LESSON 2-7

Proving Segment Relationships

Then

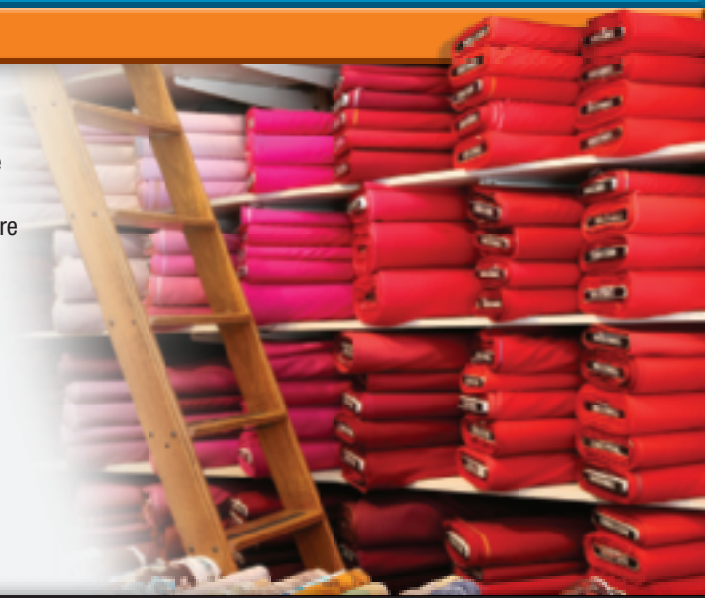
- You wrote algebraic and two-column proofs.

Now

- 1 Write proofs involving segment addition.
- 2 Write proofs involving segment congruence.

Why?

- Emma works at a fabric store after school. She measures a length of fabric by holding the straight edge of the fabric against a yardstick. To measure lengths such as 39 inches, which is longer than the yardstick, she marks a length of 36 inches. From the end of that mark, she measures an additional length of 3 inches. This ensures that the total length of fabric is 36 + 3 inches or 39 inches.



Common Core State Standards

Content Standards

G.CO.9 Prove theorems about lines and angles.

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

Mathematical Practices

2 Reason abstractly and quantitatively.

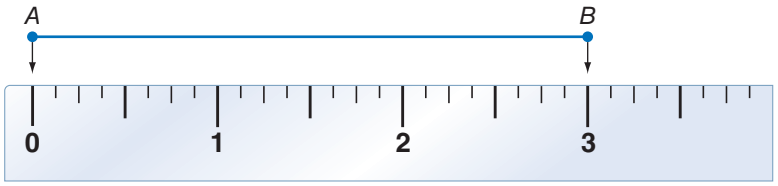
3 Construct viable arguments and critique the reasoning of others.

1 Ruler Postulate In Lesson 1-2, you measured segments with a ruler by matching the mark for zero with one endpoint and then finding the number on the ruler that corresponded to the other endpoint. This illustrates the Ruler Postulate.

Postulate 2.8 Ruler Postulate

Words The points on any line or line segment can be put into one-to-one correspondence with real numbers.

Symbols Given any two points A and B on a line, if A corresponds to zero, then B corresponds to a positive real number.

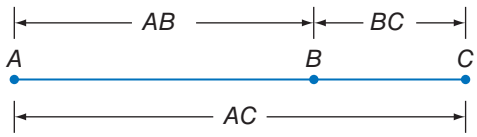


In Lesson 1-2, you also learned about what it means for a point to be *between* two other points. This relationship can be expressed as the Segment Addition Postulate.

Postulate 2.9 Segment Addition Postulate

Words If A, B, and C are collinear, then point B is between A and C if and only if $AB + BC = AC$.

Symbols

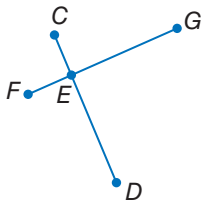


The Segment Addition Postulate is used as a justification in many geometric proofs.

Richard Nowitz/National Geographic/Getty Images

Example 1 Use the Segment Addition Postulate

Prove that if $\overline{CE} \cong \overline{FE}$ and $\overline{ED} \cong \overline{EG}$ then $\overline{CD} \cong \overline{FG}$.



Given: $\overline{CE} \cong \overline{FE}$; $\overline{ED} \cong \overline{EG}$

Prove: $\overline{CD} \cong \overline{FG}$

Proof:

Statements	Reasons
1. $\overline{CE} \cong \overline{FE}$; $\overline{ED} \cong \overline{EG}$	**1.** Given
2. $CE = FE$; $ED = EG$	**2.** Definition of congruence
3. $CE + ED = CD$	**3.** Segment Addition Postulate
4. $FE + EG = CD$	**4.** Substitution (Steps 2 & 3)
5. $FE + EG = FG$	**5.** Segment Addition Postulate
6. $CD = FG$	**6.** Substitution (Steps 4 & 5)
7. $\overline{CD} \cong \overline{FG}$	**7.** Definition of congruence

ReadingMath

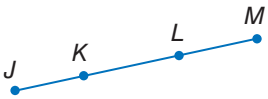
Substitution Property The Substitution Property of Equality is often just written as *Substitution*.

GuidedPractice

Copy and complete the proof.

1. Given: $\overline{JL} \cong \overline{KM}$

Prove: $\overline{JK} \cong \overline{LM}$



Proof:

Statements	Reasons
a. $\overline{JL} \cong \overline{KM}$	**a.** Given
b. $JL = KM$	**b.** ____?____
c. $JK + KL =$ ____?____; $KL + LM =$ ____?____	**c.** Segment Addition Postulate
d. $JK + KL = KL + LM$	**d.** ____?____
e. $JK + KL - KL = KL + LM - KL$	**e.** Subtraction Property of Equality
f. ____?____	**f.** Substitution
g. $\overline{JK} \cong \overline{LM}$	**g.** Definition of congruence

2 Segment Congruence

In Lesson 2-6, you saw that segment measures are reflexive, symmetric, and transitive. Since segments with the same measure are congruent, congruence of segments is also reflexive, symmetric, and transitive.

Theorem 2.2 Properties of Segment Congruence

Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$
Symmetric Property of Congruence	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive Property of Congruence	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

VocabularyLink

Symmetric

Everyday Use balanced or proportional

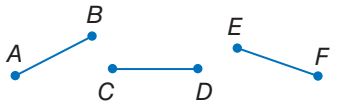
Math Use If $a = b$, then $b = a$.

You will prove the Symmetric and Reflexive Properties in Exercises 6 and 7, respectively.

Proof Transitive Property of Congruence

Given: $\overline{AB} \cong \overline{CD}$; $\overline{CD} \cong \overline{EF}$

Prove: $\overline{AB} \cong \overline{EF}$



Paragraph Proof:

Since $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, $AB = CD$ and $CD = EF$ by the definition of congruent segments. By the Transitive Property of Equality, $AB = EF$. Thus, $\overline{AB} \cong \overline{EF}$ by the definition of congruence.

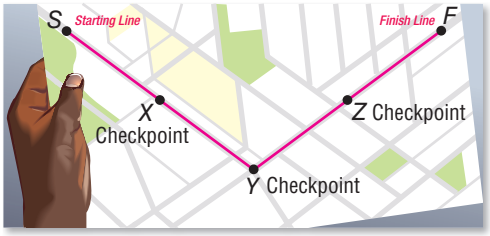
Real-World Link

According to a recent poll, 70% of teens who volunteer began doing so before age 12. Others said they would volunteer if given more opportunities to do so.

Source: Youth Service America

Real-World Example 2 Proof Using Segment Congruence

VOLUNTEERING **The route for a charity fitness run is shown. Checkpoints *X* and *Z* are the midpoints between the starting line and Checkpoint *Y* and Checkpoint *Y* and the finish line *F*, respectively. If Checkpoint *Y* is the same distance from Checkpoints *X* and *Z*, prove that the route from Checkpoint *Z* to the finish line is congruent to the route from the starting line to Checkpoint *X*.**



Given: X is the midpoint of $\overline{SY}$. Z is the midpoint of $\overline{YF}$. $XY = YZ$

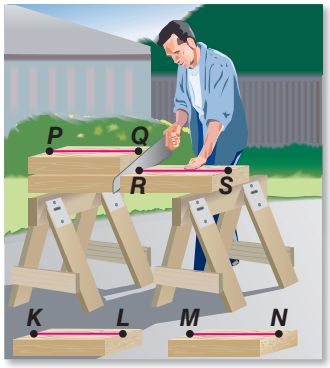
Prove: $\overline{ZF} \cong \overline{SX}$

Two-Column Proof:

Statements	Reasons
1. X is the midpoint of $\overline{SY}$. Z is the midpoint of $\overline{YF}$. $XY = YZ$	1. Given
2. $\overline{SX} \cong \overline{XY}$; $\overline{YZ} \cong \overline{ZF}$	2. Definition of midpoint
3. $\overline{XY} \cong \overline{YZ}$	3. Definition of congruence
4. $\overline{SX} \cong \overline{YZ}$	4. Transitive Property of Congruence
5. $\overline{SX} \cong \overline{ZF}$	5. Transitive Property of Congruence
6. $\overline{ZF} \cong \overline{SX}$	6. Symmetric Property of Congruence

Guided Practice

2. CARPENTRY A carpenter cuts a 2″ × 4″ board to a desired length. He then uses this board as a pattern to cut a second board congruent to the first. Similarly, he uses the second board to cut a third board and the third board to cut a fourth board. Prove that the last board cut has the same measure as the first.



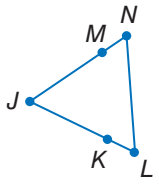
Jim West/age fotostock

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1

1. CCSS **ARGUMENTS** Copy and complete the proof.



Given: $\overline{LK} \cong \overline{NM}$, $\overline{KJ} \cong \overline{MJ}$

Prove: $\overline{LJ} \cong \overline{NJ}$

Proof:

Statements	Reasons
a. $\overline{LK} \cong \overline{NM}$, $\overline{KJ} \cong \overline{MJ}$	a. ?
b. ?	b. Def. of congruent segments
c. $LK + KJ = NM + MJ$	c. ?
d. ?	d. Segment Addition Postulate
e. $LJ = NJ$	e. ?
f. $\overline{LJ} \cong \overline{NJ}$	f. ?

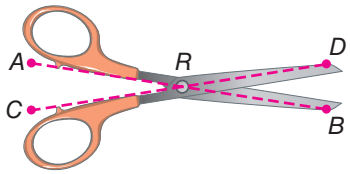
Example 2

2. **PROOF** Prove the following.

Given: $\overline{WX} \cong \overline{YZ}$

Prove: $\overline{WY} \cong \overline{XZ}$

3 **SCISSORS** Refer to the diagram shown. $\overline{AR}$ is congruent to $\overline{CR}$. $\overline{DR}$ is congruent to $\overline{BR}$. Prove that $AR + DR = CR + BR$.

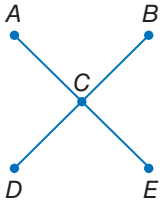


Practice and Problem Solving

Extra Practice is on page R2.

Example 1

4. CCSS **ARGUMENTS** Copy and complete the proof.



Given: C is the midpoint of $\overline{AE}$.

C is the midpoint of $\overline{BD}$.

$\overline{AE} \cong \overline{BD}$

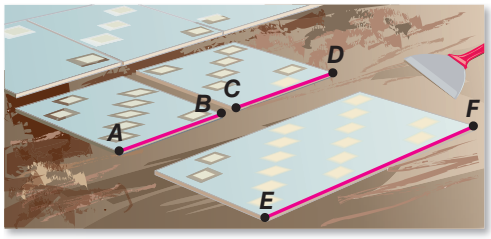
Prove: $\overline{AC} \cong \overline{CD}$

Proof:

Statements	Reasons
a. ?	a. Given
b. $AC = CE$, $BC = CD$	b. ?
c. $AE = BD$	c. ?
d. ?	d. Segment Addition Postulate
e. $AC + CE = BC + CD$	e. ?
f. $AC + AC = CD + CD$	f. ?
g. ?	g. Simplify.
h. ?	h. Division Property
i. $\overline{AC} \cong \overline{CD}$	i. ?

Example 2

5. TILING A tile setter cuts a piece of tile to a desired length. He then uses this tile as a pattern to cut a second tile congruent to the first. He uses the first two tiles to cut a third tile whose length is the sum of the measures of the first two tiles. Prove that the measure of the third tile is twice the measure of the first tile.



CCSS **ARGUMENTS** **Prove each theorem.**

6. Symmetric Property of Congruence (Theorem 2.2)

7 Reflexive Property of Congruence (Theorem 2.2)

8. TRAVEL Four cities in New York are connected by Interstate 90: Buffalo, Utica, Albany, and Syracuse. Buffalo is the farthest west.

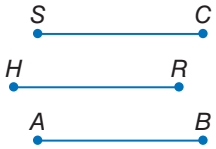
- Albany is 126 miles from Syracuse and 263 miles from Buffalo.
- Buffalo is 137 miles from Syracuse and 184 miles from Utica.

a. Draw a diagram to represent the locations of the cities in relation to each other and the distances between each city. Assume that Interstate 90 is straight.

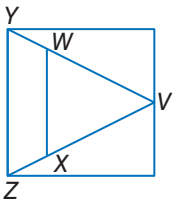
b. Write a paragraph proof to support your conclusion.

PROOF **Prove the following.**

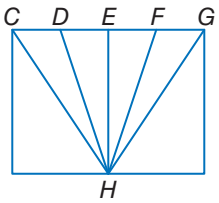
9. If $\overline{SC} \cong \overline{HR}$ and $\overline{HR} \cong \overline{AB}$, then $\overline{SC} \cong \overline{AB}$.



10. If $\overline{VZ} \cong \overline{VY}$ and $\overline{WY} \cong \overline{XZ}$, then $\overline{VW} \cong \overline{VX}$.



11. If E is the midpoint of $\overline{DF}$ and $\overline{CD} \cong \overline{FG}$, then $\overline{CE} \cong \overline{EG}$.

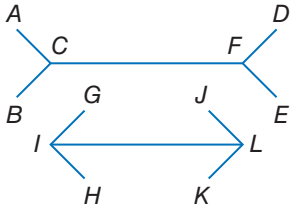


12. If B is the midpoint of $\overline{AC}$, D is the midpoint of $\overline{CE}$, and $\overline{AB} \cong \overline{DE}$, then $AE = 4AB$.

13. OPTICAL ILLUSION $\overline{AC} \cong \overline{GI}$, $\overline{FE} \cong \overline{LK}$, and $AC + CF + FE = GI + IL + LK$.

a. Prove that $\overline{CF} \cong \overline{IL}$.

b. Justify your proof using measurement. Explain your method.

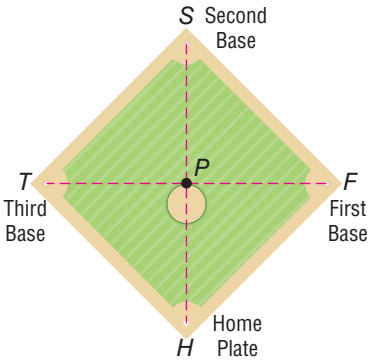


14. CONSTRUCTION Construct a segment that is twice as long as $\overline{PQ}$. Explain how the Segment Addition Postulate can be used to justify your construction.

15 BASEBALL Use the diagram of a baseball diamond shown.

a. On the diagram, $\overline{SH} \cong \overline{TF}$. P is the midpoint of $\overline{SH}$ and $\overline{TF}$. Using a two-column proof, prove that $\overline{SP} \cong \overline{TP}$.

b. The distance from home plate to second base is 127.3 feet. What is the distance from first base to second base?



16. MULTIPLE REPRESENTATIONS A is the midpoint of $\overline{PQ}$, B is the midpoint of $\overline{PA}$, and C is the midpoint of $\overline{PB}$.

a. Geometric Make a sketch to represent this situation.

b. Algebraic Make a conjecture as to the algebraic relationship between PC and PQ.

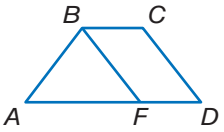
c. Geometric Copy segment $\overline{PQ}$ from your sketch. Then construct points B and C on $\overline{PQ}$. Explain how you can use your construction to support your conjecture.

d. Concrete Use a ruler to draw a segment congruent to $\overline{PQ}$ from your sketch and to draw points B and C on $\overline{PQ}$. Use your drawing to support your conjecture.

e. Logical Prove your conjecture.

H.O.T. Problems Use Higher-Order Thinking Skills

17. CCSS CRITIQUE In the diagram, $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{BF}$. Examine the conclusions made by Leslie and Shantice. Is either of them correct?



Leslie

Since $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{BF}$, then $\overline{AB} \cong \overline{AF}$ by the Transitive Property of Congruence

Shantice

Since $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{BF}$, then $\overline{AB} \cong \overline{BF}$ by the Reflexive Property of Congruence.

18. CHALLENGE $ABCD$ is a square. Prove that $\overline{AC} \cong \overline{BD}$.

19. WRITING IN MATH Does there exist an Addition Property of Congruence? Explain.

20. REASONING Classify the following statement as *true* or *false*. If false, provide a counterexample.

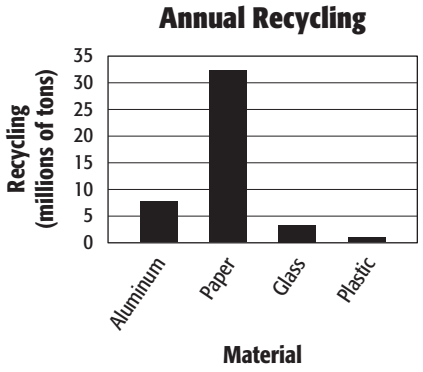
If A, B, C, D, and E are collinear with B between A and C, C between B and D, and D between C and E, and AC = BD = CE, then AB = BC = DE.

21. OPEN ENDED Draw a representation of the Segment Addition Postulate in which the segment is two inches long, contains four collinear points, and contains no congruent segments.

22. WRITING IN MATH Compare and contrast paragraph proofs and two-column proofs.

Standardized Test Practice

23. ALGEBRA The chart below shows annual recycling by material in the United States. About how many pounds of aluminum are recycled each year?



A 7.5
B 15,000
C 7,500,000
D 15,000,000,000

24. ALGEBRA Which expression is equivalent to $\frac{12x^{-4}}{4x^{-8}}$?

F $\frac{1}{3x^4}$
G $3x^4$
H $8x^2$
J $\frac{x^4}{3}$

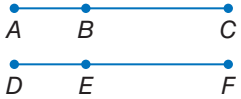
25. SHORT RESPONSE The measures of two complementary angles are in the ratio 4 : 1. What is the measure of the smaller angle?

26. SAT/ACT Julie can word process 40 words per minute. How many minutes will it take Julie to word process 200 words?

A 0.5
B 2
C 5
D 10
E 12

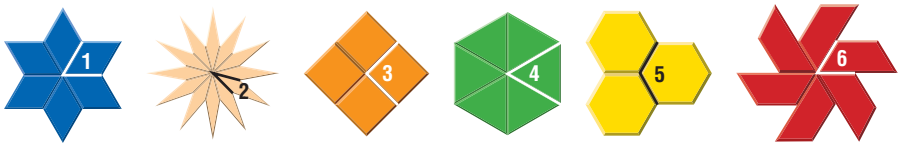
Spiral Review

27. PROOF **Write a two-column proof.** (Lesson 2-6)
Given: $AC = DF$
$AB = DE$
Prove: $BC = EF$



28. MODELS Brian is using six squares of cardboard to form a rectangular prism. What geometric figure do the pieces of cardboard represent, and how many lines will be formed by their intersections? (Lesson 2-5)

29. PATTERN BLOCKS Pattern blocks can be arranged to fit in a circular pattern without leaving spaces. Remember that the measurement around a full circle is 360°. Determine the degree measure of the numbered angles shown below. (Lesson 1-4)



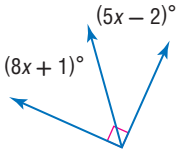
Simplify. (Lesson 0-9)

30. $\sqrt{48}$
31. $\sqrt{162}$
32. $\sqrt{25a^6b^4}$
33. $\sqrt{45xy^8}$

Skills Review

ALGEBRA **Find x.**

34.



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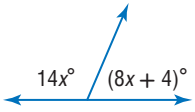

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