
40. OPEN ENDED The area of a polygon is 35 square units. The height is 7 units. Draw three different triangles and three different parallelograms that meet these requirements. Label the base and height on each.

41. WRITING IN MATH Describe two different ways you could use measurement to find the area of parallelogram $PQRS$.



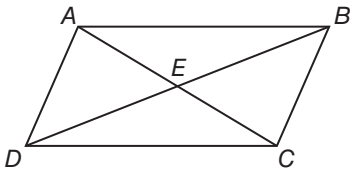
Standardized Test Practice

42. What is the area, in square units, of the parallelogram shown?

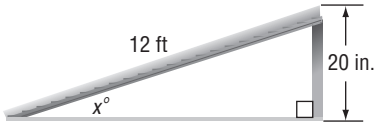


A 12 **C** 32
B 20 **D** 40

43. GRIDDED RESPONSE In parallelogram $ABCD$, $\overline{BD}$ and $\overline{AC}$ intersect at E. If $AE = 9$, $BE = 3x - 7$, and $DE = x + 5$, find x.



44. A wheelchair ramp is built that is 20 inches high and has a length of 12 feet as shown. What is the measure of the angle x that the ramp makes with the ground, to the *nearest* degree?



Note: Not drawn to scale.

F 8 **H** 37
G 16 **J** 53

45. SAT/ACT The formula for converting a Celsius temperature to a Fahrenheit temperature is $F = \frac{9}{5}C + 32$, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius. Which of the following is the Celsius equivalent to a temperature of 86° Fahrenheit?

A 15.7° C **D** 122.8° C
B 30° C **E** 186.8° C
C 65.5° C

Spiral Review

Write the equation of each circle. (Lesson 10-8)

46. center at origin, $r = 3$

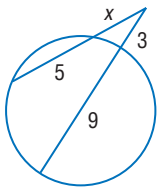
47. center at origin, $d = 12$

48. center at $(-3, -10)$, $d = 24$

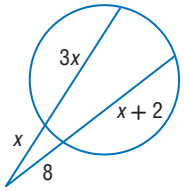
49. center at $(1, -4)$, $r = \sqrt{17}$

Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent. (Lesson 10-7)

50.



51.



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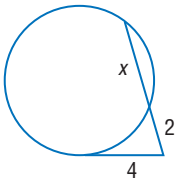
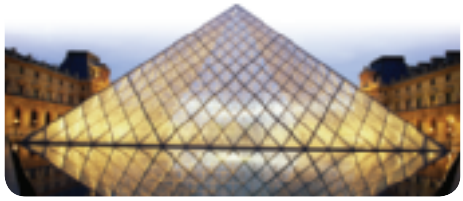


53. ARCHITECTURE The Louvre Pyramid is the main entrance to the Louvre Museum in Paris, France. The structure consists mainly of quadrilateral-shaped glass segments, as shown in the photo at the right. Describe one method that could be used to prove that the shapes of the segments are parallelograms. (Lesson 6-3)



Cosmo Condina/Stone/Getty Images

Skills Review

Evaluate each expression if $a = 2$, $b = 6$, and $c = 3$.

54. $\frac{1}{2}ac$ **55.** $\frac{1}{2}cb$ **56.** $\frac{1}{2}b(2a + c)$ **57.** $\frac{1}{2}c(b + a)$ **58.** $\frac{1}{2}a(2c + b)$