Areas of Circles and Sectors

∷Then	∷Now	∷Why?
● You found the circumference of a circle.	**1** Find areas of circles. **2** Find areas of sectors of circles.	● To determine whether a medium or large pizza is a better value, you can compare the cost per square inch. Divide the cost of each pizza by its area.

NewVocabulary
sector of a circle
segment of a circle

**Common Core State Standards**

Content Standards
G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

Mathematical Practices
1 Make sense of problems and persevere in solving them.
6 Attend to precision.

1 Areas of Circles In Lesson 10-1, you learned that the formula for the circumference C of a circle with radius r is given by $C = 2\pi r$. You can use this formula to develop the formula for the area of a circle.
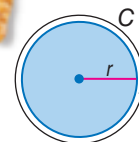
Below, a circle with radius r and circumference C has been divided into congruent pieces and then rearranged to form a figure that resembles a parallelogram.
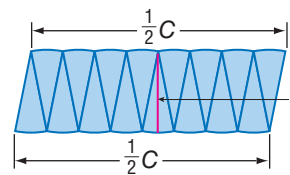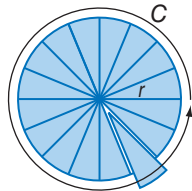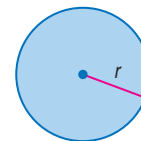
As the number of congruent pieces increases, the rearranged figure more closely approaches a parallelogram. The base of the parallelogram is $\frac{1}{2}C$ and the height is r, so its area is $\frac{1}{2}C \cdot r$. Since $C = 2\pi r$, the area of the parallelogram is also $\frac{1}{2}(2\pi r)r$ or πr^2.

> **Key**Concept **Area of a Circle**
>
> **Words** The area A of a circle is equal to π times the square of the radius r.
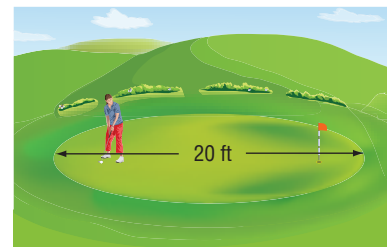>
> **Symbols** $A = \pi r^2$

Real-World Example 1 Area of a Circle

SPORTS What is the area of the circular putting green shown to the nearest square foot?

The diameter is 20 feet, so the radius is 10 feet.

$$A = \pi r^2 \qquad \text{Area of a circle}$$
$$= \pi(10)^2 \qquad r = 10$$
$$\approx 314 \qquad \text{Use a calculator.}$$

So, the area is about 314 square feet.

▶ **Guided**Practice

1. **SPORTS** An archery target has a radius of 12 inches. What is the area of the target to the nearest square inch?

Example 2 Use the Area of a Circle to Find a Missing Measure

ALGEBRA **Find the radius of a circle with an area of 95 square centimeters.**

$A = \pi r^2$ Area of a circle

$95 = \pi r^2$ $A = 95$

$\dfrac{95}{\pi} = r^2$ Divide each side by π.

$5.5 \approx r$ Use a calculator. Take the positive square root of each side.

The radius of the circle is about 5.5 centimeters.

▶ GuidedPractice

2. ALGEBRA The area of a circle is 196π square yards. Find the diameter.

ReviewVocabulary

central angle an angle with a vertex in the center of a circle and with sides that contain two radii of the circle

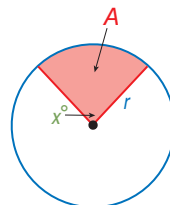**arc** a portion of a circle defined by two endpoints

2 Areas of Sectors A slice of a circular pizza is an example of a sector of a circle. A **sector of a circle** is a region of a circle bounded by a central angle and its intercepted major or minor arc. The formula for the area of a sector is similar to the formula for arc length.

KeyConcept Area of a Sector

The ratio of the **area _A_ of a sector** to the **area of the whole circle, πr^2**, is equal to the ratio of the **degree measure of the intercepted arc _x_** to 360.

Proportion: $\dfrac{A}{\pi r^2} = \dfrac{x}{360}$

Equation: $A = \dfrac{x}{360} \cdot \pi r^2$

🌐 Real-World Example 3 Area of a Sector

PIZZA A circular pizza has a diameter of 12 inches and is cut into 8 congruent slices. What is the area of one slice to the nearest hundredth?

Step 1 Find the arc measure of a pizza slice.

Since the pizza is equally divided into 8 slices, each slice will have an arc measure of 360 ÷ 8 or 45.

Step 2 Find the radius of the pizza. Use this measure to find the area of the sector, or slice.

The diameter is 12 inches, so the radius is 6 inches.

$A = \dfrac{x}{360} \cdot \pi r^2$ Area of a sector

$= \dfrac{45}{360} \cdot \pi (6)^2$ $x = 45$ and $r = 6$

≈ 14.14 Use a calculator.

So, the area of one slice of this pizza is about 14.14 square inches.

Real-World Link

About 3 billion pizzas are sold each year in the United States. That is equivalent to about 46 slices per person annually.

Source: ThinkQuest Library

Picturenet/Blend Images/Getty Images

▶ **Guided**Practice

Find the area of the shaded sector. Round to the nearest tenth.

3A.

3B.

3C.
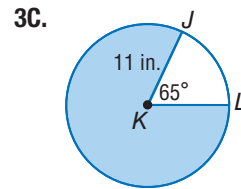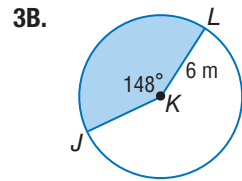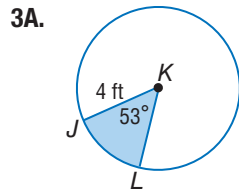
3D. CRAFTS The color wheel at the right is a tool that artists use to organize color schemes. If the diameter of the wheel is 10 inches and each of the 12 sections is congruent, find the approximate area covered by green hues.

Check Your Understanding

🟢 **= Step-by-Step Solutions begin on page R14.**

Example 1   **CONSTRUCTION** Find the area of each circle. Round to the nearest tenth.

1.

21 yd

2.

0.4 km

Example 2   **Find the indicated measure. Round to the nearest tenth.**

3 Find the diameter of a circle with an area of 74 square millimeters.

4. The area of a circle is 88 square inches. Find the radius.

Example 3   **Find the area of each shaded sector. Round to the nearest tenth.**

5.

3 in.
57°

6.

7 cm
92°
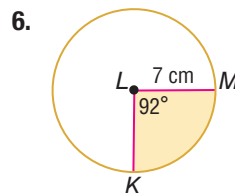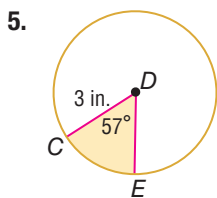
7. BAKING Chelsea is baking pies for a fundraiser at her school. She divides each 9-inch pie into 6 equal slices.

   a. What is the area, in square inches, for each slice of pie?

   b. If each slice costs $0.25 to make and she sells 8 pies at $1.25 for each slice, how much money will she raise?
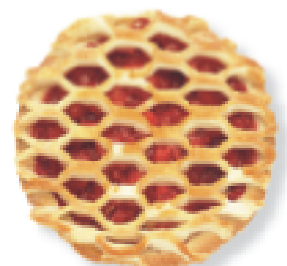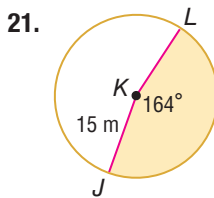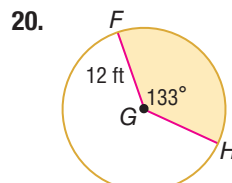
Example 1   **CCSS** **MODELING** Find the area of each circle. Round to the nearest tenth.

8.

3 ft

9.

5 yd

10.

8 ft

11.



4.25 in.

12.

18 in.

13.

10 ft
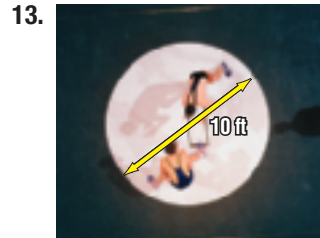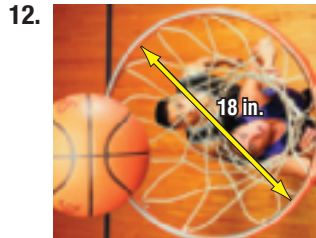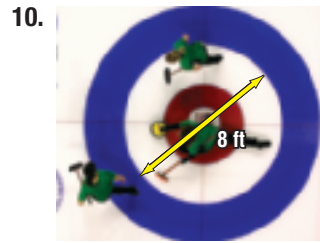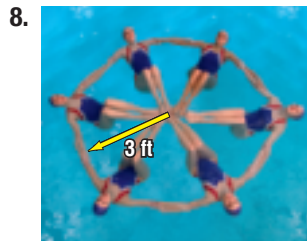
Example 2   **Find the indicated measure. Round to the nearest tenth, if necessary.**

14. The area of a circle is 68 square centimeters. Find the diameter.

15. Find the diameter of a circle with an area of 94 square millimeters.

16. The area of a circle is 112 square inches. Find the radius.

17. Find the radius of a circle with an area of 206 square feet.

Example 3   **Find the area of each shaded sector. Round to the nearest tenth, if necessary.**

18.

5 in.
46°
A
C
B

19

S
8 cm 72°
R
T

20.

F
12 ft
133°
G
H

21.

L
K
164°
15 m
J

22.

Z
11.2 mm
55° Y
X

23.

P
10.8 in.
81°
M   Q
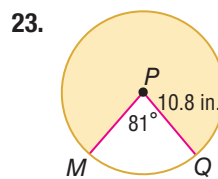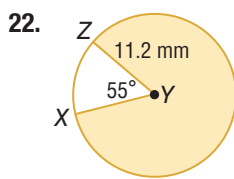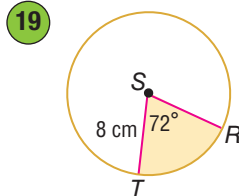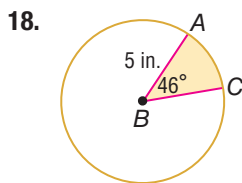
24. **MUSIC** The music preferences of students at Thomas Jefferson High are shown in the circle graph. Find the area of each sector and the degree measure of each intercepted arc if the radius of the circle is 1 unit.

Country
10%
Classical
2%
Alternative
14%
Rap
48%
Rock &
Roll
26%
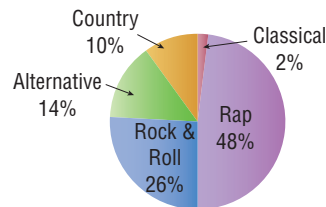
25. **JEWELRY** A jeweler makes a pair of earrings by cutting two 50° sectors from a silver disk.

a. Find the area of each sector.

b. If the weight of the silver disk is 2.3 grams, how many milligrams does the silver wedge for each earring weigh?
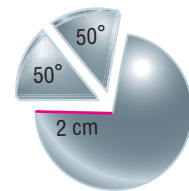

50°
50°
2 cm

26. PROM The table shows the results of a survey of students to determine their preference for a prom theme.

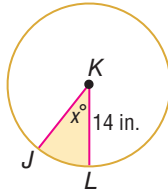| Theme	Percent
An Evening of Stars	11
Mardi Gras	32
Springtime in Paris	8
Night in Times Square	47
Undecided	2

a. Create a circle graph with a diameter of 2 inches to represent these data.

b. Find the area of each theme's sector in your graph. Round to the nearest hundredth of an inch.

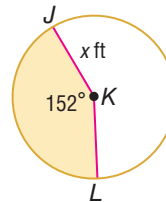**CCSS SENSE-MAKING** The area A of each shaded region is given. Find x.
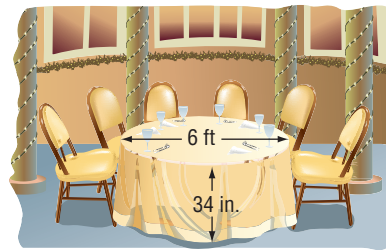
27. $A = 66 \text{ cm}^2$
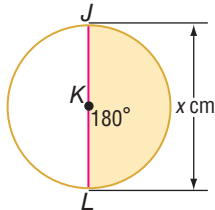


**28.** $A = 94 \text{ in}^2$

29. $A = 128 \text{ ft}^2$

30. CRAFTS Luna is making tablecloths with the dimensions shown for a club banquet. Find the area of each tablecloth in square feet if each one is to just reach the floor.

31 TREES The age of a living tree can be determined by multiplying the diameter of the tree by its growth factor, or rate of growth.

a. What is the diameter of a tree with a circumference of 2.5 feet?

b. If the growth factor of the tree is 4.5, what is the age of the tree?

Find the area of the shaded region. Round to the nearest tenth.

32.
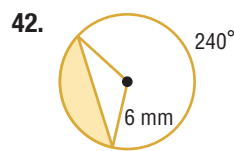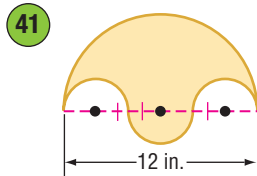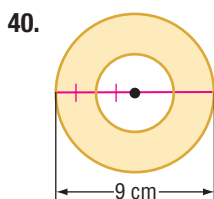

33.

34.

35.

36.

37.
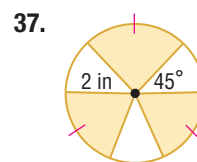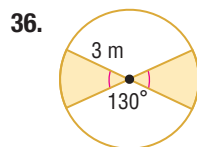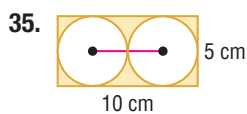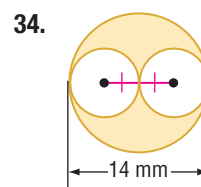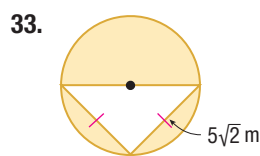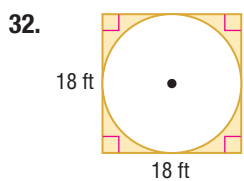
38. COORDINATE GEOMETRY What is the area of sector ABC shown on the graph?

39. ALGEBRA The figure shown below is a sector of a circle. If the perimeter of the figure is 22 millimeters, find its area in square millimeters.
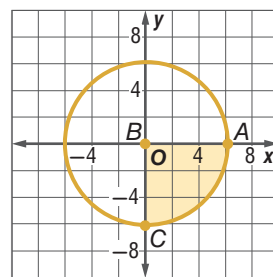
Find the area of each shaded region.

40.

9 cm

41

12 in.

42.

240°
6 mm

43. ⟐ **MULTIPLE REPRESENTATIONS** In this problem, you will investigate segments of circles. A **segment of a circle** is the region bounded by an arc and a chord.

a. **Algebraic** Write an equation for the area A of a segment of a circle with a radius r and a central angle of $x°$. (*Hint*: Use trigonometry to find the base and height of the triangle.)
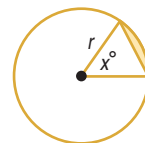
b. **Tabular** Calculate and record in a table ten values of A for x-values ranging from 10 to 90 if r is 12 inches. Round to the nearest tenth.

c. **Graphical** Graph the data from your table with the x-values on the horizontal axis and the A-values on the vertical axis.

d. **Analytical** Use your graph to predict the value of A when x is 63. Then use the formula you generated in part **a** to calculate the value of A when x is 63. How do the values compare?

H.O.T. Problems   Use Higher-Order Thinking Skills

44. **ERROR ANALYSIS** Kristen and Chase want to find the area of the shaded region in the circle shown. Is either of them correct? Explain your reasoning.

58°
8 in.

Kristen

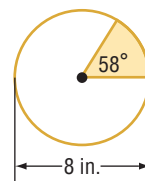$A = \dfrac{x}{360} \cdot \pi r^2$

$= \dfrac{58}{360} \cdot \pi(8)^2$

$= 32.4 \text{ in}^2$
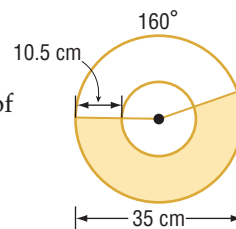
Chase

$A = \dfrac{x}{360} \cdot \pi r^2$

$= \dfrac{58}{360} \cdot \pi(4)^2$

$= 8.1 \text{ in}^2$

45. **CHALLENGE** Find the area of the shaded region. Round to the nearest tenth.

46. **CCSS ARGUMENTS** Refer to Exercise 43. Is the area of a sector of a circle *sometimes*, *always*, or *never* greater than the area of its corresponding segment?

160°
10.5 cm
35 cm

47. **WRITING IN MATH** Describe two methods you could use to find the area of the shaded region of the circle. Which method do you think is more efficient? Explain your reasoning.
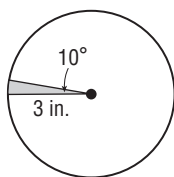

r
x°
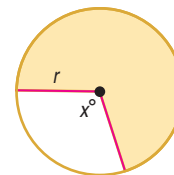
48. **CHALLENGE** Derive the formula for the area of a sector of a circle using the formula for arc length.

49. **WRITING IN MATH** If the radius of a circle doubles, will the measure of a sector of that circle double? Will it double if the arc measure of that sector doubles?

50. What is the area of the sector?

10°

3 in.

A $\frac{9\pi}{10}$ in^2

C $\frac{\pi}{4}$ in^2

B $\frac{3\pi}{5}$ in^2

D $\frac{\pi}{6}$ in^2

51. SHORT RESPONSE $\overleftrightarrow{MN}$ and $\overleftrightarrow{PQ}$ intersect at T. Find the value of x for which $m\angle MTQ = 2x + 5$ and $m\angle PTM = x + 7$. What are the degree measures of $\angle MTQ$ and $\angle PTM$?

52. ALGEBRA Raphael bowled 4 games and had a mean score of 130. He then bowled two more games with scores of 180 and 230. What was his mean score for all 6 games?

F 90   **H** 180

G 155   **J** 185

53. SAT/ACT The diagonals of rectangle $ABCD$ each have a length of 56 feet. If $m\angle BAC = 42°$, what is the length of $\overline{AB}$ to the nearest tenth of a foot?
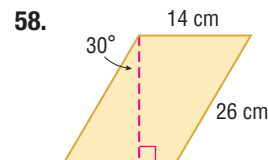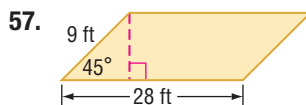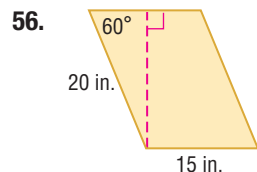

56 ft
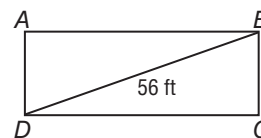
A 80.5   **D** 50.4

B 75.4   **E** 41.6

C 56.3

Find each missing length. (Lesson 11-2)

54. One diagonal of a kite is half as long as the other diagonal. If the area of the kite is 188 square inches, what are the lengths of the diagonals?

55. The area of a rhombus is 175 square centimeters. If one diagonal is two times as long as the other, what are the lengths of the diagonals?
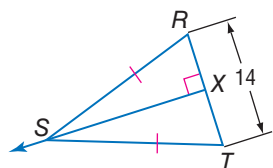
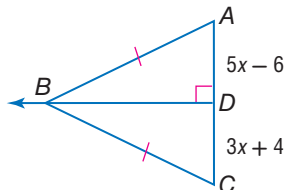**Find the area of each parallelogram. Round to the nearest tenth if necessary.** (Lesson 11-1)

56.

60°

20 in.

15 in.

57.

9 ft

45°

28 ft

58.

14 cm

30°

26 cm

Find each measure.

59. XT

R

X 14

S

T

60. AC

A

$5x - 6$

B

D

$3x + 4$

C

61. JK

J

$9x - 5$

L

$6x + 7$

N

K