

Then

- You used inductive reasoning to analyze patterns and make conjectures.

Now

- Use the Law of Detachment.
- Use the Law of Syllogism.

Why?

- When detectives are trying to solve a case, they use techniques like fingerprinting to analyze evidence. Then they use this evidence to eliminate suspects and eventually identify the person responsible for the crime.



New Vocabulary

deductive reasoning
valid
Law of Detachment
Law of Syllogism

1 Law of Detachment The process that detectives use to identify who is most likely responsible for a crime is called deductive reasoning. Unlike inductive reasoning, which uses a pattern of examples or observations to make a conjecture, **deductive reasoning** uses facts, rules, definitions, or properties to reach logical conclusions from given statements.

 **Real-World Example 1 Inductive and Deductive Reasoning**

Determine whether each conclusion is based on *inductive* or *deductive* reasoning.

- a. Every time Katie has worn her favorite socks to a softball game, she has gotten at least one hit. Katie is wearing her favorite socks to a game tonight, so she concludes that she will get at least one hit.

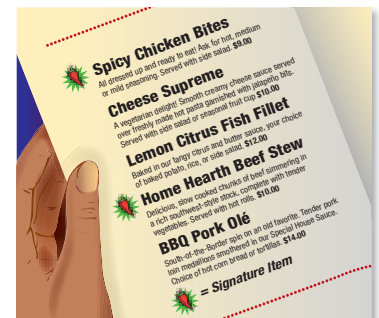
Katie is basing her conclusion on a pattern of observations, so she is using inductive reasoning.

- b. If John is late making his car insurance payment, he will be assessed a late fee of \$50. John's payment is late this month, so he concludes that he will be assessed a late fee of \$50.

John is basing his conclusion on facts provided to him by his insurance company, so he is using deductive reasoning.

 **Guided Practice**

- 1A. All of the signature items on the restaurant's menu shown are noted with a special symbol. Kevin orders a menu item that has this symbol next to it, so he concludes that the menu item that he has ordered is a signature item.
- 1B. None of the students who ride Raul's bus own a car. Ebony rides a bus to school, so Raul concludes that Ebony does not own a car.



While one counterexample is enough to disprove a conjecture reached using inductive reasoning, it is not a logically correct, or **valid**, method of proving a conjecture. To prove a conjecture requires deductive reasoning. One valid form of deductive reasoning is the **Law of Detachment**.



KeyConcept Law of Detachment

Words If $p \rightarrow q$ is a true statement and p is true, then q is true.

Example *Given:* If **a car is out of gas**, then **it will not start**.
Sarah's **car is out of gas**.

Valid Conclusion: Sarah's **car will not start**.

As long as the facts given are true, the conclusion reached using deductive reasoning will also be true.



Example 2 Law of Detachment

Determine whether each conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

- a. **Given:** If two angles form a linear pair, then their noncommon sides are opposite rays.
 $\angle AED$ and $\angle AEB$ form a linear pair.

Conclusion: \overrightarrow{ED} and \overrightarrow{EB} are opposite rays.

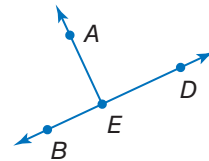
Step 1 Identify the hypothesis p and the conclusion q of the true conditional.

p : **Two angles form a linear pair.**

q : **Their noncommon sides are opposite rays.**

Step 2 Analyze the conclusion.

The given statement **$\angle AED$ and $\angle AEB$ form a linear pair** satisfies the hypothesis, so p is true. By the Law of Detachment, **\overrightarrow{ED} and \overrightarrow{EB} are opposite rays**, which matches q , is a true or valid conclusion.



- b. **Given:** If Mika goes to the beach, she will wear sunscreen.
Mika is wearing sunscreen.

Conclusion: Mika is at the beach.

Step 1 p : **Mika goes to the beach.**

q : **Mika wears sunscreen.**

Step 2 The given statement *Mika is wearing sunscreen* satisfies the conclusion q of the true conditional. However, knowing that a conditional statement and its conclusion are true does not make the hypothesis true. Mika could be wearing sunscreen because she is at the pool. The conclusion is invalid.

GuidedPractice

- 2A. Given:** If three points are noncollinear, they determine a plane.
Points A , B , and C lie in plane G .

Conclusion: Points A , B , and C are noncollinear.

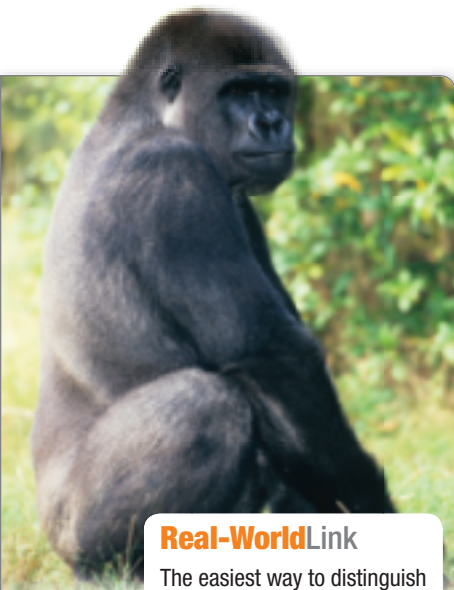
- 2B. Given:** If a student turns in a permission slip, then the student can go on the field trip.
Felipe turned in his permission slip.

Conclusion: Felipe can go on the field trip.

StudyTip

Given Information From this point forward in this text, all given information can be assumed true.





Real-WorldLink

The easiest way to distinguish monkeys from other primates is to look for a tail. Most monkey species have tails, but apes do not.

Source: Encyclopaedia Britannica

You can also use a Venn diagram to test the validity of a conclusion.



Example 3 Judge Conclusions Using Venn Diagrams

NATURE Determine whether each conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning using a Venn diagram.

Given: If a primate is an ape, then it does not have a tail.
Koko is a primate who does not have a tail.

Conclusion: Koko is an ape.

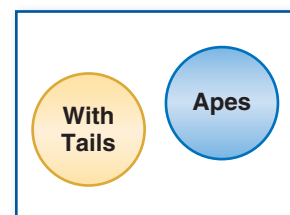
Understand Draw a Venn diagram. According to the conditional, an ape does not have a tail, so draw a circle for apes that does not intersect the circle for primates with tails.

Plan Since we are only given that Koko does not have a tail, we can only conclude that Koko belongs outside the circle for primates with tails.

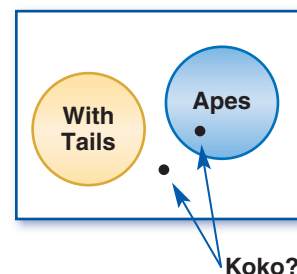
Solve This could put her in the area inside or outside of the Apes circle, so the conclusion is invalid.

Check From the given information, we know that apes are primates that do not have tails. We also know that Koko is a primate that does not have a tail. It is possible for Koko to be a primate without a tail and still not be an ape. Therefore, the conclusion is invalid. ✓

Primates



Primates



StudyTip

CCSS Arguments An *argument* consists of reasons, proof, or evidence to support a position. A *logical argument* such as the one shown is supported by the rules of logic. This is different from a *statistical argument*, which is supported by examples or data.

GuidedPractice

3. Given: If a figure is a square, then it is a polygon.
Figure *A* is a square.

Conclusion: Figure *A* is a polygon.

2 Law of Syllogism The **Law of Syllogism** is another valid form of deductive reasoning. This law allows you to draw conclusions from two true conditional statements when the conclusion of one statement is the hypothesis of the other.

KeyConcept Law of Syllogism

Words If $p \rightarrow q$ and $q \rightarrow r$ are true statements, then $p \rightarrow r$ is a true statement.

Example *Given:* If **you get a job**, then **you will earn money**.
If **you earn money**, then **you will buy a car**.

Valid Conclusion: If **you get a job**, then **you will buy a car**.

It is important to remember that if the conclusion of the first statement is *not* the hypothesis of the second statement, no valid conclusion can be drawn.





Standardized Test Example 4 Law of Syllogism

Determine which statement follows logically from the given statements.

(1) If you like musicals, then you enjoy theater productions.

(2) If you are an actor, then you enjoy theater productions.

- A If you are an actor, then you like musicals.
- B If you like musicals, then you are an actor.
- C If you do not enjoy musicals, then you are not an actor.
- D There is no valid conclusion.

Read the Test Item

Let p , q , and r represent the parts of the given conditional statements.

p : You like musicals.

q : You enjoy theater productions.

r : You are an actor.

Solve the Test Item

Analyze the logic of the given conditional statement using symbols.

Statement (1): $p \rightarrow q$

Statement (2): $r \rightarrow q$

Both statements are considered true. However, the Law of Syllogism does not apply since q , the conclusion of the Statement (1), is not the hypothesis of the second statement. While choices A, B, and C may be true, the logic used to draw these conclusions is not valid. Therefore, choice D is correct.

Guided Practice

4. Determine which statement follows logically from the given statements.

(1) If you do not get enough sleep, then you will be tired.

(2) If you are tired, then you will not do well on the test.

F If you are tired, then you will not get enough sleep.

G If you do not get enough sleep, then you will not do well on the test.

H If you do not do well on the test, then you did not get enough sleep.

J There is no valid conclusion.

Test-Taking Tip

True vs. Valid Conclusions

A true conclusion is not the same as a valid conclusion. True conclusions that are reached using invalid deductive reasoning are still invalid.



Example 5 Apply Laws of Deductive Reasoning

Draw a valid conclusion from the given statements, if possible. Then state whether your conclusion was drawn using the Law of Detachment or the Law of Syllogism. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning.

Given: If you are 16 years old, then you can apply for a driver's license. Nate is 16 years old.

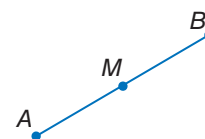
p : You are 16 years old.

q : You can apply for a driver's license.

Since Nate is 16 years old satisfies the hypothesis p is true. By the Law of Detachment, a valid conclusion is *Nate can apply for a driver's license*.

Guided Practice

5. Given: The midpoint divides a segment into two congruent segments. If two segments are congruent, then their measures are equal. M is the midpoint of \overline{AB} .





Example 1 Determine whether each conclusion is based on *inductive* or *deductive* reasoning.

- 1 Students at Olivia’s high school must have a B average in order to participate in sports. Olivia has a B average, so she concludes that she can participate in sports at school.
2. Holly notices that every Saturday, her neighbor mows his lawn. Today is Saturday. Holly concludes her neighbor will mow his lawn.

Example 2 Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

3. **Given:** If a number is divisible by 4, then the number is divisible by 2. 12 is divisible by 4.

Conclusion: 12 is divisible by 2.

4. **Given:** If Elan stays up late, he will be tired the next day. Elan is tired.

Conclusion: Elan stayed up late.

Example 3 Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning using a Venn diagram.

5. **Given:** If a beach is public, then it does not have a lifeguard. Bayview does not have a lifeguard.

Conclusion: Bayview is a public beach.

6. **Given:** If students pass an entrance exam, they will be accepted into college. Latisha passed the entrance exam.

Conclusion: Latisha will be accepted into college.



Example 4 7. **MULTIPLE CHOICE** Determine which statement follows logically from the given statements.

- (1) If a triangle is a right triangle, then it has an angle that measures 90.
- (2) If a triangle has an angle that measures 90, then its acute angles are complementary.
 - A If a triangle is not a right triangle, then it has an angle that measures 90.
 - B If an angle of a triangle measures 90, then its acute angles are not complementary.
 - C If a triangle is a right triangle, then its acute angles are complementary.
 - D If a triangle has an angle that measures 90, then it is not a right triangle.

Example 5 **CCSS ARGUMENTS** Draw a valid conclusion from the given statements, if possible. Then state whether your conclusion was drawn using the Law of Detachment or the Law of Syllogism. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning.


8. **Given:** If Dalila finishes her chores, she will receive her allowance.
If Dalila receives her allowance, she will buy a CD.

9. **Given:** Vertical angles are congruent.
 $\angle 1 \cong \angle 2$



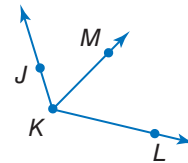
Example 1 Determine whether each conclusion is based on *inductive* or *deductive* reasoning.

10. At Fumio’s school if you are late five times, you will receive a detention. Fumio has been late to school five times; therefore he will receive a detention.
11. A dental assistant notices a patient has never been on time for an appointment. She concludes the patient will be late for her next appointment.
12. A person must have a membership to work out at a gym. Jesse is working out at a gym. Jesse has a membership to the gym.
13. If Eduardo decides to go to a concert tonight, he will miss football practice. Tonight, Eduardo went to a concert. Eduardo missed football practice.
14. Every Wednesday Lucy’s mother calls. Today is Wednesday, so Lucy concludes her mother will call.
15. Whenever Juanita has attended a tutoring session she notices that her grades have improved. Juanita attends a tutoring session and she concludes her grades will improve.

Example 2  **CRITIQUE** Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

16. **Given:** Right angles are congruent. $\angle 1$ and $\angle 2$ are right angles.
Conclusion: $\angle 1 \cong \angle 2$
17. **Given:** If a figure is a square, it has four right angles. Figure $ABCD$ has four right angles.
Conclusion: Figure $ABCD$ is a square.

18. **Given:** An angle bisector divides an angle into two congruent angles.
 \overrightarrow{KM} is an angle bisector of $\angle JKL$.



- Conclusion:** $\angle JKM \cong \angle MKL$
19. **Given:** If you leave your lights on while your car is off, your battery will die. Your battery is dead.
Conclusion: You left your lights on while the car was off.
20. **Given:** If Dante obtains a part-time job, he can afford a car payment. Dante can afford a car payment.
Conclusion: Dante obtained a part-time job.
21. **Given:** If 75% of the prom tickets are sold, the prom will be held at the country club. 75% of the prom tickets were sold.
Conclusion: The prom will be held at the country club.

22. **COMPUTER GAMES** Refer to the game ratings at the right. Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

Given: If a title is rated E, then it has content that may be suitable for ages 6 and older. Cesar buys a computer game that he believes is suitable for his little sister, who is 7.

Conclusion: The game Cesar purchased has a rating of E.

Game Ratings	
Rating	Age
EC	3 and older
E	6 and older
E10+	10 and older
T	13 and older
M	17 and older



Example 3 Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning using a Venn diagram.

23. Given: If the temperature drops below 32°F, it may snow. The temperature did not drop below 32°F on Monday.

Conclusion: It did not snow on Monday.

24. Given: If a person is a Missouri resident, he or she does not live by a beach. Michelle does not live by the beach.

Conclusion: Michelle is a Missouri resident.

25. Given: Some nurses wear blue uniforms. Sabrina is a nurse.

Conclusion: Sabrina wears a blue uniform.

26. Given: All vegetarians do not eat meat. Theo is a vegetarian.

Conclusion: Theo does not eat meat.

27 TRANSPORTATION There are many types of vehicles and they are classified using different sets of criteria. Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning using a Venn diagram.

Given: If a vehicle is a sport-utility vehicle, then it is a four-wheel-drive car built on a truck chassis. Ms. Rodriguez has just purchased a vehicle that has four-wheel drive.

Conclusion: Ms. Rodriguez has just purchased a sport-utility vehicle.

Examples 4–5 **28. GOLF** Zach Johnson won the Masters Tournament in 2007. Use the Law of Syllogism to draw a valid conclusion from each set of statements, if possible. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning.

(1) If Zach Johnson's score is lower than the other golfers at the end of the tournament, then he wins the tournament.

(2) If a golfer wins the Masters Tournament, then he gets a green jacket.

CCSS ARGUMENTS Use the Law of Syllogism to draw a valid conclusion from each set of statements, if possible. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning.

29. If you interview for a job, then you wear a suit.

If you interview for a job, then you will update your resume.

30. If Tina has a grade point average of 3.0 or greater, she will be on the honor roll.

If Tina is on the honor roll, then she will have her name in the school paper.

31. If two lines are perpendicular, then they intersect to form right angles.

Lines r and s form right angles.

32. If the measure of an angle is between 90 and 180, then it is obtuse.

If an angle is obtuse, then it is not acute.

33. If two lines in a plane are not parallel, then they intersect.

If two lines intersect, then they intersect in a point.

34. If a number ends in 0, then it is divisible by 2.

If a number ends in 4, then it is divisible by 2.



Draw a valid conclusion from the given statements, if possible. Then state whether your conclusion was drawn using the Law of Detachment or the Law of Syllogism. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning.

35. **Given:** If a figure is a square, then all the sides are congruent.

Figure $ABCD$ is a square.

36. **Given:** If two angles are complementary, the sum of the measures of the angles is 90.

$\angle 1$ and $\angle 2$ are complements of each other.

37. **Given:** Ballet dancers like classical music.

If you like classical music, then you enjoy the opera.

38. **Given:** If you are athletic, then you enjoy sports.

If you are competitive, then you enjoy sports.

39. **Given:** If a polygon is regular, then all of its sides are congruent.

All sides of polygon $WXYZ$ are congruent.

40. **Given:** If Bob completes a course with a grade of C, then he will not receive credit.

If Bob does not receive credit, he will have to take the course again.

41. **DATA ANALYSIS** The table shows the number of at bats and hits for some of the members of the Florida Marlins in a recent season.

- Construct a scatter plot to represent the data.
- Predict the number of hits a player with 300 at bats would get. Identify and explain your reasoning.
- Did the player with 157 at bats or the player with 240 at bats get more hits? What type of reasoning did you use? Explain.

At Bats	Hits
13	6
576	195
240	79
502	139
157	36
64	11

Source: ESPN

H.O.T. Problems Use Higher-Order Thinking Skills

42. **WRITING IN MATH** Explain why the Law of Syllogism cannot be used to draw a conclusion from these conditionals.

If you wear winter gloves, then you will have warm hands.

If you do not have warm hands, then your gloves are too thin.

43. **CHALLENGE** Use the symbols from Lesson 2-2 for *conjunction* and *disjunction*, and the symbol for *implies* from Lesson 2-3 to represent the Law of Detachment and the Law of Syllogism symbolically. Let p represent the hypothesis, and let q represent the conclusion.

44. **OPEN ENDED** Write a pair of statements in which the Law of Syllogism can be used to reach a valid conclusion. Specify the conclusion that can be reached.

45. **CCSS REASONING** Students in Mr. Kendrick's class are divided into two groups for an activity. Students in group A must always tell the truth. Students in group B must always lie. Jonah and Janeka are in Mr. Kendrick's class. When asked if he and Janeka are in group A or B, Jonah says, "We are both in Group B." To which group does each student belong? Explain your reasoning.

46. **WRITING IN MATH** Compare and contrast inductive and deductive reasoning when making conclusions and proving conjectures.



Standardized Test Practice

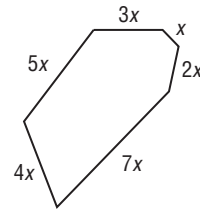
47. Determine which statement follows logically from the given statements.
If you order two burritos, then you also get nachos.
Michael ordered two burritos.
- A Michael ordered one burrito.
 B Michael will order two burritos.
 C Michael ordered nachos.
 D Michael got nachos.

48. **ALGEBRA** Solve for x :

$$4(x + 2) = x - 1$$

- F -3
 G -5
 H -6
 J -8

49. **SHORT RESPONSE** If the perimeter of the figure shown is 52 units, what is the value of x ?



50. **SAT/ACT** If 30% of x is 50, then 60% of x is

- A 300
 B 250
 C 175
 D 150
 E 100

Spiral Review

51. **TIME** All states in the United States observe daylight savings time except for Arizona and Hawaii. (Lesson 2-3)
- a. Write a true conditional statement in if-then form for daylight savings time.
 b. Write the converse of the true conditional statement. State whether the statement is *true* or *false*. If false, find a counterexample.

Construct a truth table for each compound statement. (Lesson 2-2)

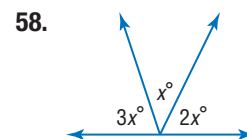
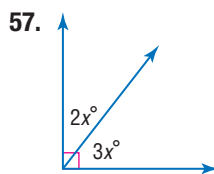
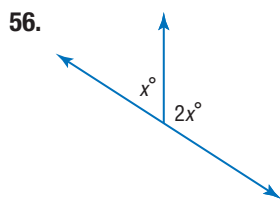
52. a and b

53. $\sim p$ or $\sim q$

54. k and $\sim m$

55. $\sim y$ or z

ALGEBRA Find x . (Lesson 1-5)



Skills Review

Determine whether each statement can be assumed from the figure. Explain.

59. $\angle DAB$ is a right angle.
 60. $\angle AEB \cong \angle DEC$
 61. $\angle ADB$ and $\angle BDC$ are complementary.
 62. $\angle DAE \cong \angle ADE$
 63. $\overline{AB} \perp \overline{BC}$
 64. $\angle AEB$ and $\angle BEC$ are supplementary.

