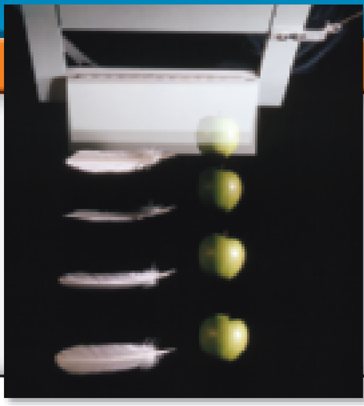
LESSON 2-5

Postulates and Paragraph Proofs



Then

- You used deductive reasoning by applying the Law of Detachment and the Law of Syllogism.

Now

1. Identify and use basic postulates about points, lines, and planes.
2. Write paragraph proofs.

Why?

- If a feather and an apple are dropped from the same height in a vacuum chamber, the two objects will fall at the same rate. This demonstrates one of Sir Isaac Newton's laws of gravity and inertia. These laws are accepted as fundamental truths of physics. Some laws in geometry also must be assumed or accepted as true.

NewVocabulary

postulate
axiom
proof
theorem
deductive argument
paragraph proof
informal proof

Common Core State Standards

Content Standards

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

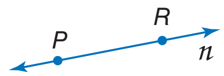
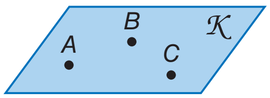
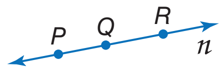
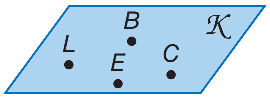
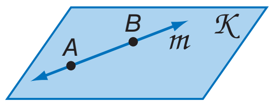
Mathematical Practices

2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.

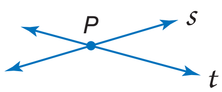
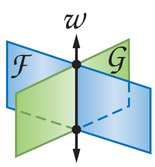
1 Points, Lines, and Planes

A **postulate** or **axiom** is a statement that is accepted as true without proof. Basic ideas about points, lines, and planes can be stated as postulates.

Postulates Points, Lines, and Planes

Words	Example	
2.1 Through any two points, there is exactly one line.		Line n is the only line through points P and R.
2.2 Through any three noncollinear points, there is exactly one plane.		Plane $\mathcal{K}$ is the only plane through noncollinear points A, B, and C.
2.3 A line contains at least two points.		Line n contains points P, Q, and R.
2.4 A plane contains at least three noncollinear points.		Plane $\mathcal{K}$ contains noncollinear points L, B, C, and E.
2.5 If two points lie in a plane, then the entire line containing those points lies in that plane.		Points A and B lie in plane $\mathcal{K}$ and line m contains points A and B, so line m is in plane $\mathcal{K}$.

KeyConcept Intersections of Lines and Planes

Words	Example	
2.6 If two lines intersect, then their intersection is exactly one point.		Lines s and t intersect at point P.
2.7 If two planes intersect, then their intersection is a line.		Planes $\mathcal{F}$ and $\mathcal{G}$ intersect in line w.

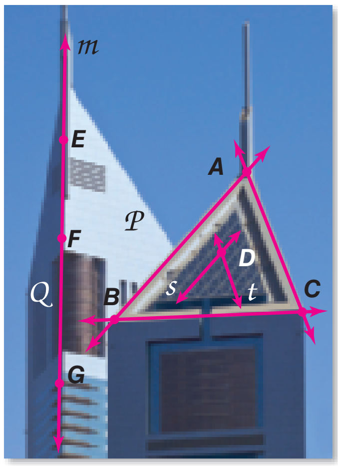
Jim Sugar/CORBIS

> **StudyTip**
> **Undefined Terms** Recall from Lesson 1-1 that points, lines, and planes are *undefined terms*. The postulates that you have learned in this lesson describe special relationships between them.

These additional postulates form a foundation for proofs and reasoning about points, lines, and planes.

Real-World Example 1 Identifying Postulates

ARCHITECTURE Explain how the picture illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.



a. Line *m* contains points *F* and *G*. Point *E* can also be on line *m*.

The edge of the building is a straight line *m*. Points *E*, *F*, and *G* lie along this edge, so they lie along a line *m*. Postulate 2.3, which states that a line contains at least two points, shows that this is true.

b. Lines *s* and *t* intersect at point *D*.

The lattice on the window of the building forms intersecting lines. Lines *s* and *t* of this lattice intersect at only one location, point *D*. Postulate 2.6, which states that if two lines intersect, then their intersection is exactly one point, shows that this is true.

GuidedPractice

1A. Points *A*, *B*, and *C* determine a plane. **1B.** Planes *P* and *Q* intersect in line *m*.

You can use postulates to explain your reasoning when analyzing statements.

Example 2 Analyze Statements Using Postulates

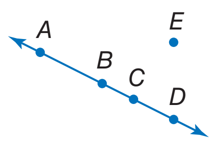
Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

a. If two coplanar lines intersect, then the point of intersection lies in the same plane as the two lines.

Always; Postulate 2.5 states that if two points lie in a plane, then the entire line containing those points lies in that plane. So, since both points lie in the plane, any point on those lines, including their point of intersection, also lies in the plane.

b. Four points are noncollinear.

Sometimes; Postulate 2.3 states that a line contains at least two points. This means that a line can contain two *or more* points. So four points can be noncollinear, like *A*, *E*, *C*, and *D*, or collinear, like points *A*, *B*, *C*, and *D*.



GuidedPractice

2A. Two intersecting lines determine a plane. **2B.** Three lines intersect in two points.

> **StudyTip**
> **Axiomatic System** An axiomatic system is a set of axioms, from which some or all axioms can be used to logically derive theorems.

2 Paragraph Proofs

To prove a conjecture, you use deductive reasoning to move from a hypothesis to the conclusion of the conjecture you are trying to prove. This is done by writing a **proof**, which is a logical argument in which each statement you make is supported by a statement that is accepted as true.

Jochem Wijnands/age fotostock

Once a statement or conjecture has been proven, it is called a **theorem**, and it can be used as a reason to justify statements in other proofs.

KeyConcept The Proof Process

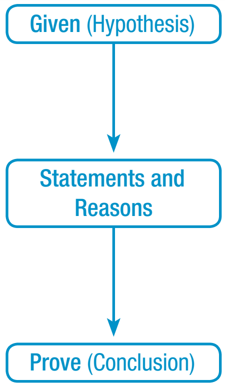
Step 1 List the given information and, if possible, draw a diagram to illustrate this information.

Step 2 State the theorem or conjecture to be proven.

Step 3 Create a **deductive argument** by forming a logical chain of statements linking the given to what you are trying to prove.

Step 4 Justify each statement with a reason. Reasons include definitions, algebraic properties, postulates, and theorems.

Step 5 State what it is that you have proven.



> **StudyTip**
> **Proposition** A *proposition* is a statement that makes an assertion that is either false or true. In mathematics, a proposition is usually used to mean a true assertion and can be synonymous with theorem.

One method of proving statements and conjectures, a **paragraph proof**, involves writing a paragraph to explain why a conjecture for a given situation is true. Paragraph proofs are also called **informal proofs**, although the term *informal* is not meant to imply that this form of proof is any less valid than any other type of proof.

Example 3 Write a Paragraph Proof

Given that M is the midpoint of $\overline{XY}$ write a paragraph proof to show that $\overline{XM} \cong \overline{MY}$.

Steps 1 and 2 →

Given: M is the midpoint of $\overline{XY}$.

Prove: $\overline{XM} \cong \overline{MY}$

Steps 3 and 4 →

If M is the midpoint of $\overline{XY}$, then from the definition of midpoint of a segment, we know that $XM = MY$. This means that $\overline{XM}$ and $\overline{MY}$ have the same measure. By the definition of congruence, if two segments have the same measure, then they are congruent.

Step 5 →

Thus, $\overline{XM} \cong \overline{MY}$.

> **Problem-SolvingTip**
> **Work Backward** One strategy for writing a proof is to *work backward*. Start with what you are trying to prove, and work backward step by step until you reach the given information.

GuidedPractice

3. Given that C is between A and B and $\overline{AC} \cong \overline{CB}$, write a paragraph proof to show that C is the midpoint of $\overline{AB}$.

Once a conjecture has been proven true, it can be stated as a theorem and used in other proofs. The conjecture in Example 3 is known as the Midpoint Theorem.

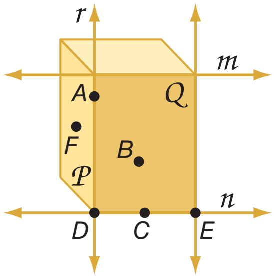
Theorem 2.1 Midpoint Theorem

If M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **Explain how the figure illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.**



1. Planes $\mathcal{P}$ and Q intersect in line r.

2. Lines r and n intersect at point D.

3. Line n contains points C, D, and E.

4. Plane $\mathcal{P}$ contains the points A, F, and D.

5. Line n lies in plane Q.

6. Line r is the only line through points A and D.

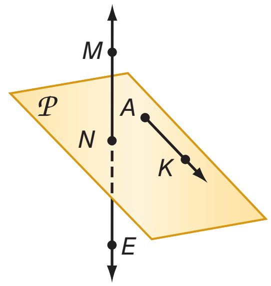
Example 2 **Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.**

7. The intersection of three planes is a line.

8. Line r contains only point $\mathcal{P}$.

9. Through two points, there is exactly one line.

In the figure, $\overrightarrow{AK}$ is in plane $\mathcal{P}$ and M is on $\overleftrightarrow{NE}$. State the postulate that can be used to show each statement is true.



10. M, K, and N are coplanar.

11. $\overleftrightarrow{NE}$ contains points N and M.

12. N and K are collinear.

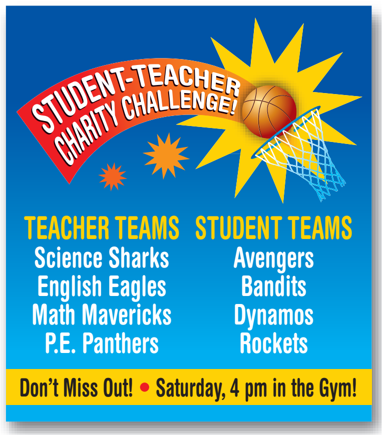
13. Points N, K, and A are coplanar.

14. SPORTS Each year, Jennifer's school hosts a student vs. teacher basketball tournament to raise money for charity. This year, there are eight teams participating in the tournament. During the first round, each team plays all of the other teams.

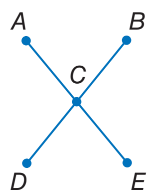
a. How many games will be played in the first round?

b. Draw a diagram to model the number of first round games. Which postulate can be used to justify your diagram?

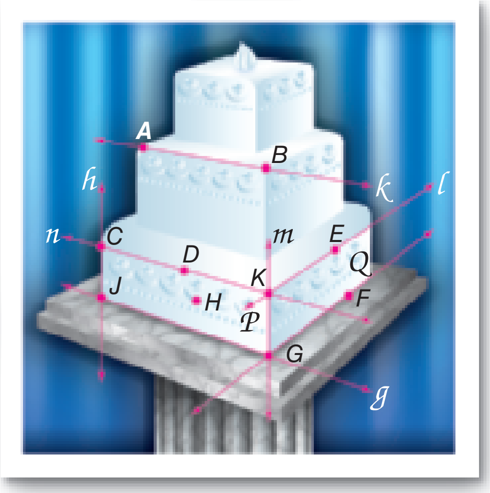
c. Find a numerical method that you could use regardless of the number of the teams in the tournament to calculate the number of games in the first round.



Example 3 **15. CCSS ARGUMENTS** In the figure at the right, $\overline{AE} \cong \overline{DB}$ and C is the midpoint of $\overline{AE}$ and $\overline{DB}$. Write a paragraph proof to show that $AC = CB$.



Example 1 **CAKES** **Explain how the picture illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.**



16. Lines n and ℓ intersect at point K.

17. Planes $\mathcal{P}$ and $\mathcal{Q}$ intersect in line m.

18. Points D, K, and H determine a plane.

19. Point D is also on the line n through points C and K.

20. Points D and H are collinear.

21. Points E, F, and G are coplanar.

22. $\overleftrightarrow{EF}$ lies in plane $\mathcal{Q}$.

23. Lines h and g intersect at point J.

Example 2 **Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.**

24. There is exactly one plane that contains noncollinear points A, B, and C.

25. There are at least three lines through points J and K.

26. If points M, N, and P lie in plane $\mathcal{X}$, then they are collinear.

27. Points X and Y are in plane $\mathcal{Z}$. Any point collinear with X and Y is in plane $\mathcal{Z}$.

28. The intersection of two planes can be a point.

29. Points A, B, and C determine a plane.

Example 3 **30.** **PROOF** Point Y is the midpoint of $\overline{XZ}$. Z is the midpoint of $\overline{YW}$. Prove that $\overline{XY} \cong \overline{ZW}$.

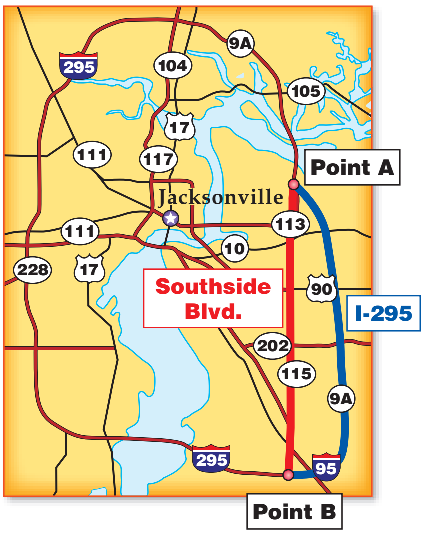
31. **PROOF** Point L is the midpoint of $\overline{JK}$. $\overline{JK}$ intersects $\overline{MK}$ at K. If $\overline{MK} \cong \overline{JL}$, prove that $\overline{LK} \cong \overline{MK}$.

32. **CCSS ARGUMENTS** Last weekend, Emilio and his friends spent Saturday afternoon at the park. There were several people there with bikes and skateboards. There were a total of 11 bikes and skateboards that had a total of 36 wheels. Use a paragraph proof to show how many bikes and how many skateboards there were.

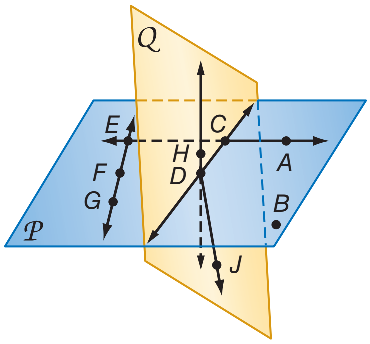
33. **DRIVING** Keisha is traveling from point A to point B. Two possible routes are shown on the map. Assume that the speed limit on Southside Boulevard is 55 miles per hour and the speed limit on I–295 is 70 miles per hour.

a. Which of the two routes covers the shortest distance? Explain your reasoning.

b. If the distance from point A to point B along Southside Boulevard is 10.5 miles and the distance along I-295 is 11.6 miles, which route is faster, assuming that Keisha drives the speed limit?



In the figure at the right, $\overleftrightarrow{CD}$ and $\overleftrightarrow{CE}$ lie in plane $\mathcal{P}$ and $\overleftrightarrow{DH}$ and $\overleftrightarrow{DJ}$ lie in plane Q. State the postulate that can be used to show each statement is true.



34. Points C and B are collinear.

35. $\overleftrightarrow{EG}$ contains points E, F, and G.

36. $\overleftrightarrow{DA}$ lies in plane $\mathcal{P}$.

37. Points D and F are collinear.

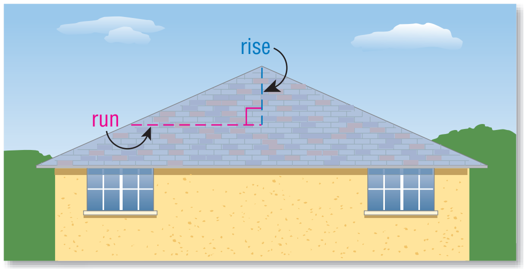
38. Points C, D, and B are coplanar.

39. Plane Q contains the points C, H, D, and J.

40. $\overleftrightarrow{AC}$ and $\overleftrightarrow{FG}$ intersect at point E.

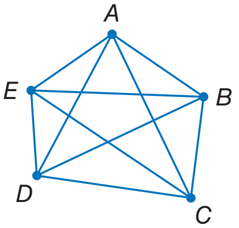
41. Plane $\mathcal{P}$ and plane Q intersect at $\overleftrightarrow{CD}$.

42. **CCSS ARGUMENTS** Roofs are designed based on the materials used to ensure that water does not leak into the buildings they cover. Some roofs are constructed from waterproof material, and others are constructed for watershed, or gravity removal of water. The pitch of a roof is the rise over the run, which is generally measured in rise per foot of run. Use the statements below to write a paragraph proof justifying the following statement: The pitch of the roof in Den's design is not steep enough.



- Waterproof roofs should have a minimum slope of $\frac{1}{4}$ inch per foot.
- Watershed roofs should have a minimum slope of 4 inches per foot.
- Den is designing a house with a watershed roof.
- The pitch in Den's design is 2 inches per foot.

43. **NETWORKS** Diego is setting up a network of multiple computers so that each computer is connected to every other. The diagram at the right illustrates this network if Diego has 5 computers.



a. Draw diagrams of the networks if Diego has 2, 3, 4, or 6 computers.

b. Create a table with the number of computers and the number of connections for the diagrams you drew.

c. If there are n computers in the network, write an expression for the number of computers to which each of the computers is connected.

d. If there are n computers in the network, write an expression for the number of connections there are.

44. **SENSE-MAKING** The photo is of the rotunda in the capitol building in St. Paul, Minnesota. A rotunda is a round building, usually covered by a dome. Use Postulate 2.1 to help you answer parts **a–c.**

a. If you were standing in the middle of the rotunda, which arched exit is the closest to you?

b. What information did you use to formulate your answer?

c. What term describes the shortest distance from the center of a circle to a point on the circle?

H.O.T. Problems Use Higher-Order Thinking Skills

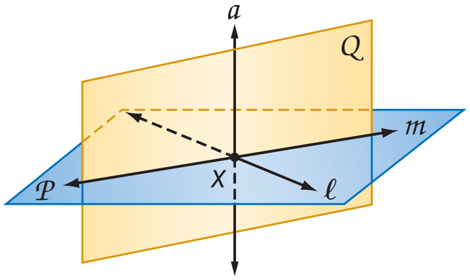
45. **ERROR ANALYSIS** Omari and Lisa were working on a paragraph proof to prove that if $\overline{AB}$ is congruent to $\overline{BD}$ and A, B, and D are collinear, then B is the midpoint of $\overline{AD}$. Each student started his or her proof in a different way. Is either of them correct? Explain your reasoning.

Omari	Lisa
If B is the midpoint of $\overline{AB}$, then B divides $\overline{AD}$ into two congruent segments.	$\overline{AB}$ is congruent to $\overline{BD}$ and A, B, and D are collinear.

46. **OPEN ENDED** Draw a figure that satisfies five of the seven postulates you have learned. Explain which postulates you chose and how your figure satisfies each postulate.

47. **CHALLENGE** Use the following true statement and the definitions and postulates you have learned to answer each question.

Two planes are perpendicular if and only if one plane contains a line perpendicular to the second plane.



a. Through a given point, there passes one and only one plane perpendicular to a given line. If plane Q is perpendicular to line ℓ at point X and line ℓ lies in plane $\mathcal{P}$, what must also be true?

b. Through a given point, there passes one and only one line perpendicular to a given plane. If plane Q is perpendicular to plane $\mathcal{P}$ at point X and line a lies in plane Q, what must also be true?

REASONING **Determine if each statement is *sometimes, always,* or *never* true. Explain your reasoning or provide a counterexample.**

48. Through any three points, there is exactly one plane.

49. Three coplanar lines have two points of intersection.

50. **WRITING IN MATH** How does writing a proof require logical thinking?

Standardized Test Practice

51. ALGEBRA Which is one of the solutions of the equation $3x^2 - 5x + 1 = 0$?

A $\frac{5 + \sqrt{13}}{6}$

B $\frac{-5 - \sqrt{13}}{6}$

C $\frac{5}{6} - \sqrt{13}$

D $-\frac{5}{6} + \sqrt{13}$

52. GRIDDED RESPONSE Steve has 20 marbles in a bag, all the same size and shape. There are 8 red, 2 blue, and 10 yellow marbles in the bag. He will select a marble from the bag at random. What is the probability that the marble Steve selects will be yellow?

53. Which statement *cannot* be true?

F Three noncollinear points determine a plane.

G Two lines intersect in exactly one point.

H At least two lines can contain the same two points.

J A midpoint divides a segment into two congruent segments.

54. SAT/ACT What is the greatest number of regions that can be formed if 3 distinct lines intersect a circle?

A 3

B 4

C 5

D 6

E 7

Spiral Review

Determine whether a valid conclusion can be reached from the two true statements using the Law of Detachment or the Law of Syllogism. If a valid conclusion is possible, state it and the law that is used. If a valid conclusion does not follow, write *no conclusion*. (Lesson 2-4)

55. (1) If two angles are vertical, then they do not form a linear pair.

(2) If two angles form a linear pair, then they are not congruent.

56. (1) If an angle is acute, then its measure is less than 90.

(2) $\angle EFG$ is acute.

Write each statement in if-then form. (Lesson 2-3)

57. Happy people rarely correct their faults.

58. A champion is afraid of losing.

Use the following statements to write a compound statement for each conjunction. Then find its truth value. Explain your reasoning. (Lesson 2-2)

***p*: *M* is on $\overline{AB}$.**

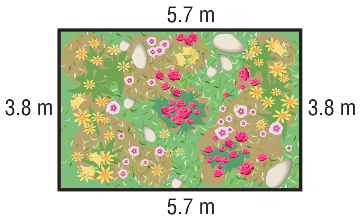
***q*: $AM + MB = AB$**

***r*: *M* is the midpoint of $\overline{AB}$.**

59. $p \wedge q$

60. $\sim p \vee \sim r$

61. GARDENING A landscape designer is putting black plastic edging around a rectangular flower garden that has length 5.7 meters and width 3.8 meters. The edging is sold in 5-meter lengths. Find the perimeter of the garden and determine how much edging the designer should buy. (Lesson 1-6)



62. HEIGHT Taylor is 5 feet 8 inches tall. How many inches tall is Taylor? (Lesson 0-1)

Skills Review

ALGEBRA Solve each equation.

63. $4x - 3 = 19$

64. $\frac{1}{3}x + 6 = 14$

65. $5(x^2 + 2) = 30$