LESSON 11-2

Areas of Trapezoids, Rhombi, and Kites

Then
- You found areas of triangles and parallelograms.

Now
1. Find areas of trapezoids.
2. Find areas of rhombi and kites.

Why?
- Brianna has turned her hobby of making designer handbags and totes into a small business. Among her designs is a trapezoid-shaped handbag. To estimate the amount of material needed to produce each handbag, she needs to calculate the area of a trapezoid.

New Vocabulary
height of a trapezoid

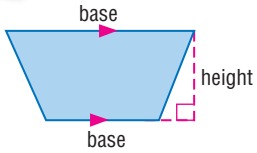
Common Core State Standards

Content Standards
G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

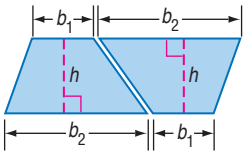
Mathematical Practices
1 Make sense of problems and persevere in solving them.
7 Look for and make use of structure.

1 Areas of Trapezoids

In Lesson 6-6, you learned that a *trapezoid* is a quadrilateral with exactly one pair of parallel sides. These parallel sides are called *bases*. The **height of a trapezoid** is the perpendicular distance between its bases.



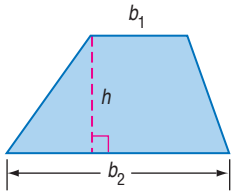
In the figure below, a glide reflection of the first trapezoid results in two congruent trapezoids that fit together to form a parallelogram.



The area of the parallelogram is the product of the height h and the sum of the two bases, b_1 and b_2. The area of one trapezoid is one half the area of the parallelogram.

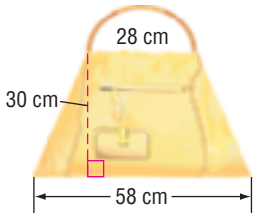
KeyConcept Area of a Trapezoid

Words	The area A of a trapezoid is one half the product of the height h and the sum of its bases, b_1 and b_2.
Symbols	$A = \frac{1}{2}h(b_1 + b_2)$



Real-World Example 1 Area of a Trapezoid

CRAFTS **One of Brianna's trapezoid-shaped totes is shown. Find the amount of material used to make the side shown.**



$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

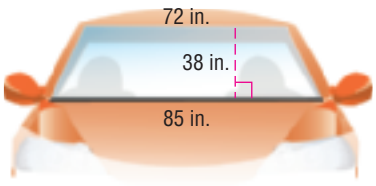
$= \frac{1}{2}(30)(28 + 58)$ $h = 30$, $b_1 = 28$, $b_2 = 58$

$= 1290$ Simplify.

The tote requires 1290 square centimeters.

GuidedPractice

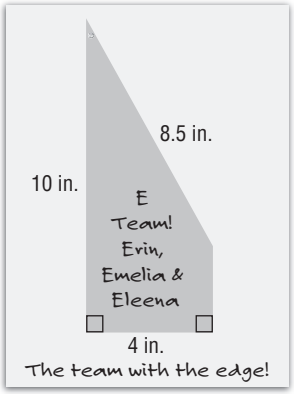
1. AUTOMOBILES Find the area of glass used to make the windshield of a van shown at the right.



Juice Images/age fotostock

Standardized Test Example 2 Area of a Trapezoid

SHORT RESPONSE **Emelia designed the pennant shown for her team. Find the area of the shaded portion of her team's pennant.**

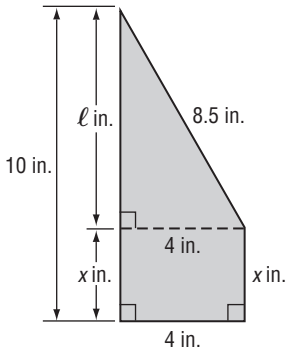


Read the Test Item

You are given a trapezoid with one base measuring 10 inches, a height of 4 inches, and a third side measuring 8.5 inches. To find the area of the trapezoid, first find the measure of the other base.

Solve the Test Item

Draw the segment shown to form a right triangle and a rectangle. The triangle has a hypotenuse of 8.5 inches and legs of 4 and ℓ inches. The rectangle has a length of 4 inches and a width of x inches.



Use the Pythagorean Theorem to find ℓ.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$\ell^2 + 4^2 = 8.5^2$ $a = \ell$, $b = 4$, and $c = 8.5$

$\ell^2 + 16 = 72.25$ Simplify.

$\ell^2 = 56.25$ Subtract 16 from each side.

$\ell = 7.5$ Take the positive square root of each side.

By Segment Addition, $\ell + x = 10$. So, $7.5 + x = 10$ and $x = 2.5$. The width of the rectangle is also the measure of the second base of the trapezoid.

$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$= \frac{1}{2}(4)(10 + 2.5)$ $h = 4$, $b_1 = 10$, and $b_2 = 2.5$

$= 25$ Simplify.

So the pennant has an area of 25 square inches.

CHECK The area of the trapezoid is the sum of the areas of the right triangle and rectangle. The area of the triangle is $\frac{1}{2}(4)(7.5)$ or 15 square inches. The area of the rectangle is (4)(2.5) or 10 square inches. So the area of the trapezoid is 15 + 10 or 25 square inches. ✓

Test-TakingTip

Separating Figures To solve some area problems, you need to draw in parallel and/or perpendicular lines to find information not provided.

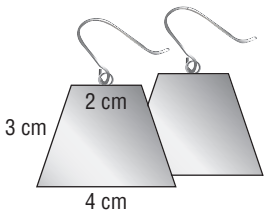
Real-WorldCareer

Craft Artist Craft artists create their art by hand to sell or exhibit. They work with a wide variety of materials including textiles, woods, metal, and ceramics.

Most artists receive some type of postsecondary training, and about 63% are self-employed. Craft artists make up about 3% of all artists.

GuidedPractice

2. SHORT RESPONSE Owen designed the silver earrings shown that are shaped like isosceles trapezoids. What is the area of each earring?

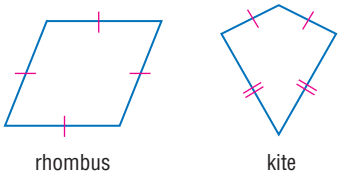


Review Vocabulary

diagonal a segment that connects any two nonconsecutive vertices in a polygon

2 Areas of Rhombi and Kites

Recall from Lessons 6-5 and 6-6 that a *rhombus* is a parallelogram with all four sides congruent and a *kite* is a quadrilateral with exactly two pairs of consecutive congruent sides.

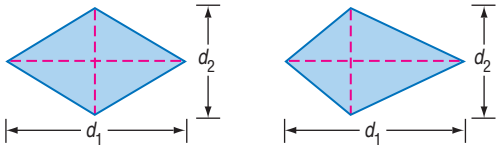


The areas of rhombi and kites are related to the lengths of their diagonals.

KeyConcept Area of a Rhombus or Kite

Words The area A of a rhombus or kite is one half the product of the lengths of its diagonals, d_1 and d_2.

Symbols $A = \frac{1}{2}d_1d_2$

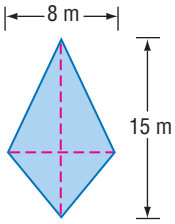


You will derive the formulas for the area of a kite and the area of a rhombus in Exercises 23 and 24.

Example 3 Area of a Rhombus and a Kite

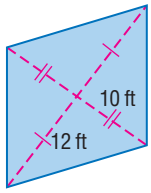
Find the area of each rhombus or kite.

a.



$$A = \frac{1}{2}d_1d_2 \quad \text{Area of a kite}$$
$$= \frac{1}{2}(8)(15) \quad d_1 = 8 \text{ and } d_2 = 15$$
$$= 60 \text{ m}^2 \quad \text{Simplify.}$$

b.



Step 1 Find the length of each diagonal.

Since the diagonals of a rhombus bisect each other, then lengths of the diagonals are 12 + 12 or 24 feet and 10 + 10 or 20 feet.

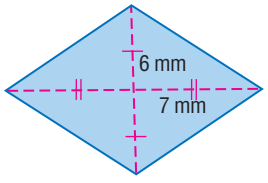
Step 2 Find the area of the rhombus.

$$A = \frac{1}{2}d_1d_2 \quad \text{Area of a rhombus}$$
$$= \frac{1}{2}(24)(20) \quad d_1 = 24 \text{ and } d_2 = 20$$
$$= 240 \text{ ft}^2 \quad \text{Simplify.}$$

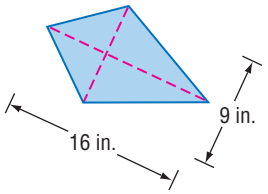
GuidedPractice

Find the area of each rhombus or kite.

3A.



3B.



Apic/Hulton Archive/Getty Images

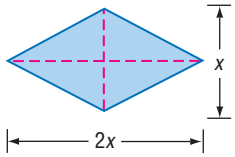
Math History Link

Heron of Alexandria (c. 10–70 A.D.) Heron was a mathematician and engineer in Roman Egypt. He developed a formula for finding the area of a triangle if the lengths of the sides are known.

You can use algebra to solve for unknown measures in trapezoids, rhombi, and kites.

Example 4 Use Area to Find Missing Measures

ALGEBRA **One diagonal of a rhombus is twice as long as the other diagonal. If the area of the rhombus is 169 square millimeters, what are the lengths of the diagonals?**



Step 1 Write an expression to represent each measure.

Let x represent the length of one diagonal. Then the length of the other diagonal is $2x$.

Step 2 Use the formula for the area of a rhombus to find x.

$A = \frac{1}{2}d_1d_2$ Area of a rhombus

$169 = \frac{1}{2}(x)(2x)$ $A = 169$, $d_1 = x$, and $d_2 = 2x$

$169 = x^2$ Simplify.

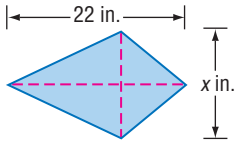
$13 = x$ Take the positive square root of each side.

So the lengths of the diagonals are 13 millimeters and 2(13) or 26 millimeters.

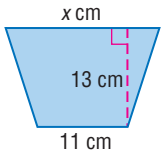
GuidedPractice

ALGEBRA **Find x.**

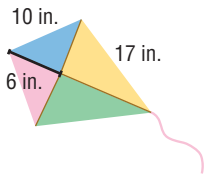
4A. $A = 92$ in^2



4B. $A = 177$ cm^2



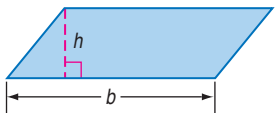
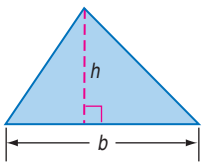
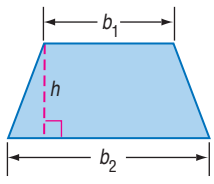
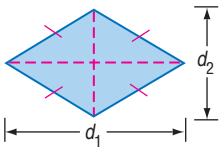
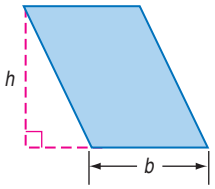
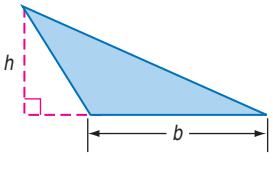
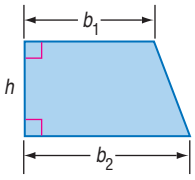
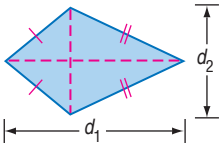
4C. **ALGEBRA** What is the area of the kite shown?



StudyTip

Kites Recall from Lesson 6-6 that the diagonals of kites are perpendicular.

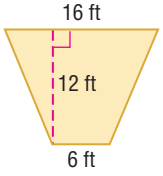
ConceptSummary Areas of Polygons

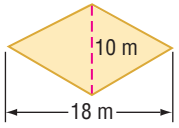
Parallelogram	Triangles	Trapezoids	Rhombi and Kites
			
			
$A = bh$	$A = \frac{1}{2}bh$	$A = \frac{1}{2}h(b_1 + b_2)$	$A = \frac{1}{2}d_1d_2$

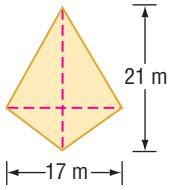
Check Your Understanding

= Step-by-Step Solutions begin on page R14.

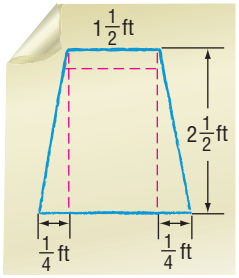
Examples 1–3 Find the area of each trapezoid, rhombus, or kite.

1. 

2. 

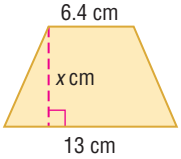
3. 

4. **SHORT RESPONSE** Suki is doing fashion design at 4-H Club. Her first project is to make a simple A-line skirt. How much fabric will she need according to the design at the right?

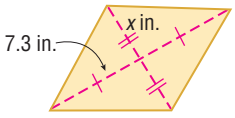


Example 4 **ALGEBRA** Find x.

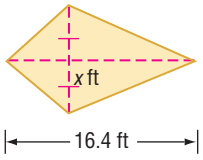
5. $A = 78 \text{ cm}^2$



6. $A = 96 \text{ in}^2$



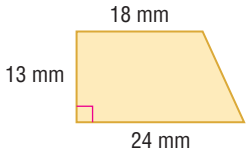
7. $A = 104 \text{ ft}^2$

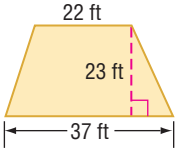


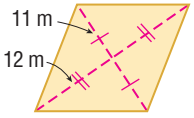
Practice and Problem Solving

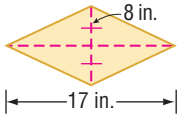
Extra Practice is on page R11.

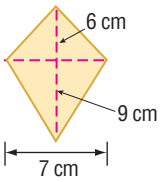
Examples 1–3 CCSS **STRUCTURE** Find the area of each trapezoid, rhombus, or kite.

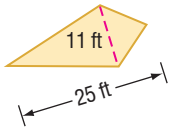
8. 

9. 

10. 

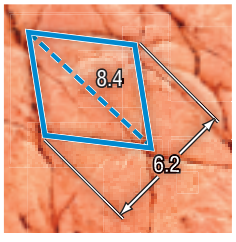
11. 

12. 

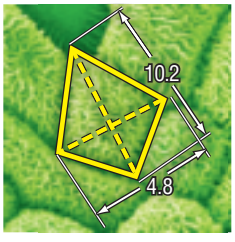
13. 

MICROSCOPES **Find the area of the identified portion of each magnified image. Assume that the identified portion is either a trapezoid, rhombus, or kite. Measures are provided in microns.**

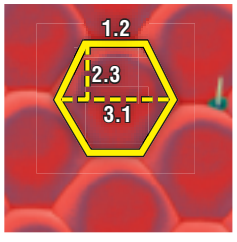
14. human skin



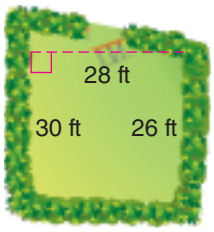
15. heartleaf plant



16. eye of a fly



17. JOBS Jimmy works on his neighbors' yards after school to earn extra money to buy a car. He is going to plant grass seed in Mr. Troyer's yard. What is the area of the yard?



Example 4 **ALGEBRA Find each missing length.**

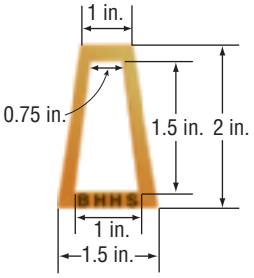
18. One diagonal of a kite is twice as long as the other diagonal. If the area of the kite is 240 square inches, what are the lengths of the diagonals?

19 The area of a rhombus is 168 square centimeters. If one diagonal is three times as long as the other, what are the lengths of the diagonals?

20. A trapezoid has base lengths of 12 and 14 feet with an area of 322 square feet. What is the height of the trapezoid?

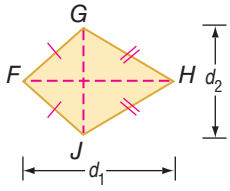
21. A trapezoid has a height of 8 meters, a base length of 12 meters, and an area of 64 square meters. What is the length of the other base?

22. HONORS Estella has been asked to join an honor society at school. Before the first meeting, new members are asked to sand and stain the front side of a piece of wood in the shape of an isosceles trapezoid. What is the surface area that Estella will need to sand and stain?

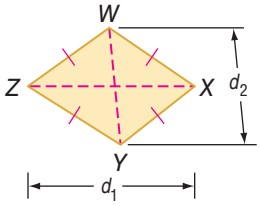


For each figure, provide a justification showing that $A = \frac{1}{2}d_1d_2$.

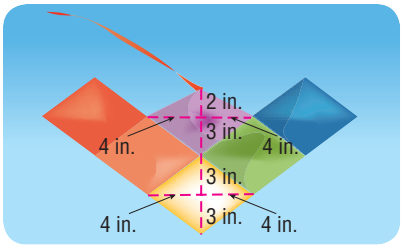
23.



24.



25. CRAFTS Ashanti is in a kite competition. The yellow, red, orange, green, and blue pieces of her kite design shown are congruent rhombi.



a. How much fabric of each color does she need to buy?

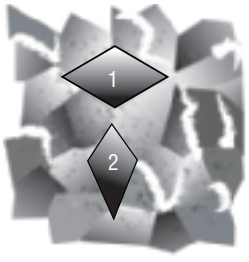
b. Competition rules require that the total area of each kite be no greater than 200 square inches. Does Ashanti's kite meet this requirement? Explain.

CCSS **SENSE-MAKING Find the area of each quadrilateral with the given vertices.**

26. $A(-8, 6)$, $B(-5, 8)$, $C(-2, 6)$, and $D(-5, 0)$

27. $W(3, 0)$, $X(0, 3)$, $Y(-3, 0)$, and $Z(0, -3)$

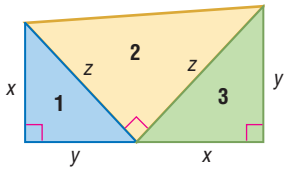
28. METALS When magnified in very powerful microscopes, some metals are composed of grains that have various polygonal shapes.



a. What is the area of figure 1 if the grain has a height of 4 microns and bases with lengths of 5 and 6 microns?

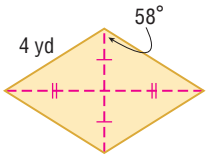
b. If figure 2 has perpendicular diagonal lengths of 3.8 microns and 4.9 microns, what is the area of the grain?

29. PROOF The figure at the right is a trapezoid that consists of two congruent right triangles and an isosceles triangle. In 1876, James A. Garfield, the 20th president of the United States, discovered a proof of the Pythagorean Theorem using this diagram. Prove that $x^2 + y^2 = z^2$.

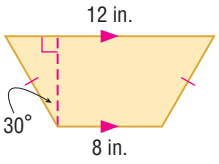


DIMENSIONAL ANALYSIS **Find the perimeter and area of each figure in feet. Round to the nearest tenth, if necessary.**

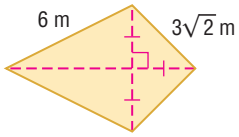
30.



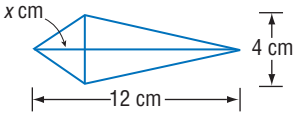
31



32.



33. MULTIPLE REPRESENTATIONS In this problem, you will investigate perimeters of kites.



a. Geometric Draw a kite like the one shown if $x = 2$.

b. Geometric Repeat the process in part **a** for three x-values between 2 and 10 and for an x-value of 10.

c. Tabular Measure and record in a table the perimeter of each kite, along with the x-value.

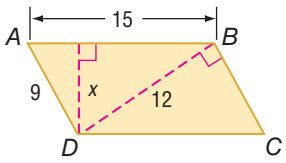
d. Graphical Graph the perimeter versus the x-value using the data from your table.

e. Analytical Make a conjecture about the value of x that will minimize the perimeter of the kite. What is the significance of this value?

H.O.T. Problems Use Higher-Order Thinking Skills

34. CCSS CRITIQUE Antonio and Madeline want to draw a trapezoid that has a height of 4 units and an area of 18 square units. Antonio says that only one trapezoid will meet the criteria. Madeline disagrees and thinks that she can draw several different trapezoids with a height of 4 units and an area of 18 square units. Is either of them correct? Explain your reasoning.

35. CHALLENGE Find x in parallelogram $ABCD$.



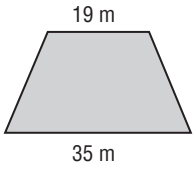
36. OPEN ENDED Draw a kite and a rhombus with an area of 6 square inches. Label and justify your drawings.

37. REASONING If the areas of two rhombi are equal, are the perimeters *sometimes*, *always*, or *never* equal? Explain.

38. WRITING IN MATH How can you use trigonometry to find the area of a figure?

Standardized Test Practice

39. The lengths of the bases of an isosceles trapezoid are shown below.



If the perimeter is 74 meters, what is its area?

A 162 m^2 **C** 332.5 m^2
B 270 m^2 **D** 342.25 m^2

40. SHORT RESPONSE One diagonal of a rhombus is three times as long as the other diagonal. If the area of the rhombus is 54 square millimeters, what are the lengths of the diagonals?

41. ALGEBRA What is the effect on the graph of the equation $y = \frac{1}{2}x$ when the equation is changed to $y = -2x$?

F The graph is moved 1 unit down.
G The graph is moved 1 unit up.
H The graph is rotated 45° about the origin.
J The graph is rotated 90° about the origin.

42. A regular hexagon is divided into 6 congruent triangles. If the perimeter of the hexagon is 48 centimeters, what is the height of each triangle?

A 4 cm **C** $6\sqrt{3}$ cm **E** $8\sqrt{3}$ cm
B $4\sqrt{3}$ cm **D** 8 cm

Spiral Review

COORDINATE GEOMETRY **Find the area of each figure.** (Lesson 11-1)

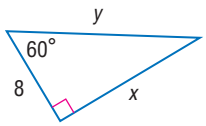
43. $\triangle JKL$ with $J(-4, 3)$, $K(-9, -1)$, and $L(-4, -4)$

44. $\square RSTV$ with $R(-5, 7)$, $S(2, 7)$, $T(0, 2)$, and $V(-7, 2)$

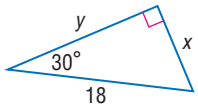
45. WEATHER Meteorologists track severe storms using Doppler radar. A polar grid is used to measure distances as the storms progress. If the center of the radar screen is the origin and each ring is 10 miles farther from the center, what is the equation of the fourth ring? (Lesson 10-8)

Find x and y. (Lesson 8-3)

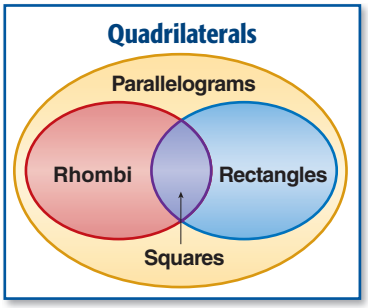
46.



47.



Use the Venn diagram to determine whether each statement is *always*, *sometimes*, or *never* true. (Lesson 6-5)



48. A parallelogram is a square.

49. A square is a rhombus.

50. A rectangle is a parallelogram.

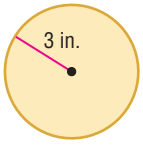
51. A rhombus is a rectangle but not a square.

52. A rhombus is a square.

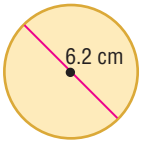
Skills Review

Find the circumference and area of each figure. Round to the nearest tenth.

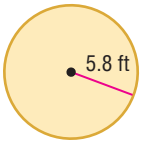
53.



54.



55.



NOAA