LESSON 2-2

Logic

Then

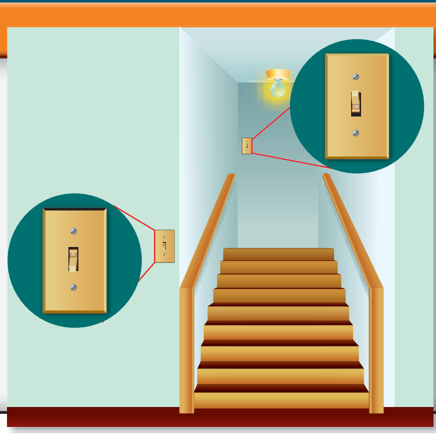
- You found counterexamples for false conjectures.

Now

1. Determine truth values of negations, conjunctions, and disjunctions, and represent them using Venn diagrams.
2. Find counterexamples.

Why?

- Many electrical circuits operate by evaluating a series of tests that are either true or false. For example, a single light can be controlled by two different switches connected on a circuit. The positions of both switches, either up or down, determine whether the light is on or off.



New Vocabulary

statement
truth value
negation
compound statement
conjunction
disjunction
truth table

1 Determine Truth Values

A **statement** is a sentence that is either true or false. The **truth value** of a statement is either true (T) or false (F). Statements are often represented using a letter such as p or q.

p: **A rectangle is a quadrilateral.** Truth value: T

The **negation** of a statement has the opposite meaning, as well as an opposite truth value. For example, the negation of the statement above is *not* p or $\sim p$.

$\sim p$: **A rectangle is** not **a quadrilateral.** Truth value: F

Two or more statements joined by the word *and* or *or* form a **compound statement**. A compound statement using the word *and* is called a **conjunction**. A conjunction is true only when both statements that form it are true.

p: **A rectangle is a quadrilateral.** Truth value: T

q: **A rectangle is convex.** Truth value: T

p and q: **A rectangle is a quadrilateral,** and **a rectangle is convex.**

Since both p and q are true, the conjunction p and q, also written $p \wedge q$, is true.

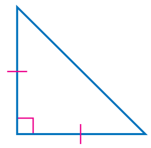
Example 1 Truth Values of Conjunctions

Use the following statements to write a compound statement for each conjunction. Then find its truth value. Explain your reasoning.

p: **The figure is a triangle.**

q: **The figure has two congruent sides.**

r: **The figure has three acute angles.**



a. p **and** r

p and r: The figure is a triangle, and the figure has three acute angles.

Although p is true, r is false. So, p and r is false.

b. $q \wedge \sim r$

$q \wedge \sim r$: The figure has two congruent sides, and the figure does not have three acute angles.

Both q and $\sim r$ are true, so $q \wedge \sim r$ is true.

Guided Practice

1A. $p \wedge q$

1B. not p and not r

A compound statement that uses the word *or* is called a **disjunction**.

p: Malik studies geometry

q: Malik studies chemistry.

p or *q*: Malik studies geometry, or Malik studies chemistry.

A disjunction is true if at least one of the statements is true. If Malik studies either geometry or chemistry or both subjects, the disjunction p or q, also written as $p \vee q$, is true. If Malik studies neither geometry nor chemistry, p or q is false.

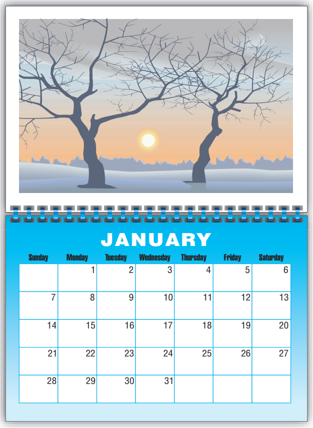
Example 2 Truth Values of Disjunctions

Use the following statements to write a compound statement for each disconjunction. Then find its truth value. Explain your reasoning.

***p*: January is a fall month.**

***q*: January has only 30 days.**

***r*: January 1 is the first day of a new year.**



a. p **or** r

q or r: January has only 30 days, or January 1 is the first day of a new year.

q or r is true because r is true. It does not matter that q is false.

b. $p \vee q$

$p \vee q$: January is a fall month, or January has only 30 days.

Since both p and q are false, $p \vee q$ is false.

c. $\sim p \vee r$

$\sim p \vee r$: January is *not* a fall month, or January 1 is the first day of a new year.

Not p or r is true, because not p is true and r is true.

GuidedPractice

2A. r or p **2B.** $q \vee \sim r$ **2C.** $p \vee \sim q$

WatchOut!

Negation Just as the opposite of an integer is not always negative, the negation of a statement is not always false. The negation of a statement has the opposite truth value of the original statement.

ConceptSummary Negation, Conjunction, Disconjunction

Statement	Words	Symbols
negation	a statement that has the opposite meaning and truth value of an original statement	$\sim p$, read not p
conjunction	a compound statement formed by joining two or more statements using the word *and*	$p \wedge q$, read p and q
disconjunction	a compound statement formed by joining two or more statements using the word *or*	$p \vee q$, read p or q

A convenient method for organizing the truth values of statements is to use a **truth table**. Truth tables can be used to determine truth values of negations and compound statements.

Negation	
p	$\sim p$
T	F
F	T

Conjunction		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction		
p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Study Tip

Truth Tables The truth tables for a conjunction and a disjunction are easier to recall if you remember the following.

- A conjunction is true only when both statements are true.
- A disjunction is false only when both statements are false.

You can use the truth values for negation, conjunction, and disjunction to construct truth tables for more complex compound statements.

Example 3 Construct Truth Tables

Construct a truth table for $\sim p \vee q$.

1 Make columns with headings that include each original statement, any negations of these statements, and the compound statement itself.

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

4 Use the truth values for each part of the compound statement to determine the truth value of the statement.

2 List the possible combinations of truth values.

3 Use the truth values of p to determine the truth values of its negation.

Guided Practice

3. Construct a truth table for $\sim p \wedge \sim q$.

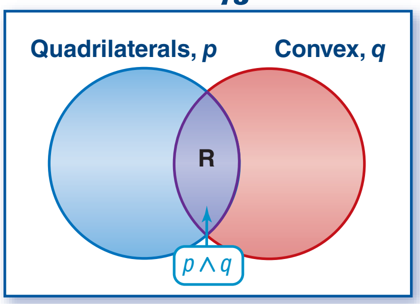
2 Venn Diagrams

Conjunctions can be illustrated with Venn diagrams. Consider the conjunction given at the beginning of the lesson.

p and q: **A rectangle is a quadrilateral,** and **a rectangle is convex.**

The Venn diagram shows that a rectangle (R) is located in the *intersection* of the set of quadrilaterals and the set of convex polygons. In other words, rectangles must be in the set containing quadrilaterals *and* in the set of convex polygons.

All Polygons



Vocabulary Link

Intersection

Everyday Use the point at which two or more objects overlap

Math Use The intersection of two sets is the set of elements that are common to both.

A disjunction can also be illustrated with a Venn diagram. Consider the following statements.

p: A figure is a quadrilateral.

q: A figure is convex.

p or q: A figure is a quadrilateral or convex.

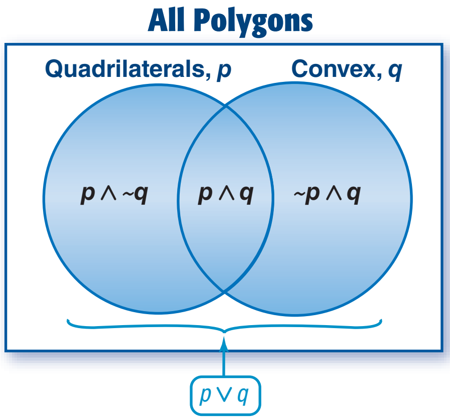
VocabularyLink

Union

Everyday Use the joining of two or more objects

Math Use The union of two sets is the set of elements that appear in either of the sets.

In the Venn diagram, the disjunction is represented by the *union* of the two sets. The union includes all polygons that are quadrilaterals, convex, *or* both.

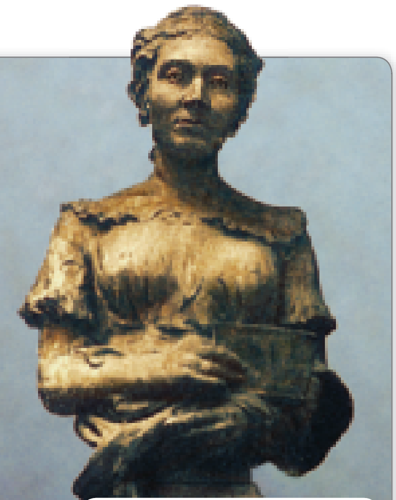


The disjunction includes these three regions:

$p \wedge {\sim}q$ quadrilaterals that are *not* convex

${\sim}p \wedge q$ convex polygons that are *not* quadrilaterals

$p \wedge q$ polygons that are both quadrilaterals and convex



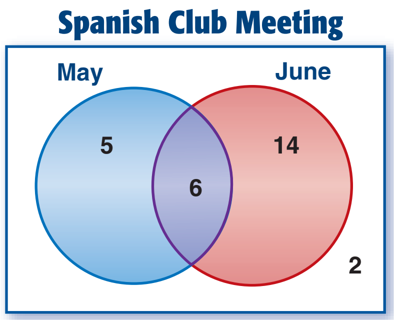
Math History Link

Sophie Germain

(1776–1831) Sophie Germain was born in Paris, France. Like Goldbach, she studied relationships involving prime numbers. In order to pursue her passion for mathematics, she assumed a man's identity.

Real-World Example 4 Use Venn Diagrams

SCHEDULING The Venn diagram shows the number of people who can or cannot attend the May or the June Spanish Club meetings.



a. How many people can attend the May or the June meeting?

The people who can attend either the May meeting or the June meeting are represented by the union of the sets. There are $5 + 6 + 14$ or 25 people who can attend either night.

b. How many people can attend both the May and the June meetings?

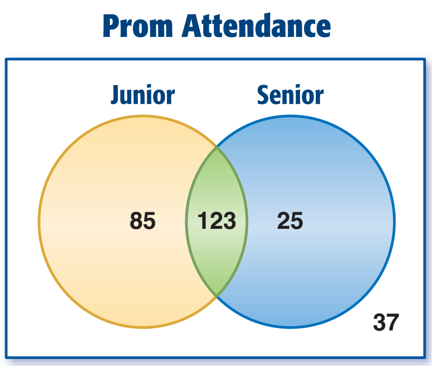
The people who can attend both the May and the June meetings are represented by the intersection of the two sets. There are 6 people who can attend both meetings.

c. Describe the meetings that the 14 people located in the nonintersecting portion of the June region can attend.

These 14 people can attend the June meeting but not the May meeting.

Guided Practice

4. PROM The Venn diagram shows the number of graduates last year who did or did not attend their junior or senior prom.



A. How many graduates attended their senior but not their junior prom?

B. How many graduates attended their junior and senior proms?

C. How many graduates did not attend either of their proms?

D. How many students graduated last year? Explain your reasoning.

Stock Montage/Hulton Archive/Getty Images

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Examples 1–2 **Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain your reasoning.**

p: A week has seven days.

q: There are 20 hours in a day.

r: There are 60 minutes in an hour.

1. p and r **2.** $p \wedge q$ **3.** $q \vee r$

4. $\sim p$ or q **5.** $p \vee r$ **6.** $\sim p \wedge \sim r$

Example 3

7. Copy and complete the truth table at the right.

p	q	$\sim q$	$p \vee \sim q$
T	T	F	
T	F		
F	T		
F	F		

Construct a truth table for each compound statement.

8. $p \wedge q$ **9.** $\sim p \vee \sim q$

Example 4

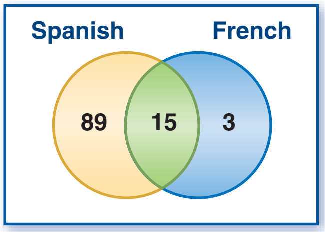
10. **CLASSES** Refer to the Venn diagram that represents the foreign language classes students selected in high school.

a. How many students chose only Spanish?

b. How many students chose Spanish and French?

c. Describe the class(es) the three people in the nonintersecting portion of the French region chose.

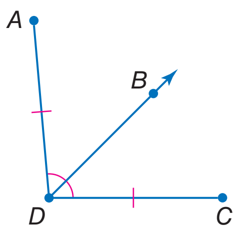
Foreign Language Classes Selected



Practice and Problem Solving

Extra Practice is on page R2.

Examples 1–2 **Use the following statements and figure to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain your reasoning.**



p: $\overrightarrow{DB}$ is the angle bisector of $\angle ADC$.

q: Points C, D, and B are collinear.

r: $\overline{AD} \cong \overline{DC}$

11. p and r **12.** q or p **13.** r or $\sim p$

14. r and q **15.** $\sim p$ or $\sim r$ **16.** $\sim p$ and $\sim r$

CCSS **REASONING** **Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain your reasoning.**

p: Springfield is the capital of Illinois.

q: Illinois borders the Atlantic Ocean.

r: Illinois shares a border with Kentucky.

s: Illinois is to the west of Missouri.

17. $p \wedge r$ **18.** $p \wedge q$ **19.** $\sim r \vee s$

20. $r \vee q$ **21.** $\sim p \wedge \sim r$ **22.** $\sim s \vee \sim p$

Example 3 **Copy and complete each truth table.**

23.

p	q	$\sim p$	$\sim p \wedge q$
T		F	
T		F	
F		T	
F		T	

24.

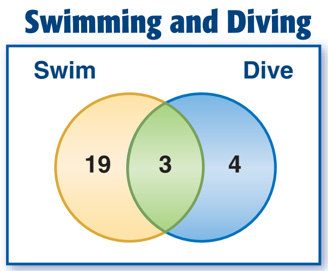
p	q	$-p$	$\sim q$	$\sim p \vee q$
T			F	
T			T	
F			F	
F			T	

Construct a truth table for each compound statement.

25. $p \wedge r$ **26.** $r \wedge q$ **27.** $p \vee r$

28. $q \vee r$ **29.** $\sim p \wedge r$ **30.** $\sim q \vee \sim r$

Example 4 **31** **WATER SPORTS** Refer to the Venn diagram that represents the number of students who swim and dive at a high school.



a. How many students dive?

b. How many students participate in swimming or diving or both?

c. How many students swim and dive?

32. **CCSS REASONING** Venus has switches at the top and bottom of her stairs to control the light for the stairwell. She notices that when the upstairs switch is up and the downstairs switch is down, the light is turned on.

Position of Switch		Light On
Upstairs	Downstairs	
up		
up	down	T

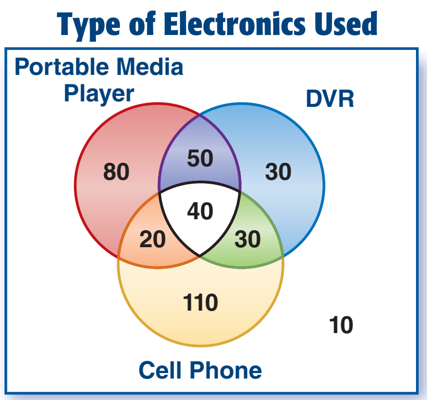
a. Copy and complete the truth table.

b. If both the upstairs and downstairs switches are in the up position, will the light be on? Explain your reasoning.

c. If the upstairs switch is in the down position and the downstairs switch is in the up position, will the light be on?

d. In general, how should the two switches be positioned so that the light is on?

33. **ELECTRONICS** A group of 330 teens were surveyed about what type of electronics they used. They chose from a cell phone, a portable media player, and a DVR. The results are shown in the Venn diagram.



a. How many teens used only a portable media player and DVR?

b. How many said they used all three types of electronics?

c. How many said they used only a cell phone?

d. How many teens said they used only a portable media player and a cell phone?

e. Describe the electronics that the 10 teens outside of the regions use.

Construct a truth table for each compound statement. Determine the truth value of each compound statement if the given statements are true.

34. $p \wedge (q \wedge r)$; p, q

35 $p \wedge (\sim q \vee r)$; p, r

36. $(\sim p \vee q) \wedge r$; q, r

37. $p \wedge (\sim q \wedge \sim r)$; p, q, r

38. $\sim p \wedge (\sim q \wedge \sim r)$; p, q, r

39. $(\sim p \vee q) \vee \sim r$; p, q

40. CCSS **REASONING** A travel agency surveyed 70 of their clients who had visited Europe about international travel. Of the 70 clients who had visited Europe, 60 had traveled to England, France, or both. Of those 60 clients, 45 had visited England, and 50 had visited France.

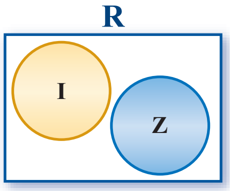
a. Make a Venn diagram to show the results of the survey.

b. If p represents a client who has visited England and q represents a client who has visited France, write a compound statement to represent each area of the Venn diagram. Include the compound statements on your Venn diagram.

c. What is the probability that a randomly chosen participant in the survey will have visited both England and France? Explain your reasoning.

H.O.T. Problems Use Higher-Order Thinking Skills

41. **REASONING** Irrational numbers and integers both belong to the set of real numbers (R). Based upon the Venn diagram, is it *sometimes*, *always*, or *never* true that integers (Z) are irrational numbers (I)? Explain your reasoning.



CHALLENGE To negate a statement containing the words *all* or *for every*, you can use the phrase *at least one* or *there exists*. To negate a statement containing the phrase *there exists*, use the phrase *for all* or *for every*.

p: All polygons are convex.

$\sim p$: *At least one* polygon is *not* convex.

q: *There exists* a problem that has no solution.

$\sim q$: *For every* problem, there is a solution.

Sometimes these phrases may be implied. For example, *The square of a real number is nonnegative* implies the following conditional and its negation.

p: *For every* real number x, $x^2 \geq 0$.

$\sim p$: *There exists* a real number x such that $x^2 < 0$.

Use the information above to write the negation of each statement.

42. Every student at Hammond High School has a locker.

43. All squares are rectangles.

44. There exists a real number x such that $x^2 = x$.

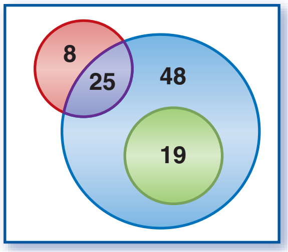
45. There exists a student who has at least one class in C-Wing.

46. Every real number has a real square root.

47. There exists a segment that has no midpoint.

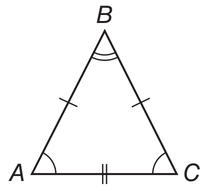
48. **WRITING IN MATH** Describe a situation that might be depicted using the Venn diagram shown.

49. **OPEN ENDED** Write a compound statement that results in a true conjunction.



Standardized Test Practice

50. Which statement about $\triangle ABC$ has the same truth value as $AB = BC$?



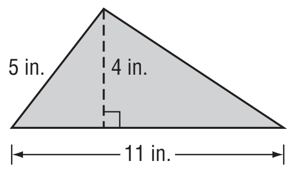
A $m\angle A = m\angle C$

B $m\angle A = m\angle B$

C $AC = BC$

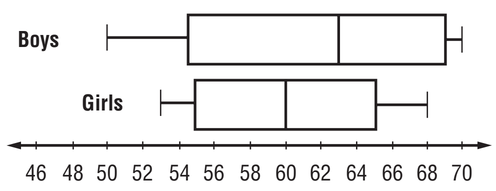
D $AB = AC$

51. EXTENDED RESPONSE What is the area of the triangle shown below? Explain how you found your answer.



52. STATISTICS The box-and-whisker plot below represents the heights of 9th graders at a certain high school. How much greater was the median height of the boys than the median height of the girls?

Heights of 9th Graders (inches)



F 3 inches

G 4 inches

H 5 inches

J 6 inches

53. SAT/ACT Heather, Teresa, and Nina went shopping for new clothes. Heather spent twice as much as Teresa, and Nina spent three times what Heather spent. If they spent a total of \$300, how much did Teresa spend?

A \$33.33

B \$50.00

C \$66.33

D \$100.00

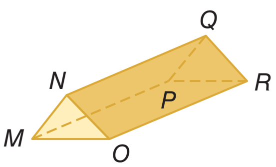
E \$104.33

Spiral Review

54. LUNCH For the past four Tuesdays, Jason's school has served chicken sandwiches for lunch. Jason assumes that chicken sandwiches will be served for lunch on the next Tuesday. What type of reasoning did he use? Explain. (Lesson 2-1)

Identify each solid. Name the bases, faces, edges, and vertices. (Lesson 1-7)

55.



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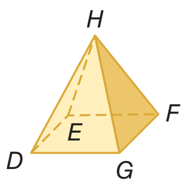

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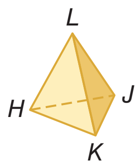

ALGEBRA **Solve each equation.** (Lesson 0-5)

58. $\frac{y}{2} - 7 = 5$

59. $3x + 9 = 6$

60. $4(m - 5) = 12$

61. $6(w + 7) = 0$

62. $2x - 7 = 11$

63. $\frac{y}{5} + 4 = 9$

Skills Review

ALGEBRA **Evaluate each expression for the given values.**

64. $2y + 3x$ if $y = 3$ and $x = -1$

65. $4d - c$ if $d = 4$ and $c = 2$

66. $m^2 + 7n$ if $m = 4$ and $n = -2$

67. $ab - 2a$ if $a = -2$ and $b = -3$