Conditional Statements

: · Then	: · Now	: · Why?
● You used logic and Venn diagrams to determine truth values of negations, conjunctions, and disjunctions.	● **1** Analyze statements in if-then form. **2** Write the converse, inverse, and contrapositive of if-then statements.	● Call centers route calls to the appropriate departments using menus that allow callers to choose from a number of options. The recorded directions are frequently in the form of conditional statements.

NewVocabulary
conditional statement
if-then statement
hypothesis
conclusion
related conditionals
converse
inverse
contrapositive
logically equivalent

1 **If-Then Statements** A **conditional statement** is a statement that can be written in *if-then form*. The direction given below is an example of a conditional statement.

If you would like to speak to a representative, **then** you will press 0 now.

KeyConcept Conditional Statement

Words	Symbols	Model
An **if-then statement** is of the form *if p, then q*.	$p \rightarrow q$ read *if p then q,* or *p implies q*	
The **hypothesis** of a conditional statement is the phrase immediately following the word *if*.	p	$p \rightarrow q$
The **conclusion** of a conditional statement is the phrase immediately following the word *then*.	q	

When a conditional statement is written as an if-then statement, you can quickly identify its hypothesis and conclusion.

Example 1 Identify the Hypothesis and Conclusion

Identify the hypothesis and conclusion of each conditional statement.

a. **If the forecast is rain, then I will take an umbrella.**
   Hypothesis: The forecast is rain.
   Conclusion: I will take an umbrella.

b. **A number is divisible by 10 if its last digit is a 0.**
   Hypothesis: The last digit of a number is zero.
   Conclusion: The number is divisible by 10.

▶ **GuidedPractice**

1A. If a polygon has six sides, then it is a hexagon.

1B. Another performance will be scheduled if the first one is sold out.

Tom Grill/CORBIS

Many conditional statements are written without using the words *if* and *then*. To write these statements in if-then form, identify the hypothesis and conclusion.

Points will be deducted from any **paper turned in after Wednesday's deadline.**

        Conclusion   Hypothesis

If **a paper is turned in after Wednesday's deadline**, then **points will be deducted.**

Remember, the conclusion depends upon the hypothesis.

Example 2 Write a Conditional in If-Then Form

Identify the hypothesis and conclusion for each conditional statement. Then write the statement in if-then form.

a. A mammal is a warm-blooded animal.

   Hypothesis: An animal is a mammal.

   Conclusion: It is warm-blooded.

   If an animal is a mammal, then it is warm-blooded.

b. A prism with bases that are regular polygons is a regular prism.

   Hypothesis: A prism has bases that are regular polygons.

   Conclusion: It is a regular prism.

   If a prism has bases that are regular polygons, then it is a regular prism.

▶ **Guided**Practice

2A. Four quarters can be exchanged for a $1 bill.

2B. The sum of the measures of two supplementary angles is 180.

The hypothesis and the conclusion of a conditional statement can have a truth value of true or false, as can the conditional statement itself. Consider the following conditional.

If **Tom finishes his homework**, then **he will clean his room**.

Hypothesis	Conclusion	Conditional	
Tom finishes his homework.	Tom cleans his room.	If Tom finishes his homework, then he will clean his room.	
T	T	T	If Tom *does* finish his homework and he *does* clean his room, then the conditional is true.
T	F	F	If Tom does *not* clean his room after he *does* finish his homework, then he has not fulfilled his promise and the conditional is false.
F	T	?	The conditional only indicates what will happen if Tom *does* finish his homework. He could clean his room or not clean his room if he does *not* finish his homework.
F	F	?	

When the hypothesis of a conditional is not met, the truth of a conditional cannot be determined. When the truth of a conditional statement cannot be determined, it is considered true by default.

The results from the previous page can be used to create a truth table for conditional statements.

Conditional Statements		
p	q	$p \rightarrow q$
T	T	T
T	**F**	**F**
F	T	T
F	F	T

Notice that a conditional is false *only* when its hypothesis is true and its conclusion is false.

Notice too that when a hypothesis is false, the conditional will *always* be considered true, regardless of whether the conclusion is true or false.

To show that a conditional is true, you must show that for each case when the hypothesis is true, the conclusion is also true. To show that a conditional is false, you only need to find one counterexample.

Example 3 Truth Values of Conditionals

Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

a. **If you divide an integer by another integer, the result is also an integer.**

Counterexample: When you divide 1 by 2, the result is 0.5.

Since 0.5 is not an integer, the conclusion is false.
Since you can find a counterexample, the conditional statement is false.

b. **If next month is August, then this month is July.**

When the hypothesis is true, the conclusion is also true, since August is the month that follows July. So, the conditional statement is true.

c. **If a triangle has four sides, then it is concave.**

The hypothesis is false, since a triangle can never have four sides. A conditional with a false hypothesis is always true.

GuidedPractice

3A. If $\angle A$ is an acute angle, then $m\angle A$ is 35.   **3B.** If $\sqrt{x} = -1$, then $(-1)^2 = -1$.

2 **Related Conditionals** There are other statements that are based on a given conditional statement. These are known as **related conditionals**.

KeyConcept Related Conditionals

Words	Symbols	Examples
A conditional statement is a statement that can be written in the form *if p, then q*.	$p \rightarrow q$	If $m\angle A$ is 35, then $\angle A$ is an acute angle.
The **converse** is formed by exchanging the hypothesis and conclusion of the conditional.	$q \rightarrow p$	If $\angle A$ is an acute angle, then $m\angle A$ is 35.
The **inverse** is formed by negating both the hypothesis and conclusion of the conditional.	$\sim p \rightarrow \sim q$	If $m\angle A$ is *not* 35, then $\angle A$ is *not* an acute angle.
The **contrapositive** is formed by negating both the hypothesis and the conclusion of the converse of the conditional.	$\sim q \rightarrow \sim p$	If $\angle A$ is *not* an acute angle, then $m\angle A$ is *not* 35.

A conditional and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional are either both true or both false. Statements with the same truth values are said to be **logically equivalent**.

KeyConcept Logically Equivalent Statements

- A conditional and its contrapositve are logically equivalent.

- The converse and inverse of a conditional are logically equivalent.

If a conditional is true, the converse may or may not be true.

You can use logical equivalence to check the true value of statements. Notice that in Example 4, both the conditional and contrapositive are true. Also, both the converse and inverse are false.

🌐 **Real-World Example 4** Related Conditionals

NATURE **Write the converse, inverse, and contrapositive of the following true statement. Then use the information at the left to determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.**

Lions are cats that can roar.

Conditional:	First, rewrite the conditional in if-then form.
	If an animal is a lion, then it is a cat that can roar.
	Based on the information at the left, this statement is true.
Converse:	If an animal is a cat that can roar, then it is a lion.
	Counterexample: A tiger is a cat that can roar, but it is not a lion.
	Therefore, the converse is false.
Inverse:	If an animal is not a lion, then it is not a cat that can roar.
	Counterexample: A tiger is not a lion, but it is a cat that can roar.
	Therefore, the inverse is false.
Contrapositive:	If an animal is not a cat that can roar, then it is not a lion.
	Based on the information at the left, this statement is true.
CHECK	Check to see that logically equivalent statements have the same truth value.
	Both the conditional and contrapositive are true. ✔
	Both the converse and inverse are false. ✔

▶ **Guided**Practice

Write the converse, inverse, and contrapositive of each true conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

4A. Two angles that have the same measure are congruent.

4B. A hamster is a rodent.

Example 1   **Identify the hypothesis and conclusion of each conditional statement.**

    1. If today is Friday, then tomorrow is Saturday.

    2. If $2x + 5 > 7$, then $x > 1$.

    3. If two angles are supplementary, then the sum of the measures of the angles is 180.

    4. If two lines form right angles, then the lines are perpendicular.

Example 2   **Write each statement in if-then form.**

    ⑤ Sixteen-year-olds are eligible to drive.

    6. Cheese contains calcium.

    7. The measure of an acute angle is between 0 and 90.

    8. Equilateral triangles are equiangular.

    9. WEATHER Various kinds of precipitation form under different conditions. Write the three conditionals below in if-then form.

      a. Moisture in the air condenses and falls to form rain.

      b. Supercooled moisture in cumulonimbus clouds forms hail.

      c. When the temperature is freezing in all or most of the atmosphere, precipitation falls as snow.

Example 3   **Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.**

    10. If $x^2 = 16$, then $x = 4$.

    11. If you live in Charlotte, then you live in North Carolina.

    12. If tomorrow is Friday, then today is Thursday.

    13. If an animal is spotted, then it is a Dalmatian.

    14. If the measure of a right angle is 95, then bees are lizards.

    15. If pigs can fly, then $2 + 5 = 7$.

Example 4   **CCSS ARGUMENTS Write the converse, inverse, and contrapositive of each true conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.**

    16. If a number is divisible by 2, then it is divisible by 4.

    17. All whole numbers are integers

Practice and Problem Solving   **Extra Practice is on page R2.**

Example 1   **Identify the hypothesis and conclusion of each conditional statement.**

    18. If two angles are adjacent, then they have a common side.

    19. If you lead, then I will follow.

    20. If $3x - 4 = 11$, then $x = 5$.

   **21.** If two angles are vertical, then they are congruent.

Identify the hypothesis and conclusion of each conditional statement.

22. If the degree measure of an angle is between 90 and 180, then the angle is obtuse.

23. "If there is no struggle, there is no progress." (Frederick Douglass)

24. If a quadrilateral has four congruent sides, then it is a square.

25. If a convex polygon has five sides, then it is a pentagon.

Example 2 **Write each statement in if-then form.**

26. Get a free water bottle with a one-year membership.

27. Everybody at the party received a gift.

28. The intersection of two planes is a line.

29. The area of a circle is πr^2.

30. Collinear points lie on the same line.

31. A right angle measures 90 degrees.

32. MUSIC Different instruments are emphasized in different types of music. Write each statement in if-then form.

- Jazz music often incorporates trumpet or saxophone.
- Rock music emphasizes guitar and drums.
- In hip-hop music, the bass is featured.

33. ART Write the following statement in if-then form: At the Andy Warhol Museum in Pittsburgh, Pennsylvania, most of the collection is Andy Warhol's artwork.

34. SCIENCE The water on Earth is constantly changing through a process called the *water cycle*. Write the three conditionals below in if-then form.

The Water Cycle
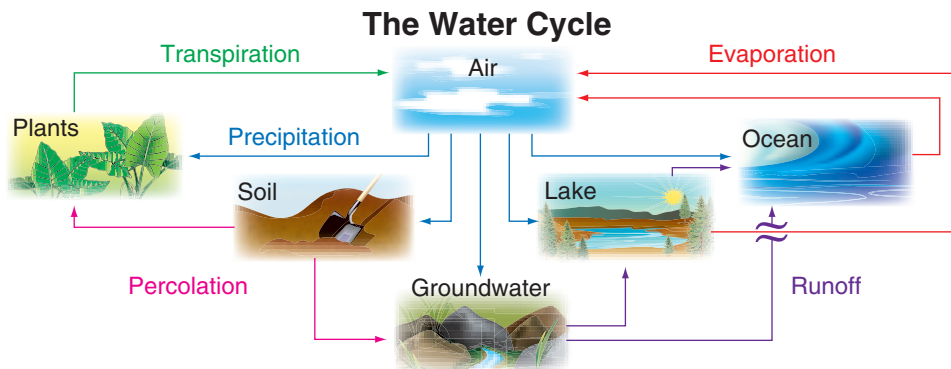
a. As runoff, water flows into bodies of water.
b. Plants return water to the air through transpiration.
c. Water bodies return water to the air through evaporation.

Example 3 **CCSS ARGUMENTS** Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

35. If a number is odd, then it is divisible by 5.

36. If a dog is an amphibian, then the season is summer.

37. If an angle is acute, then it has a measure of 45.

38. If a polygon has six sides, then it is a regular polygon.

Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

39. If an angle's measure is 25, then the measure of the angle's complement is 65.

40. If North Carolina is south of Florida, then the capital of Ohio is Columbus.

41. If red paint and blue paint mixed together make white paint, then $3 - 2 = 0$.

42. If two angles are congruent, then they are vertical angles.

43. If an animal is a bird, then it is an eagle.

44. If two angles are acute, then they are supplementary.

45. If two lines intersect, then they form right angles.

46. If a banana is blue, then an apple is a vegetable.

Example 4 **Write the converse, inverse, and contrapositive of each true conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.**

47. If you live in Chicago, you live in Illinois.

48. If a bird is an ostrich, then it cannot fly.

49. If two angles have the same measure, then the angles are congruent.

50. All squares are rectangles.

51. All congruent segments have the same length.

52. A right triangle has an angle measure of 90.

CCSS **ARGUMENTS** Write the statement indicated, and determine the truth value of each statement. If a statement is false, give a counterexample.

Animals with stripes are zebras.

**53.** conditional   **54.** converse   **55** inverse   **56.** contrapositive

57. SCIENCE Chemical compounds are grouped and described by the elements that they contain. Acids contain hydrogen (H). Bases contain hydroxide (OH). Hydrocarbons contain only hydrogen (H) and carbon (C).

Compound	Example	Chemical Formula
Acid	Hydrochloric Acid	HCl
Base	Sodium Hydroxide	NaOH
Hydrocarbon	Methane	CH_4

a. Write three conditional statements in if-then form for classifying chemical compounds.

b. Write the converse of the three true conditional statements. State whether each is *true* or *false*. If a statement is false, find a counterexample.

58. SPORTS In football, touchdowns are worth 6 points, extra point conversions are worth 2 points, and safeties are worth 2 points.

a. Write three conditional statements in if-then form for scoring in football.

b. Write the converse of the three true conditional statements. State whether each is *true* or *false*. If a statement is false, find a counterexample.

Use the Venn diagrams below to determine the truth value of each conditional. Explain your reasoning.
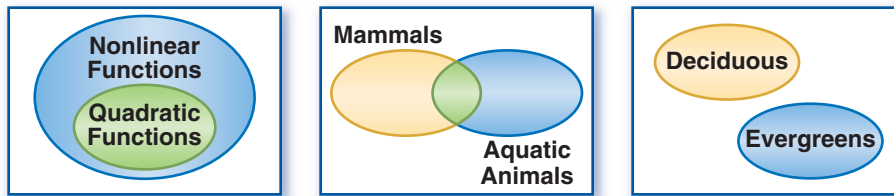
59 If a function is nonlinear, then it is quadratic.

60. If an animal is a mammal, then it cannot be aquatic.

61. If a tree is deciduous, then it is not an evergreen.

62. 🔷 MULTIPLE REPRESENTATIONS In this problem, you will investigate a law of logic by using conditionals.

 a. Logical Write three true conditional statements, using each consecutive conclusion as the hypothesis for the next statement.

 b. Graphical Create a Venn diagram to model your series of statements.

 c. Logical Write a conditional using the hypothesis of your first conditional and the conclusion of your third conditional. Is the conditional true if the hypothesis is true?

 d. Verbal Given two conditionals *If a, then b* and *If b, then c*, make a conjecture about the truth value of *c* when *a* is true. Explain your reasoning.

H.O.T. Problems   Use Higher-Order Thinking Skills

63. CCSS **CRITIQUE** Nicole and Kiri are evaluating the conditional *If 15 is a prime number, then 20 is divisible by 4*. Both think that the conditional is true, but their reasoning differs. Is either of them correct? Explain.

Nicole	Kiri
The conclusion is true, because 20 is divisible by 4, so the conditional is true.	The hypothesis is false, because 15 is not a prime number, so the conditional is true.

64. CHALLENGE You have learned that statements with the same truth value are logically equivalent. Use logical equivalence to create a truth table that summarizes the conditional, converse, inverse, and contrapositive for the statements *p* and *q*.

65. REASONING You are evaluating a conditional statement in which the hypothesis is true, but the conclusion is false. Is the inverse of the statement true or false? Explain your reasoning.

66. OPEN ENDED Write a conditional statement in which the converse, inverse, and contrapositive are all true. Explain your reasoning.

67. CHALLENGE The inverse of conditional *A* is given below. Write conditional *A*, its converse, and its contrapositive. Explain your reasoning.

 If I received a detention, then I did not arrive at school on time.

68. WRITING IN MATH Describe the relationship between a conditional, its converse, its inverse, and its contrapositive.

69. *If the sum of the measures of two angles is 90, then the angles are complementary angles.*

Which of the following is the converse of the conditional above?

A If the angles are complementary angles, then the sum of the measures of two angles is 90.

B If the angles are not complementary angles, then the sum of the measures of the angles is 90.

C If the angles are complementary angles, then the sum of the measures of the angles is not 90.

D If the angles are not complementary angles, then the sum of the measures of two angles is not 90.

70. ALGEBRA What is $\dfrac{10a^2 - 15ab}{4a^2 - 9b^2}$ reduced to lowest terms?

F $\dfrac{5a}{2a - 2b}$

H $\dfrac{a}{2a + 3b}$

G $\dfrac{5a}{2a + 3b}$

J $\dfrac{a}{2a - 3b}$

71. SHORT RESPONSE What is the standard notation for the following expression?

$$4.62 \times 10^{-3}$$

72. SAT/ACT What is the greatest common prime factor of 18 and 33?

A 1   **D** 5

B 2   **E** 11

C 3

Spiral Review

Construct a truth table for each compound statement. (Lesson 2-2)

73. p and q **74.** p or $\sim q$ **75.** $\sim p \wedge q$ **76.** $\sim p \wedge \sim q$

**Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.** (Lesson 2-1)

77. Points H, J, and K are each located on different sides of a triangle.

78. Collinear points X, Y, and Z; Z is between X and Y.

79. $R(3, -4)$, $S(-2, -4)$, and $T(0, -4)$

80. $A(-1, -7)$, $B(4, -7)$, $C(4, -3)$, and $D(-1, -3)$

81. KITES Kite making has become an art form. The kite shown is known as a diamond kite. The measures are in inches. Name all of the congruent segments in the figure. (Lesson 1-2)
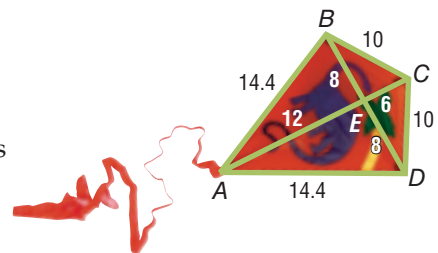
Refer to the conversion charts inside the back cover of your textbook and in Lesson 0-2. (Lesson 0-2)

82. RUNNING Ling is participating in a 5-kilometer charity run next weekend. About how many miles is the race?

83. NATURE An African elephant weighs about 9 tons. About how many kilograms is this?

84. SPORTS A football field is 120 yards long from one end zone to the other. How many feet long is a football field?

Skills Review

ALGEBRA **Identify the operation used to change Equation (1) to Equation (2).**

85. (1) $8(y - 11) = 32$

(2) $y - 11 = 4$

86. (1) $x + 9 = 4 - 3x$

(2) $4x + 9 = 4$

87. (1) $\frac{1}{3}m = 2$

(2) $m = 6$