LESSON 11-4 Areas of Regular Polygons and Composite Figures

Then

- You used inscribed and circumscribed figures and found the areas of circles.

Now

1. Find areas of regular polygons.
2. Find areas of composite figures.

Why?

- The top of the table shown is a regular hexagon. Notice that the top is composed of six congruent triangular sections. To find the area of the table top, you can find the sum of the areas of the sections.

NewVocabulary

center of a regular polygon
radius of a regular polygon
apothem
central angle of a regular polygon
composite figure

Common Core State Standards

Content Standards

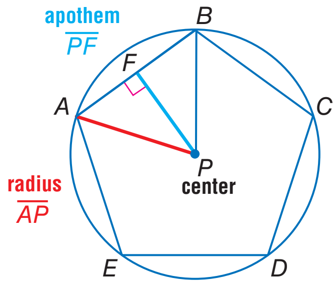
G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

Mathematical Practices

1 Make sense of problems and persevere in solving them.
6 Attend to precision.

1 Areas of Regular Polygons

In the figure, a regular pentagon is *inscribed* in ⊙P, and ⊙P is *circumscribed* about the pentagon. The **center of a regular polygon** and the **radius of a regular polygon** are also the center and the radius of its circumscribed circle.



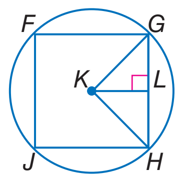
∠*APB* is a central angle of regular pentagon *ABCDE*.

A segment drawn from the center of a regular polygon perpendicular to a side of the polygon is called an **apothem**. Its length is the height of an isosceles triangle that has two radii as legs.

A **central angle of a regular polygon** has its vertex at the center of the polygon and its sides pass through consecutive vertices of the polygon. The measure of each central angle of a regular n-gon is $\frac{360}{n}$.

Example 1 Identify Segments and Angles in Regular Polygons

Square *FGHJ* is inscribed in ⊙*K*. Identify the center, a radius, an apothem, and a central angle of the polygon. Then find the measure of a central angle.

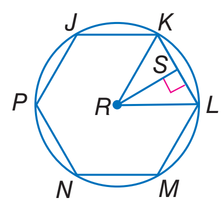


center: point K radius: $\overline{KG}$ or $\overline{KH}$

apothem: $\overline{KL}$ central angle: $\angle GKH$

A square is a regular polygon with 4 sides. Thus, the measure of each central angle of square *FGHJ* is $\frac{360}{4}$ or 90.

GuidedPractice



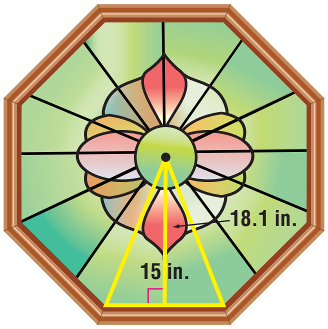
1. In the figure, regular hexagon *JKLMNP* is inscribed in ⊙*R*. Identify the center, a radius, an apothem, and a central angle of the polygon. Then find the measure of a central angle.

You can find the area of any regular n-gon by dividing the polygon into congruent isosceles triangles. This strategy is sometimes called *decomposing the polygon into triangles.*

Real-World Example 2 Area of a Regular Polygon

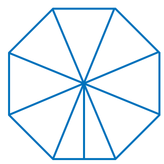
> **ReadingMath**
>
> **Apothem** Like the *radius* of a circle, the *apothem* of a polygon refers to the length of any apothem of the polygon.

ART Kang created the stained glass window shown. The window is a regular octagon with a side length of 15 inches and an apothem of 18.1 inches. What is the area covered by the window?



Step 1 Divide the polygon into congruent isosceles triangles.

Since the polygon has 8 sides, the polygon can be divided into 8 congruent isosceles triangles, each with a base of 15 inches and a height of 18.1 inches.



Step 2 Find the area of one triangle.

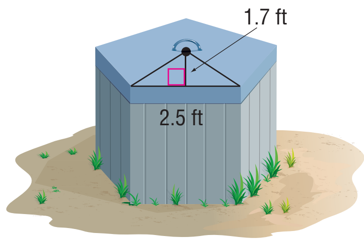
$A = \frac{1}{2}bh$	Area of a triangle
$= \frac{1}{2}(15)(18.1)$	$b = 15$ and $h = 18.1$
$= 135.75 \text{ in}^2$	Simplify.

Step 3 Multiply the area of one triangle by the total number of triangles.

Since there are 8 triangles, the area of the stained glass is 135.75 • 8 or 1086 square inches.

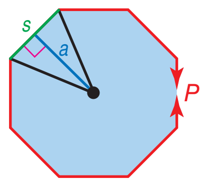
GuidedPractice

2. HOT TUBS The cover of the hot tub shown is a regular pentagon. If the side length is 2.5 feet and the apothem is 1.7 feet, find the area of the lid to the nearest tenth.



> **WatchOut!**
>
> **Area of Regular Polygon** this approach can only be applied to *regular* polygons.

From Example 2, we can develop a formula for the area of a regular n-gon with side length s and apothem a.

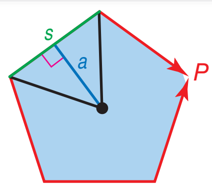


A = area of one triangle • number of triangles	
$= \frac{1}{2}$ • base • height • number of triangles	
$= \frac{1}{2} \cdot s \cdot a \cdot n$	Base of triangle is s and height is a. The number of triangles is n.
$= \frac{1}{2} \cdot a \cdot (n \cdot s)$	Commutative and Associative Properties
$= \frac{1}{2} \cdot a \cdot P$	The perimeter P of the polygon is $n \cdot s$.

KeyConcept Area of a Regular Polygon

Words The area A of a regular n-gon with side length s is one half the product of the apothem a and perimeter P.

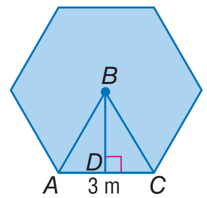
Symbols $A = \frac{1}{2}a(ns)$ or $A = \frac{1}{2}aP$.



Example 3 Use the Formula for the Area of a Regular Polygon

Find the area of each regular polygon. Round to the nearest tenth.

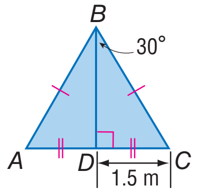
a. regular hexagon



Step 1 Find the measure of a central angle.

A regular hexagon has 6 congruent central angles, so $m\angle ABC = \frac{360}{6}$ or 60.

Step 2 Find the apothem.



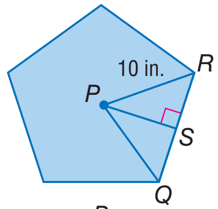
Apothem $\overline{BD}$ is the height of isosceles $\triangle ABC$. It bisects $\angle ABC$, so $m\angle DBC = 30$. It also bisects $\overline{AC}$, so $DC = 1.5$ meters.

$\triangle BDC$ is a 30°-60°-90° triangle with a shorter leg that measures 1.5 meters, so $BD = 1.5\sqrt{3}$ meters.

Step 3 Use the apothem and side length to find the area.

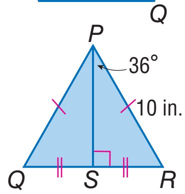
$A = \frac{1}{2}aP$	Area of a regular polygon
$= \frac{1}{2}(1.5\sqrt{3})(18)$	$a = 1.5\sqrt{3}$ and $P = 6(3)$ or 18
$\approx 23.4 \text{ m}^2$	Use a calculator.

b. regular pentagon



Step 1 A regular pentagon has 5 congruent central angles, so $m\angle QPR = \frac{360}{5}$ or 72.

Step 2 Apothem $\overline{PS}$ is the height of isosceles $\triangle RPQ$. It bisects $\angle RPQ$, so $m\angle RPS = 36$. Use trigonometric ratios to find the side length and apothem of the polygon.



$\sin 36° = \frac{SR}{10}$ $\qquad$ $\cos 36° = \frac{PS}{10}$

$10 \sin 36° = SR$ $\qquad$ $10 \cos 36° = PS$

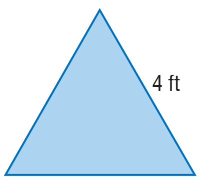
$QR = 2SR$ or $2(10 \sin 36°)$. So the pentagon's perimeter is $5 \cdot 2(10 \sin 36°)$ or $10(10 \sin 36°)$. The length of the apothem $\overline{PS}$ is $10 \cos 36°$.

Step 3

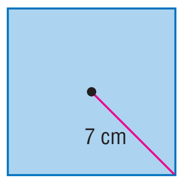
$A = \frac{1}{2}aP$	Area of a regular polygon
$= \frac{1}{2}(10 \cos 36°)[10(10 \sin 36°)]$	$a = 10 \cos 36°$, $P = 10(10 \sin 36°)$
$\approx 237.8 \text{ in}^2$	Use a calculator.

GuidedPractice

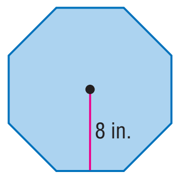
3A.



3B.



3C.



StudyTip

CCSS Precision The altitude of an isosceles triangle from its vertex to its base is also an angle bisector and median of the triangle.

2 Areas of Composite Figures

A **composite figure** is a figure that can be separated into regions that are basic figures, such as triangles, rectangles, trapezoids, and circles. To find the area of a composite figure, find the area of each basic figure and then use the Area Addition Postulate.

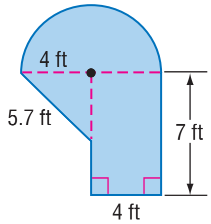
Real-WorldLink

The first miniature golf course was built in Pinehurst, North Carolina, on a private estate owned by James Barber. There are currently between 5000 and 7500 miniature golf courses in the United States.

Source: Miniature Golf Association of the United States

Example 4 Find the Area of a Composite Figure by Adding

MINIATURE GOLF The dimensions of a putting green at a miniature golf course are shown. How many square feet of carpet are needed to cover this green?



The area to be carpeted can be separated into a rectangle with a length of 4 feet and a width of 7 feet, a right triangle with a hypotenuse of 5.7 feet and a leg measuring 4 feet, and a semicircle with a radius of 4 feet.

Using the Pythagorean Theorem, the other leg of the right triangle is $\sqrt{5.7^2 - 4^2}$ or about 4.1 feet.

Area of green = **area of rectangle** + **area of triangle** + **area of semicircle.**

$$= \ell \cdot w + \frac{1}{2} \cdot b \cdot h + \frac{180}{360} \cdot \pi \cdot r^2$$

$$\approx 4 \cdot 7 + \frac{1}{2} \cdot 4 \cdot 4.1 + \frac{180}{360} \cdot \pi \cdot 4^2$$

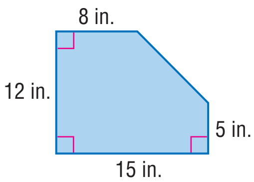
$$\approx 28 + 8.2 + 8\pi \text{ or about } 61.3 \text{ ft}^2$$

So, about 62 square feet of carpet is needed.

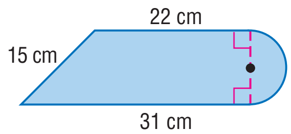
GuidedPractice

Find the area of each figure. Round to the nearest tenth if necessary.

4A.



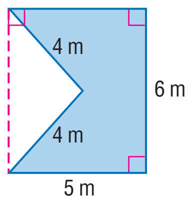
4B.



The areas of some figures can be found by subtracting the areas of basic figures.

Example 5 Find the Area of a Composite Figure by Subtracting

Find the area of the figure. Round to the nearest tenth if necessary.



To find the area of the figure, subtract the area of the triangle from the area of the rectangle.

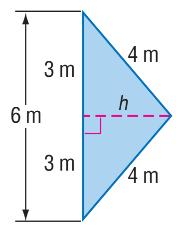
Using the Pythagorean Theorem, the height h of the triangle is $\sqrt{4^2 - 3^2}$ or $\sqrt{7}$ meters.

Area of figure = **Area of rectangle** − **Area of triangle**

$$= b \cdot h - \frac{1}{2}bh$$

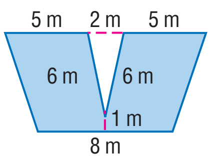
$$= 5 \cdot 6 - \frac{1}{2}(6)(\sqrt{7})$$

$$\approx 30 - 7.9 \text{ or about } 22.1 \text{ m}^2$$

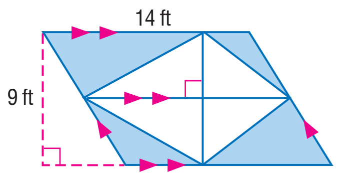


GuidedPractice

5A.



5B.

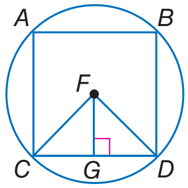


Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1

1. In the figure, square *ABDC* is inscribed in ⊙*F*. Identify the center, a radius, an apothem, and a central angle of the polygon. Then find the measure of a central angle.



Examples 2–3 **Find the area of each regular polygon. Round to the nearest tenth.**

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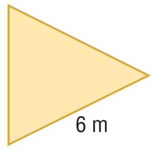

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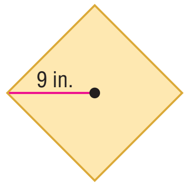
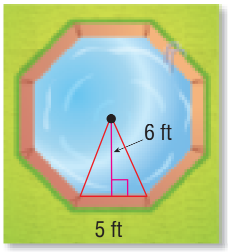
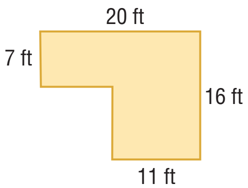


4. **POOLS** Kenton's job is to cover the community pool during fall and winter. Since the pool is in the shape of an octagon, he needs to find the area in order to have a custom cover made. If the pool has the dimensions shown at the right, what is the area of the pool?

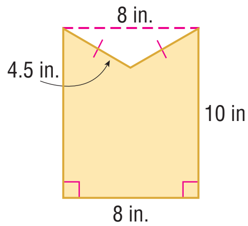


Examples 4–5 CCSS **SENSE-MAKING** **Find the area of each figure. Round to the nearest tenth if necessary.**

5.



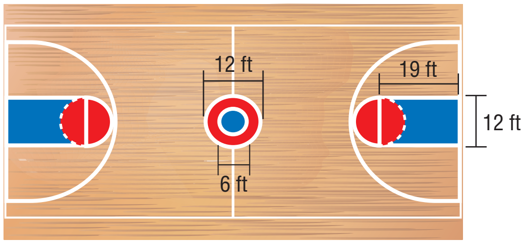
6.



7 **BASKETBALL** The basketball court in Jeff's school is painted as shown.

a. What area of the court is blue? Round to the nearest square foot.

b. What area of the court is red? Round to the nearest square foot.



Note: Art not drawn to scale.

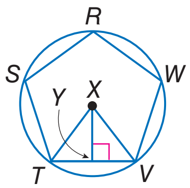
Practice and Problem Solving

Extra Practice is on page R11.

Example 1

In each figure, a regular polygon is inscribed in a circle. Identify the center, a radius, an apothem, and a central angle of each polygon. Then find the measure of a central angle.

8.



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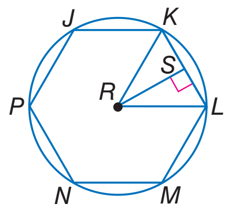

Examples 2–3 **Find the area of each regular polygon. Round to the nearest tenth.**

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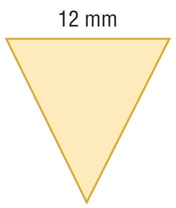

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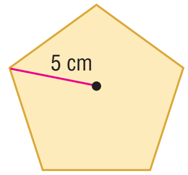

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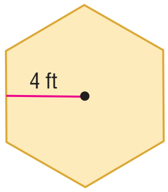

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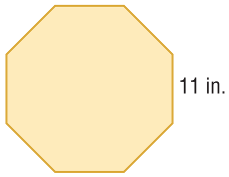
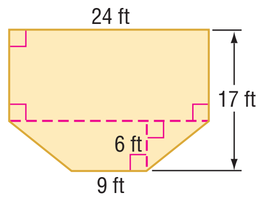


Example 4

14. CARPETING Ignacio's family is getting new carpet in their family room, and they want to determine how much the project will cost.

a. Use the floor plan shown to find the area to be carpeted.

b. If the carpet costs $4.86 per square yard, how much will the project cost?



Examples 4–5 CCSS **SENSE-MAKING** **Find the area of each figure. Round to the nearest tenth if necessary.**

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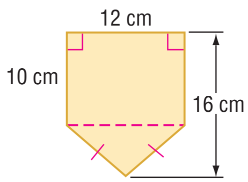

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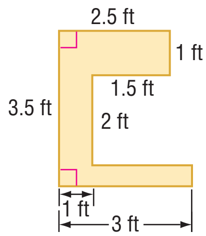

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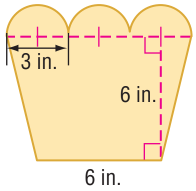

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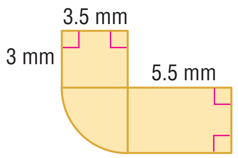

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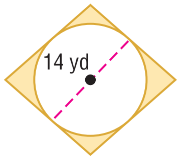

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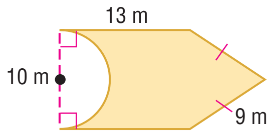
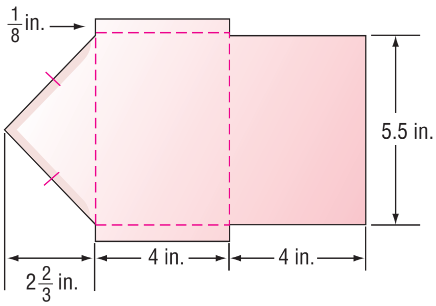


21. CRAFTS Latoya's greeting card company is making envelopes for a card from the pattern shown.

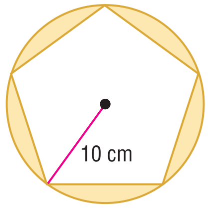
a. Find the perimeter and area of the pattern. Round to the nearest tenth.

b. If Latoya orders sheets of paper that are 2 feet by 4 feet, how many envelopes can she make per sheet?



Find the area of each shaded region formed by each circle and regular polygon. Round to the nearest tenth.

22.



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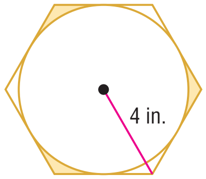

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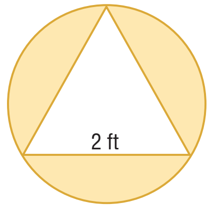
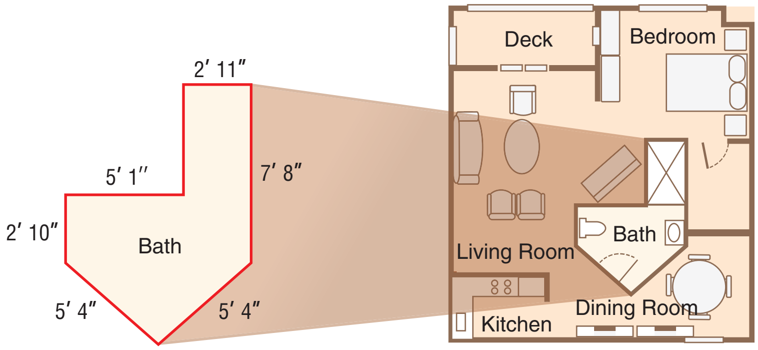


25. FLOORING JoAnn wants to lay 12″ × 12″ tile on her bathroom floor.

a. Find the area of the bathroom floor in her apartment floor plan.

b. If the tile comes in boxes of 15 and JoAnn buys no extra tile, how many boxes will she need?



Find the perimeter and area of each figure. Round to the nearest tenth, if necessary.

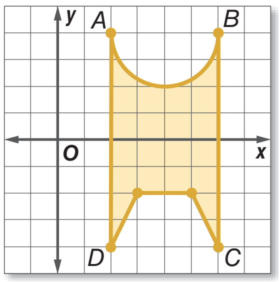
26. a regular hexagon with a side length of 12 centimeters

27. a regular pentagon circumscribed about a circle with a radius of 8 millimeters

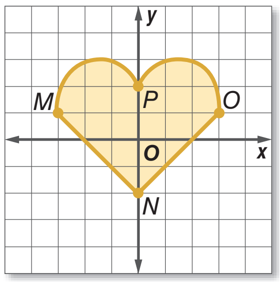
28. a regular octagon inscribed in a circle with a radius of 5 inches

CCSS PERSEVERANCE **Find the area of each shaded region. Round to the nearest tenth.**

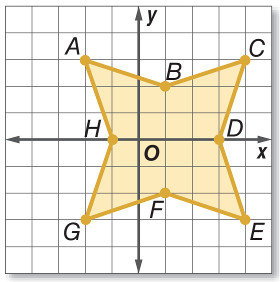
29.



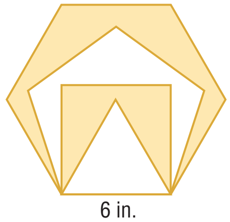
30.



31.



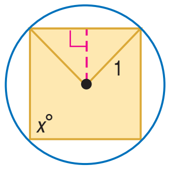
32. Find the total area of the shaded regions. Round to the nearest tenth.



33. CHANGING DIMENSIONS Calculate the area of an equilateral triangle with a perimeter of 3 inches. Calculate the areas of a square, a regular pentagon, and a regular hexagon with perimeters of 3 inches. How does the area of a regular polygon with a fixed perimeter change as the number of sides increases?

34. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the areas of regular polygons inscribed in circles.

a. Geometric Draw a circle with a radius of 1 unit and inscribe a square. Repeat twice, inscribing a regular pentagon and hexagon.



b. Algebraic Use the inscribed regular polygons from part **a** to develop a formula for the area of an inscribed regular polygon in terms of angle measure x and number of sides n.

c. Tabular Use the formula you developed in part **b** to complete the table below. Round to the nearest hundredth.

Number of Sides, n	4	5	6	8	10	20	50	100
Interior Angle Measure, x								
Area of Inscribed Regular Polygon								

d. Verbal Make a conjecture about the area of an inscribed regular polygon with a radius of 1 unit as the number of sides increases.

H.O.T. Problems Use Higher-Order Thinking Skills

35. **ERROR ANALYSIS** Chloe and Flavio want to find the area of the hexagon shown. Is either of them correct? Explain your reasoning.

Chloe

$A = \frac{1}{2}Pa$

$= \frac{1}{2}(66)(9.5)$

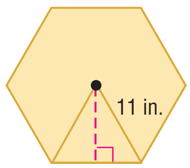
$= 313.5 \text{ in}^2$

Flavio

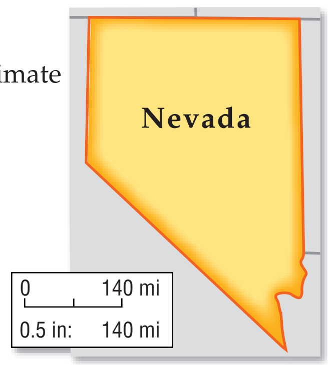
$A = \frac{1}{2}Pa$

$= \frac{1}{2}(33)(9.5)$

$= 156.8 \text{ in}^2$

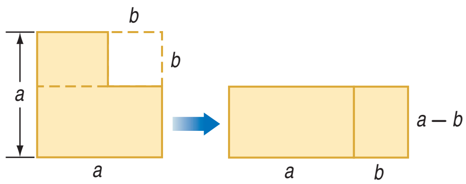


36. **SENSE-MAKING** Using the map of Nevada shown, estimate the area of the state. Explain your reasoning.



37. **OPEN ENDED** Draw a pair of composite figures that have the same area. Make one composite figure out of a rectangle and a trapezoid, and make the other composite figure out of a triangle and a rectangle. Show the area of each basic figure.

38. **WRITING IN MATH** Consider the sequence of area diagrams shown.



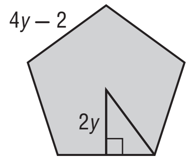
a. What algebraic theorem do the diagrams prove? Explain your reasoning.

b. Create your own sequence of diagrams to prove a different algebraic theorem.

39. **WRITING IN MATH** How can you find the area of any figure?

Standardized Test Practice

40. Which polynomial best represents the area of the regular pentagon shown below?



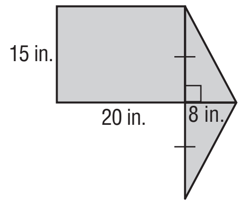
A $10y^2 - 5$ C $20y^2 + 10$
B $10y^2 + 5y$ D $20y^2 - 10y$

41. What is $27^{-\frac{2}{3}}$ in radical form?

F $\frac{1}{(\sqrt[3]{27})^2}$ H $\frac{1}{(\sqrt{27})^2}$

G $(\sqrt[3]{27})^2$ J $(\sqrt{27})^3$

42. SHORT RESPONSE Find the area of the shaded figure in square inches. Round to the nearest tenth.



43. SAT/ACT If the cos $\theta = \frac{12}{13}$ what is the value of tan θ?

A $\frac{13}{5}$ D $\frac{5}{12}$

B $\frac{12}{5}$ E $\frac{5}{13}$

C $\frac{13}{12}$

Spiral Review

Find the indicated measure. Round to the nearest tenth. (Lesson 11-3)

44. The area of a circle is 95 square feet. Find the radius.

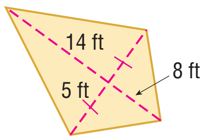
45. Find the area of a circle whose radius is 9 centimeters.

46. The area of a circle is 256 square inches. Find the diameter.

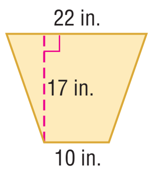
47. Find the area of a circle whose diameter is 25 millimeters.

Find the area of each trapezoid, rhombus, or kite. (Lesson 11-2)

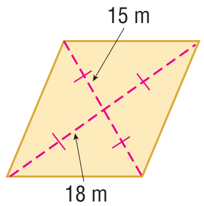
48.



49.

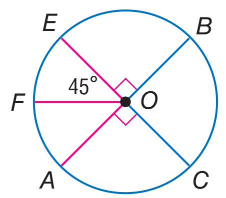


50.



$\overline{EC}$ and $\overline{AB}$ are diameters of ⊙*O*. Identify each arc as a *major arc*, *minor arc*, or *semicircle* of the circle. Then find its measure. (Lesson 10-2)

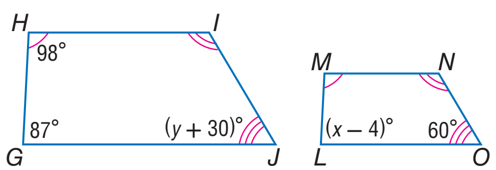
51. $m\widehat{ACB}$ **52.** $m\widehat{EB}$ **53.** $m\widehat{ACE}$



Skills Review

Each pair of polygons is similar. Find *x*.

54.



55.