LESSON 11-5

Areas of Similar Figures

Then

- You used scale factors and proportions to solve problems involving the perimeters of similar figures.

Now

- 1 Find areas of similar figures by using scale factors.
- 2 Find scale factors or missing measures given the areas of similar figures.

Why?

- Architecture firms often hire model makers to make scale models of projects that are used to sell their designs. Since the base of a model is geometrically similar to the base of the actual building it represents, their areas are related.

Common Core State Standards

Content Standards

G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★

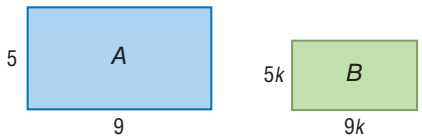
Mathematical Practices

1 Make sense of problems and persevere in solving them.

4 Model with mathematics.

1 Areas of Similar Figures

In Lesson 7-2, you learned that if two polygons are similar, then their perimeters are proportional to the scale factor between them. The areas of two similar polygons share a different relationship.



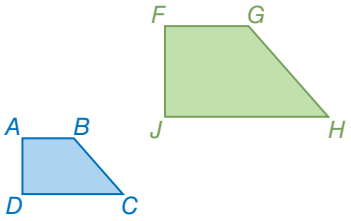
$$\frac{\text{perimeter of figure } B}{\text{perimeter of figure } A} = \frac{28k}{28} \text{ or } k$$

$$\frac{\text{area of figure } B}{\text{area of figure } A} = \frac{45k^2}{45} \text{ or } k^2$$

Theorem 11.1 Areas of Similar Polygons

Words If two polygons are similar, then their areas are proportional to the square of the scale factor between them.

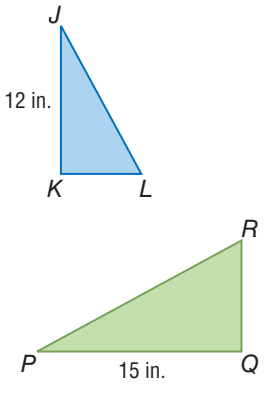
Example If $ABCD \sim FGHJ$, then $\frac{\text{area of } FGHJ}{\text{area of } ABCD} = \left(\frac{FG}{AB}\right)^2$.



You will prove Theorem 11.1 for triangles in Exercise 22.

Example 1 Find Areas of Similar Polygons

If $\triangle JKL \sim \triangle PQR$ and the area of $\triangle JKL$ is 30 square inches, find the area of $\triangle PQR$.



The scale factor between $\triangle PQR$ and $\triangle JKL$ is $\frac{15}{12}$ or $\frac{5}{4}$, so the ratio of their areas is $\left(\frac{5}{4}\right)^2$.

$\frac{\text{area of } \triangle PQR}{\text{area of } \triangle JKL} = \left(\frac{5}{4}\right)^2$ Write a proportion.

$\frac{\text{area of } \triangle PQR}{30} = \frac{25}{16}$ Area of $\triangle JKL = 30$ and $\left(\frac{5}{4}\right)^2 = \frac{25}{16}$

$\text{area of } \triangle PQR = \frac{25}{16} \cdot 30$ Multiply each side by 30.

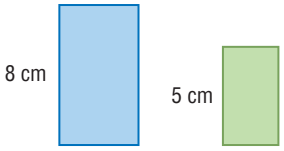
$\text{area of } \triangle PQR = 46.875$ Simplify.

So the area of $\triangle PQR$ is about 46.9 square inches.

GuidedPractice

For each pair of similar figures, find the area of the green figure.

1A.



$A = 32 \text{ cm}^2$

1B.

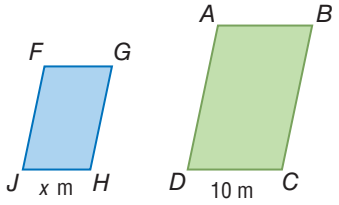
$A = 13.5 \text{ ft}^2$

2 Scale Factors and Missing Measures in Similar Figures

You can use the areas of similar figures to find the scale factor between them or a missing measure.

Example 2 Use Areas of Similar Figures

The area of $\square ABCD$ is 150 square meters. The area of $\square FGHJ$ is 54 square meters. If $\square ABCD \sim \square FGHJ$, find the scale factor of $\square FGHJ$ to $\square ABCD$ and the value of x.



Let k be the scale factor between $\square FGHJ$ and $\square ABCD$.

$$\frac{\text{area of } \square FGHJ}{\text{area of } \square ABCD} = k^2 \qquad \text{Theorem 11.1}$$

$$\frac{54}{150} = k^2 \qquad \text{Substitution}$$

$$\frac{9}{25} = k^2 \qquad \text{Simplify.}$$

$$\frac{3}{5} = k \qquad \text{Take the positive square root of each side.}$$

So the scale factor of $\square FGHJ$ to $\square ABCD$ is $\frac{3}{5}$. Use this scale factor to find the value of x.

$$\frac{JH}{DC} = k \qquad \text{The ratio of corresponding lengths of similar polygons is equal to the scale factor between the polygons.}$$

$$\frac{x}{10} = \frac{3}{5} \qquad \text{Substitution}$$

$$x = \frac{3}{5} \cdot 10 \text{ or } 6 \qquad \text{Multiply each side by 10.}$$

CHECK Confirm that $\frac{JH}{DC}$ is equal to the scale factor.

$$\frac{JH}{DC} = \frac{6}{10} = \frac{3}{5} \checkmark$$

WatchOut!

Writing Ratios When finding the ratio of the area of Figure A to the area of Figure B, be sure to write your ratio as $\frac{\text{area of figure } A}{\text{area of figure } B}$.

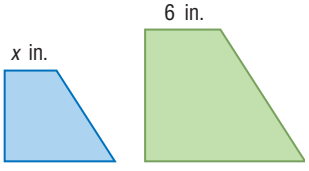
ReadingMath

Ratios Ratios can be written in different ways. For example, x to y, $x : y$, and $\frac{x}{y}$ are all representations of the ratio of x and y.

GuidedPractice

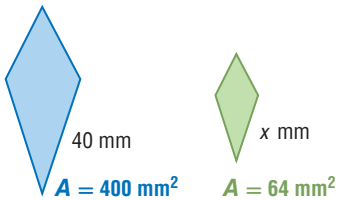
For each pair of similar figures, use the given areas to find the scale factor of the blue to the green figure. Then find x.

2A.



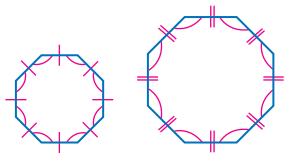
$A = 50 \text{ in}^2$ $A = 72 \text{ in}^2$

2B.



$A = 400 \text{ mm}^2$ $A = 64 \text{ mm}^2$

In Lesson 7-2, you learned that if all corresponding angles are congruent and all corresponding sides are proportional, then two polygons are similar. For this reason, all regular polygons with the same number of sides are similar.



Real-WorldLink

The Pentagon building, including its center courtyard, occupies approximately 34 acres or 1,481,000 square feet of land. Each outer wall of the regular pentagonal building is 921 feet in length.

Source: U.S. Department of Defense

Real-World Example 3 Scale Models

CRAFTS **Use the information at the left. Orlando and Mia are making a scale model of the Pentagon. If the area of the base of their model is approximately 50 square inches, about how many times the length of each outer wall of the Pentagon is the length of the outer wall of the model?**

Understand All regular pentagons are similar, so the base of the model is similar to the base of the Pentagon. You need to find the scale factor from the Pentagon to their model.

Plan The ratio of the areas of the bases of the two figures is equal to the square of the scale factor between them. Before comparing the two areas, write them so that they have the same units.

Solve Convert the area of the model's base to square feet.

$$50 \text{ in}^2 \cdot \frac{1 \text{ ft}^2}{144 \text{ in}^2} \approx 0.3472 \text{ ft}^2$$

Next, write an equation using the ratio of the two areas in square feet. Let k represent the scale factor between the two bases.

$$\frac{\text{area of model}}{\text{area of Pentagon}} = k^2 \qquad \text{Theorem 11.1}$$

$$\frac{0.3472 \text{ ft}^2}{1{,}481{,}000 \text{ ft}^2} \approx k^2 \qquad \text{Substitution}$$

$$2.34 \cdot 10^{-7} \approx k^2 \qquad \text{Simplify using a calculator.}$$

$$4.84 \cdot 10^{-4} \approx k \qquad \text{Take the positive square root of each side.}$$

$$0.0005 \approx k \qquad \text{Write in standard form.}$$

$$\frac{1}{2000} \approx k \qquad \text{Write as a simplified fraction.}$$

So the model's outer walls are about $\frac{1}{2000}$ the length of each outer wall of the Pentagon.

Check Multiply the area of the Pentagon's base by the square of this scale factor and compare to the given area of the model's base.

$$\frac{1{,}481{,}000 \text{ ft}^2}{1} \cdot \frac{144 \text{ in}^2}{1 \text{ ft}^2} \cdot \left(\frac{1}{2000}\right)^2 \approx 53 \text{ in}^2$$

This is close to the given area of 50 square inches, so our scale factor is reasonable. ✔

ReadingMath

Similar Circles Since all circles have the same shape, all circles are similar. Therefore, the areas of two circles are also related by the square of the scale factor between them.

GuidedPractice

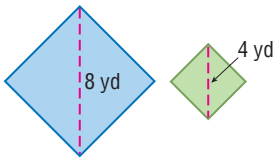
3. CRAFTS Miyoki is crocheting two circles. The area of the larger circle is to be 2.5 times the size of the smaller. If the area of the smaller circle is about 50.2 square centimeters, what is the diameter of the larger circle?

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

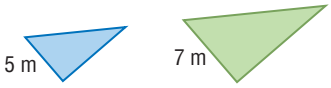
Example 1 For each pair of similar figures, find the area of the green figure.

1.



$A = 36 \text{ yd}^2$

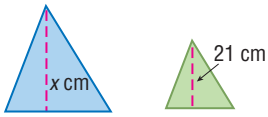
2.



$A = 40 \text{ m}^2$

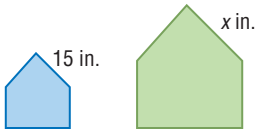
Example 2 For each pair of similar figures, use the given areas to find the scale factor from the blue to the green figure. Then find x.

3.



$A = 875 \text{ cm}^2$ $A = 315 \text{ cm}^2$

4.



$A = 153 \text{ in}^2$ $A = 272 \text{ in}^2$

Example 3

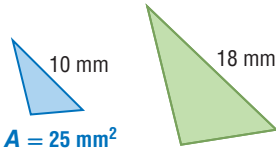
5. MEMORIES Zola has a picture frame that holds all of her school pictures. Each small opening is similar to the large opening in the center. If the center opening has an area of 33 square inches, what is the area of each small opening?

Practice and Problem Solving

Extra Practice is on page R11.

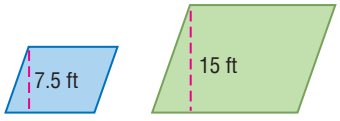
Example 1 For each pair of similar figures, find the area of the green figure.

6.



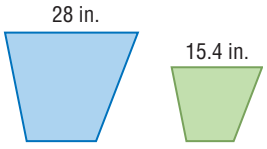
$A = 25 \text{ mm}^2$

7



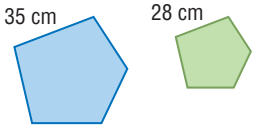
$A = 60 \text{ ft}^2$

8.



$A = 500 \text{ in}^2$

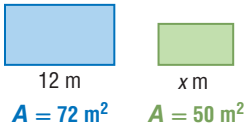
9.



$A = 1050 \text{ cm}^2$

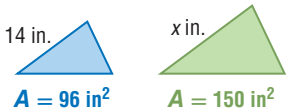
Example 2 CCSS **STRUCTURE** For each pair of similar figures, use the given areas to find the scale factor of the blue to the green figure. Then find x.

10.



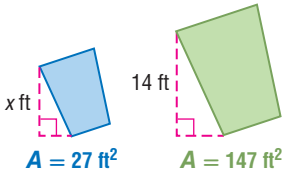
$A = 72 \text{ m}^2$ $A = 50 \text{ m}^2$

11.



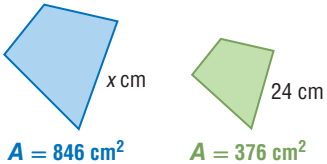
$A = 96 \text{ in}^2$ $A = 150 \text{ in}^2$

12.



$A = 27 \text{ ft}^2$ $A = 147 \text{ ft}^2$

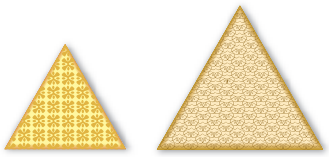
13.



$A = 846 \text{ cm}^2$ $A = 376 \text{ cm}^2$

Example 3

14. CRAFTS Marina crafts unique trivets and other kitchenware. Each trivet is an equilateral triangle. The perimeter of the small trivet is 9 inches, and the perimeter of the large trivet is 12 inches. If the area of the small trivet is about 3.9 square inches, what is the approximate area of the large trivet?



15. BAKING Kaitlyn wants to use one of two regular hexagonal cake pans for a recipe she is making. The side length of the larger pan is 4.5 inches, and the area of the base of the smaller pan is 41.6 square inches.

a. What is the side length of the smaller pan?

b. The recipe that Kaitlyn is using calls for a circular cake pan with an 8-inch diameter. Which pan should she choose? Explain your reasoning.

16. CHANGING DIMENSIONS A polygon has an area of 144 square meters.

a. If the area is doubled, how does each side length change?

b. How does each side length change if the area is tripled?

c. What is the change in each side length if the area is increased by a factor of x?

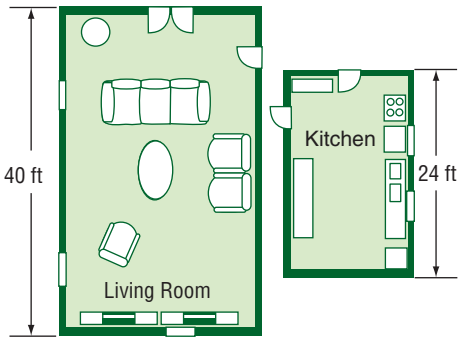
17. CHANGING DIMENSIONS A circle has a radius of 24 inches.

a. If the area is doubled, how does the radius change?

b. How does the radius change if the area is tripled?

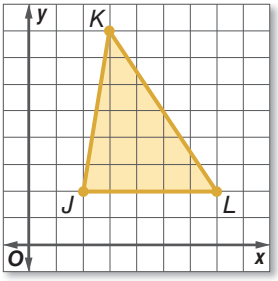
c. What is the change in the radius if the area is increased by a factor of x?

18. CCSS MODELING Federico's family is putting hardwood floors in the two geometrically similar rooms shown. If the cost of flooring is constant and the flooring for the kitchen cost \$2000, what will be the total flooring cost for the two rooms? Round to the nearest hundred dollars.

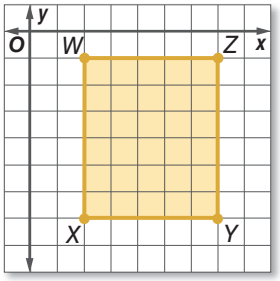


COORDINATE GEOMETRY Find the area of each figure. Use the segment length given to find the area of a similar polygon.

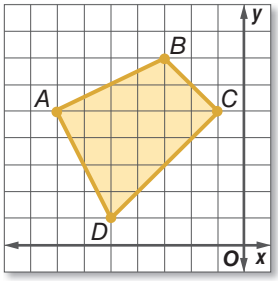
19 $J'L' = 3$



20. $W'X' = 8$



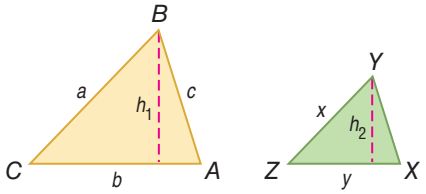
21. $B'C' = 5$



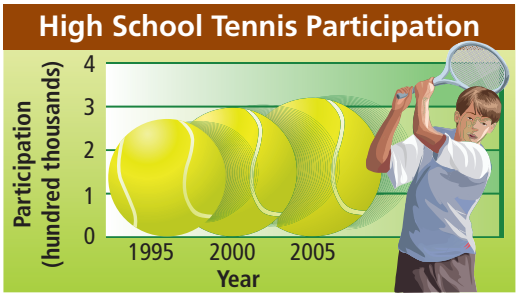
22. PROOF Write a paragraph proof.

Given: $\triangle ABC \sim \triangle XYZ$

Prove: $\dfrac{\text{area of } \triangle ABC}{\text{area of } \triangle XYZ} = \dfrac{a^2}{x^2}$



23 **STATISTICS** The graph shows the increase in high school tennis participation from 1995 to 2005.



a. Explain why the graph is misleading.

b. How could the graph be changed to more accurately represent the growth in high school tennis participation?

24. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate changing dimensions proportionally in three-dimensional figures.

a. **Tabular** Copy and complete the table below for each scale factor of a rectangular prism that is 2 inches by 3 inches by 5 inches.

Scale Factor	Length (in.)	Width (in.)	Height (in.)	Volume (in^3)	Ratio of Scaled Volume to Initial Volume
1	3	2	5		
2					
3					
4					
5					
10					

b. **Verbal** Make a conjecture about the relationship between the scale factor and the ratio of the scaled volume to the initial volume.

c. **Graphical** Make a scatter plot of the scale factor and the ratio of the scaled volume to the initial volume using the **STAT PLOT** feature on your graphing calculator. Then use the **STAT CALC** feature to approximate the function represented by the graph.

d. **Algebraic** Write an algebraic expression for the ratio of the scaled volume to the initial volume in terms of scale factor k.

H.O.T. Problems Use Higher-Order Thinking Skills

25. **CCSS** **CRITIQUE** Violeta and Gavin are trying to come up with a formula that can be used to find the area of a circle with a radius r after it has been enlarged by a scale factor k. Is either of them correct? Explain your reasoning.

Violeta	Gavin
$A = k\pi r^2$	$A = \pi(r^2)^k$

26. **CHALLENGE** If you want the area of a polygon to be x% of its original area, by what scale factor should you multiply each side length?

27. **REASONING** A regular n-gon is enlarged, and the ratio of the area of the enlarged figure to the area of the original figure is R. Write an equation relating the perimeter of the enlarged figure to the perimeter of the original figure Q.

28. **OPEN ENDED** Draw a pair of similar figures with areas that have a ratio of 4:1. Explain.

29. **WRITING IN MATH** Explain how to find the area of an enlarged polygon if you know the area of the original polygon and the scale factor of the enlargement.

Standardized Test Practice

30. $\triangle ABC \sim \triangle PRT$, $AC = 15$ inches, $PT = 6$ inches, and the area of $\triangle PRT$ is 24 square inches. Find the area of $\triangle ABC$.

A 9.6 in^2 **C** 66.7 in^2
B 60 in^2 **D** 150 in^2

31. ALGEBRA Which of the following shows $2x^2 - 18xy - 72y^2$ factored completely?

F $(2x - 18y)(x + 4y)$ **H** $(2x - 9y)(x + 4y)$
G $2(x - 9y)(x + 4y)$ **J** $2(x - 12y)(x + 3y)$

32. EXTENDED RESPONSE The measures of two complementary angles are represented by $2x + 1$ and $5x - 9$.

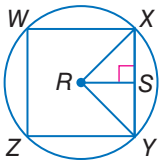
a. Write an equation that represents the relationship between the two angles.
b. Find the degree measure of each angle.

33. SAT/ACT Which of the following are the values of x for which $(x + 5)(x - 4) = 10$?

A -5 and 4 **D** 6 and -5
B 5 and 6 **E** -6 and 5
C -4 and 5

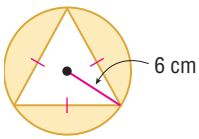
Spiral Review

34. In the figure, square $WXYZ$ is inscribed in $\odot R$. Identify the center, a radius, an apothem, and a central angle of the polygon. Then find the measure of a central angle. (Lesson 11-4)

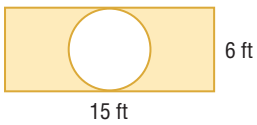


Find the area of the shaded region. Round to the nearest tenth. (Lesson 11-3)

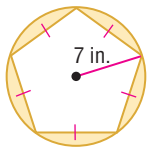
35.



36.

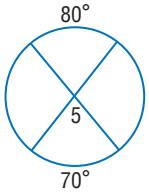


37.

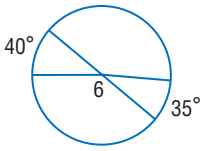


Find each measure. (Lesson 10-6)

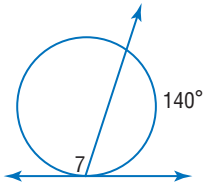
38. $m\angle 5$



39. $m\angle 6$



40. $m\angle 7$



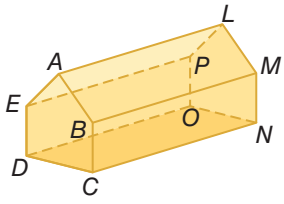
41. State whether the figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*. (Lesson 9-5)

42. YEARBOOKS Tai resized a photograph that was 8 inches by 10 inches so that it would fit in a 4-inch by 4-inch area on a yearbook page. (Lesson 7-7)

a. Find the maximum dimensions of the reduced photograph.
b. What is the percent of reduction of the length?

Skills Review

Refer to the figure at the right to identify each of the following.



43. Name all segments parallel to $\overline{AE}$.

44. Name all planes intersecting plane BCN.

45. Name all segements skew to $\overline{DC}$.