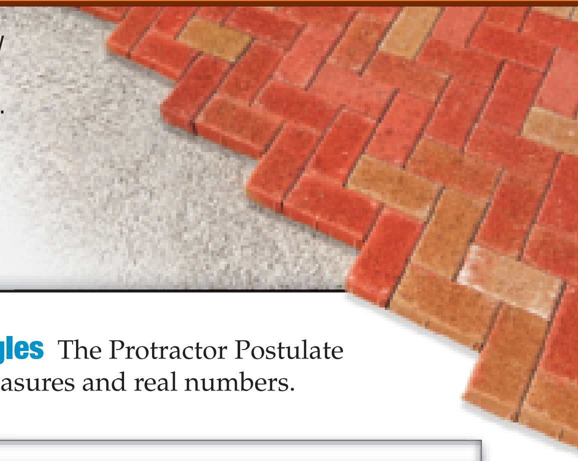
LESSON 2-8 Proving Angle Relationships

:: Then	:: Now	:: Why?
● You identified and used special pairs of angles.	● **1** Write proofs involving supplementary and complementary angles. **2** Write proofs involving congruent and right angles.	● Jamal's school is building a walkway that will include bricks with the names of graduates from each class. All of the bricks are rectangular, so when the bricks are laid, all of the angles form linear pairs.



CCSS Common Core State Standards

Content Standards
G.CO.9 Prove theorems about lines and angles.

Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.
6 Attend to precision.

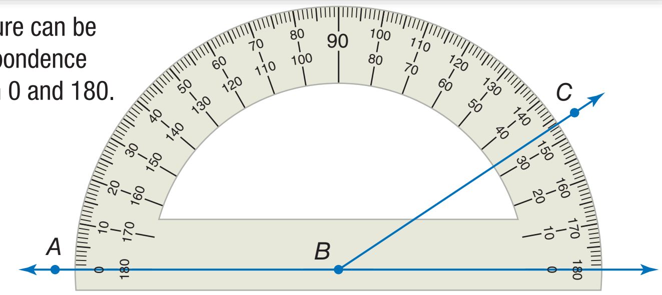
1 Supplementary and Complementary Angles

The Protractor Postulate illustrates the relationship between angle measures and real numbers.

Postulate 2.10 Protractor Postulate

Words Given any angle, the measure can be put into one-to-one correspondence with real numbers between 0 and 180.

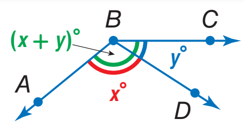
Example If $\overrightarrow{BA}$ is placed along the protractor at 0°, then the measure of $\angle ABC$ corresponds to a positive real number.



In Lesson 2-7, you learned about the Segment Addition Postulate. A similar relationship exists between the measures of angles.

Postulate 2.11 Angle Addition Postulate

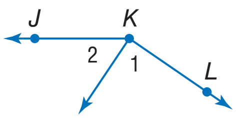
D is in the interior of $\angle ABC$ if and only if $m\angle ABD + m\angle DBC = m\angle ABC$.



Example 1 Use the Angle Addition Postulate

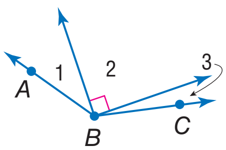
Find $m\angle 1$ if $m\angle 2 = 56$ and $m\angle JKL = 145$.

$m\angle 1 + m\angle 2 = m\angle JKL$	Angle Addition Postulate
$m\angle 1 + 56 = 145$	$m\angle 2 = 56$, $m\angle JKL = 145$
$m\angle 1 + 56 - 56 = 145 - 56$	Subtraction Property of Equality
$m\angle 1 = 89$	Substitution



GuidedPractice

1. If $m\angle 1 = 23$ and $m\angle ABC = 131$, find the measure of $\angle 3$. Justify each step.



FOTOG/Tetra Images/Photolibrary

The Angle Addition Postulate can be used with other angle relationships to provide additional theorems relating to angles.

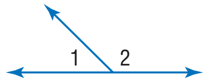
StudyTip

Linear Pair Theorem The Supplement Theorem may also be known as the *Linear Pair Theorem.*

Theorems

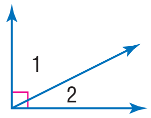
2.3 Supplement Theorem If two angles form a linear pair, then they are supplementary angles.

Example $m\angle 1 + m\angle 2 = 180$



2.4 Complement Theorem If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.

Example $m\angle 1 + m\angle 2 = 90$

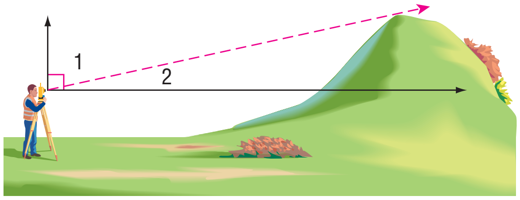


You will prove Theorems 2.3 and 2.4 in Exercises 16 and 17, respectively.

Real-World Example 2 Use Supplement or Complement

SURVEYING Using a transit, a surveyor sights the top of a hill and records an angle measure of about 73°. What is the measure of the angle the top of the hill makes with the horizon? Justify each step.

Understand Make a sketch of the situation. The surveyor is measuring the angle of his line of sight below the vertical. Draw a vertical ray and a horizontal ray from the point where the surveyor is sighting the hill, and label the angles formed. We know that the vertical and horizontal rays form a right angle.



Plan Since $\angle 1$, and $\angle 2$ form a right angle, you can use the Complement Theorem.

Solve

$m\angle 1 + m\angle 2 = 90$	Complement Theorem
$73 + m\angle 2 = 90$	$m\angle 1 = 73$
$73 + m\angle 2 - 73 = 90 - 73$	Subtraction Property of Equality
$m\angle 2 = 17$	Substitution

The top of the hill makes a 17° angle with the horizon.

Check Since we know that the sum of the angles should be 90, check your math. The sum of 17 and 73 is 90. ✓

ReviewVocabulary

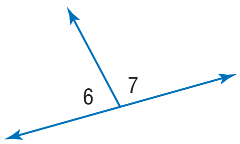
supplementary angles two angles with measures that add to 180

complementary angles two angles with measures that add to 90

linear pair a pair of adjacent angles with noncommon sides that are opposite rays

GuidedPractice

2. $\angle 6$ and $\angle 7$ form linear pair. If $m\angle 6 = 3x + 32$ and $m\angle 7 = 5x + 12$, find x, $m\angle 6$, and $m\angle 7$. Justify each step.



2 Congruent Angles

The properties of algebra that applied to the congruence of segments and the equality of their measures also hold true for the congruence of angles and the equality of their measures.

Theorem 2.5 Properties of Angle Congruence

Reflexive Property of Congruence

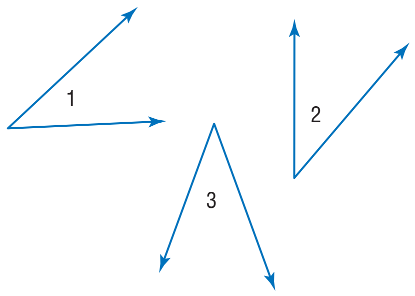
$\angle 1 \cong \angle 1$

Symmetric Property of Congruence

If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

Transitive Property of Congruence

If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

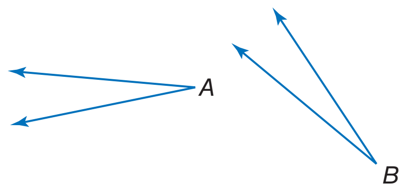


You will prove the Reflexive and Transitive Properties of Congruence in Exercises 18 and 19, respectively.

Proof Symmetric Property of Congruence

Given: $\angle A \cong \angle B$

Prove: $\angle B \cong \angle A$



Paragraph Proof:

We are given $\angle A \cong \angle B$. By the definition of congruent angles, $m\angle A = m\angle B$. Using the Symmetric Property of Equality, $m\angle B = m\angle A$. Thus, $\angle B \cong \angle A$ by the definition of congruent angles.

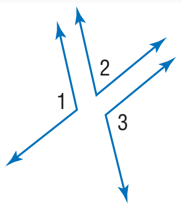
Algebraic properties can be applied to prove theorems for congruence relationships involving supplementary and complementary angles.

Theorems

2.6 Congruent Supplements Theorem
Angles supplementary to the same angle or to congruent angles are congruent.

Abbreviation *∠s suppl. to same ∠ or ≅ ∠s are ≅.*

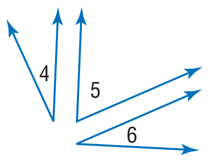
Example If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, then $\angle 1 \cong \angle 3$.



2.7 Congruent Complements Theorem
Angles complementary to the same angle or to congruent angles are congruent.

Abbreviation *∠s compl. to same ∠ or ≅ ∠s are ≅.*

Example If $m\angle 4 + m\angle 5 = 90$ and $m\angle 5 + m\angle 6 = 90$, then $\angle 4 \cong \angle 6$.



You will prove one case of Theorem 2.6 in Exercise 6.

ReadingMath

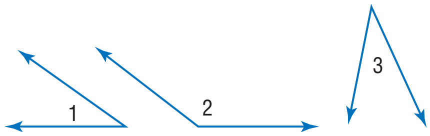
Abbreviations and Symbols

The notation ∠s means *angles*.

Proof One Case of the Congruent Supplements Theorem

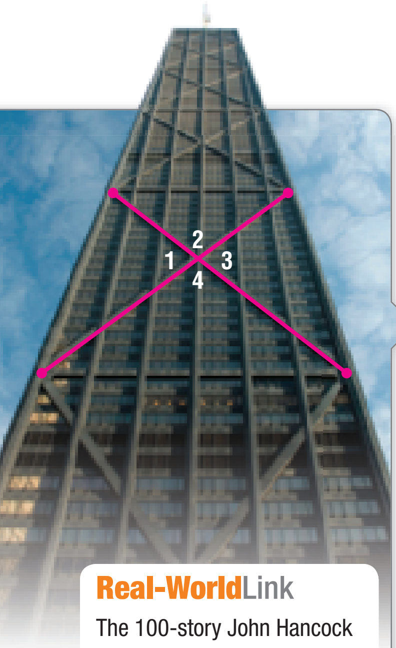
Given: $\angle 1$ and $\angle 2$ are supplementary.
$\angle 2$ and $\angle 3$ are supplementary.

Prove: $\angle 1 \cong \angle 3$



Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.	**1.** Given
2. $m\angle 1 + m\angle 2 = 180$; $m\angle 2 + m\angle 3 = 180$	**2.** Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	**3.** Substitution
4. $m\angle 2 = m\angle 2$	**4.** Reflexive Property
5. $m\angle 1 = m\angle 3$	**5.** Subtraction Property
6. $\angle 1 \cong \angle 3$	**6.** Definition of congruent angles



Real-WorldLink

The 100-story John Hancock Building uses huge X-braces in its design. These diagonals are connected to the exterior columns, making it possible for strong wind forces to be carried from the braces to the exterior columns and back.

Source: PBS

Example 3 Proofs Using Congruent Comp. or Suppl. Theorems

Prove that vertical angles 2 and 4 in the photo at the left are congruent.

Given: $\angle 2$ and $\angle 4$ are vertical angles.

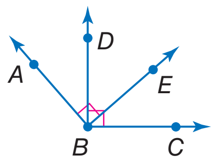
Prove: $\angle 2 \cong \angle 4$

Proof:

Statements	Reasons
1. $\angle 2$ and $\angle 4$ are vertical angles.	**1.** Given
2. $\angle 2$ and $\angle 4$ are nonadjacent angles formed by intersecting lines.	**2.** Definition of vertical angles
3. $\angle 2$ and $\angle 3$ from a linear pair. $\angle 3$ and $\angle 4$ form a linear pair.	**3.** Definition of a linear pair
4. $\angle 2$ and $\angle 3$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary.	**4.** Supplement Theorem
5. $\angle 2 \cong \angle 4$	**5.** $\measuredangle$ suppl. to same $\angle$ or $\cong$ $\measuredangle$ are $\cong$.

GuidedPractice

3. In the figure, $\angle ABE$ and $\angle DBC$ are right angles. Prove that $\angle ABD \cong \angle EBC$.



ReviewVocabulary

Vertical Angles two nonadjacent angles formed by intersecting lines

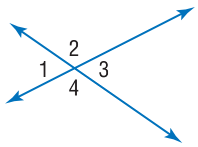
Note that in Example 3, $\angle 1$ and $\angle 3$ are vertical angles. The conclusion in the example supports the following Vertical Angles Theorem.

Theorem 2.8 Vertical Angles Theorem

If two angles are vertical angles, then they are congruent.

Abbreviation *Vert.* $\measuredangle$ *are* $\cong$.

Example $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

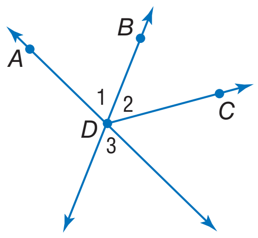


You will prove Theorem 2.8 in Exercise 28.

Yannis Emmanuel Mavromatakis/Alamy

Example 4 Use Vertical Angles

Prove that if $\overrightarrow{DB}$ bisects $\angle ADC$, then $\angle 2 \cong \angle 3$.



Given: $\overrightarrow{DB}$ bisects $\angle ADC$.

Prove: $\angle 2 \cong \angle 3$

Proof:

Statements	Reasons
1. $\overrightarrow{DB}$ bisects $\angle ADC$.	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Definition of angle bisector
3. $\angle 1$ and $\angle 3$ are vertical angles.	**3.** Definition of vertical angles
4. $\angle 3 \cong \angle 1$	**4.** Vert. $\measuredangle$s are $\cong$.
5. $\angle 3 \cong \angle 2$	**5.** Transitive Property of Congruence
6. $\angle 2 \cong \angle 3$	**6.** Symmetric Property of Congruence

GuidedPractice

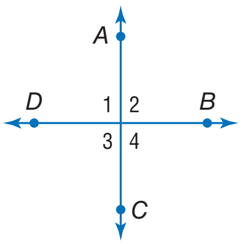
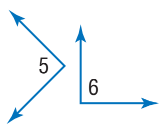
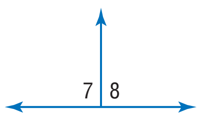
4. If $\angle 3$ and $\angle 4$ are vertical angles, $m\angle 3 = 6x + 2$, and $m\angle 4 = 8x - 14$, find $m\angle 3$ and $m\angle 4$. Justify each step.

The theorems in this lesson can be used to prove the following right angle theorems.

ReadingMath

Perpendicular Recall from Lesson 1-5 that the symbol $\perp$ means *is perpendicular to.*

Theorems Right Angle Theorems

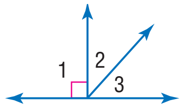
Theorem	Example
2.9 Perpendicular lines intersect to form four right angles. **Example** If $\overleftrightarrow{AC} \perp \overleftrightarrow{DB}$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are rt. $\measuredangle$s.	
2.10 All right angles are congruent. **Example** If $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are rt. $\measuredangle$s, then $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$.	
2.11 Perpendicular lines form congruent adjacent angles. **Example** If $\overleftrightarrow{AC} \perp \overleftrightarrow{DB}$, then $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 4$, $\angle 3 \cong \angle 4$, and $\angle 1 \cong \angle 3$.	
2.12 If two angles are congruent and supplementary, then each angle is a right angle. **Example** If $\angle 5 \cong \angle 6$ and $\angle 5$ is suppl. to $\angle 6$, then $\angle 5$ and $\angle 6$ are rt. $\measuredangle$s.	
2.13 If two congruent angles form a linear pair, then they are right angles. **Example** If $\angle 7$ and $\angle 8$ form a linear pair, then $\angle 7$ and $\angle 8$ are rt. $\measuredangle$s.	

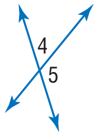
You will prove Theorems 2.9–2.13 in Exercises 22–26.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 Find the measure of each numbered angle, and name the theorems that justify your work.





1. $m\angle 2 = 26$

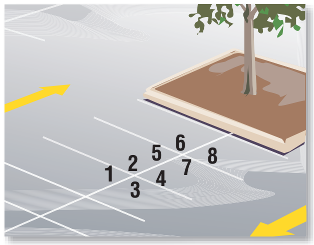
2. $m\angle 2 = x$, $m\angle 3 = x - 16$

3. $m\angle 4 = 2x$, $m\angle 5 = x + 9$

4. $m\angle 4 = 3(x - 1)$, $m\angle 5 = x + 7$

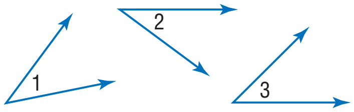
Example 2

5. **PARKING** Refer to the diagram of the parking lot at the right. Given that $\angle 2 \cong \angle 6$, prove that $\angle 4 \cong \angle 8$.



Example 3

6. **PROOF** Copy and complete the proof of one case of Theorem 2.6.



Given: $\angle 1$ and $\angle 3$ are complementary.
$\angle 2$ and $\angle 3$ are complementary.

Prove: $\angle 1 \cong \angle 2$

Proof:

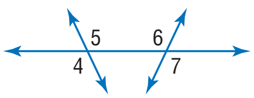
Statements	Reasons
a. $\angle 1$ and $\angle 3$ are complementary. $\angle 2$ and $\angle 3$ are complementary.	**a.** ___?___
b. $m\angle 1 + m\angle 3 = 90$; $m\angle 2 + m\angle 3 = 90$	**b.** ___?___
c. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	**c.** ___?___
d. ___?___	**d.** Reflexive Property
e. $m\angle 1 = m\angle 2$	**e.** ___?___
f. $\angle 1 \cong \angle 2$	**f.** ___?___

Example 4

7. **CCSS ARGUMENTS** Write a two-column proof.

Given: $\angle 4 \cong \angle 7$

Prove: $\angle 5 \cong \angle 6$

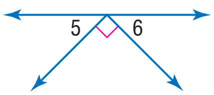


Practice and Problem Solving

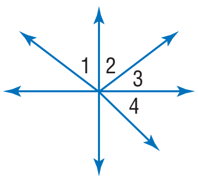
Extra Practice is on page R2.

Examples 1–3 Find the measure of each numbered angle, and name the theorems used that justify your work.

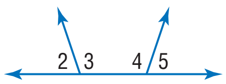
8. $m\angle 5 = m\angle 6$



9. $\angle 2$ and $\angle 3$ are complementary. $\angle 1 \cong \angle 4$ and $m\angle 2 = 28$

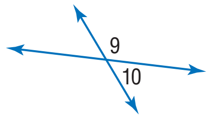


10. $\angle 2$ and $\angle 4$ and $\angle 4$ and $\angle 5$ are supplementary. $m\angle 4 = 105$

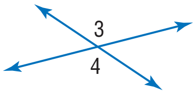


Find the measure of each numbered angle and name the theorems used that justify your work.

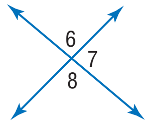
11. $m\angle 9 = 3x + 12$
$m\angle 10 = x - 24$



12. $m\angle 3 = 2x + 23$
$m\angle 4 = 5x - 112$



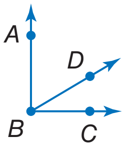
13. $m\angle 6 = 2x - 21$
$m\angle 7 = 3x - 34$



Example 4 **PROOF** **Write a two-column proof.**

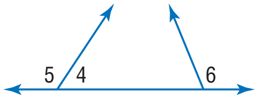
14. Given: $\angle ABC$ is a right angle.

Prove: $\angle ABD$ and $\angle CBD$ are complementary.



15. Given: $\angle 5 \cong \angle 6$

Prove: $\angle 4$ and $\angle 6$ are supplementary.



Write a proof for each theorem.

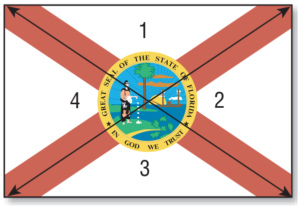
16. Supplement Theorem

17. Complement Theorem

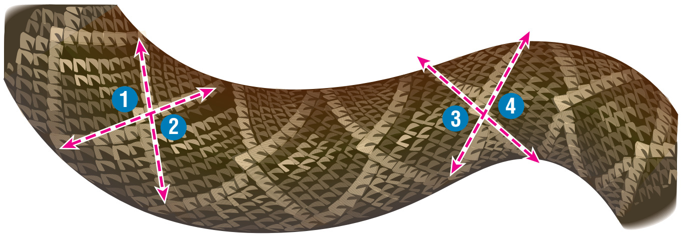
18. Reflexive Property of Angle Congruence

19. Transitive Property of Angle Congruence

20. **FLAGS** Refer to the Florida state flag at the right. Prove that the sum of the four angle measures is 360.



21. CCSS **ARGUMENTS** The diamondback rattlesnake is a pit viper with a diamond pattern on its back. An enlargement of a skin is shown below. If $\angle 1 \cong \angle 4$, prove that $\angle 2 \cong \angle 3$.



PROOF **Use the figure to write a proof of each theorem.**

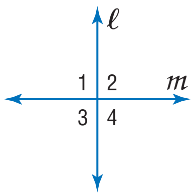
22. Theorem 2.9

23. Theorem 2.10

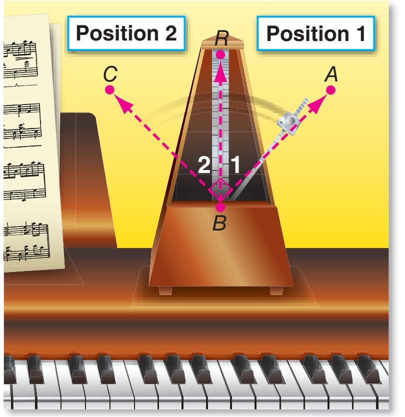
24. Theorem 2.11

25. Theorem 2.12

26. Theorem 2.13



27. **CCSS ARGUMENTS** To mark a specific tempo, the weight on the pendulum of a metronome is adjusted so that it swings at a specific rate. Suppose $\angle ABC$ in the photo is a right angle. If $m\angle 1 = 45$, write a paragraph proof to show that $\overrightarrow{BR}$ bisects $\angle ABC$.



28. **PROOF** Write a proof of Theorem 2.8.

29. **GEOGRAPHY** Utah, Colorado, Arizona, and New Mexico all share a common point on their borders called Four Corners. This is the only place where four states meet in a single point. If $\angle 2$ is a right angle, prove that lines ℓ and m are perpendicular.

30. **MULTIPLE REPRESENTATIONS** In this problem, you will explore angle relationships.

  a. **Geometric** Draw a right angle ABC. Place point D in the interior of this angle and draw $\overrightarrow{BD}$. Draw $\overrightarrow{KL}$ and construct $\angle JKL$ congruent to $\angle ABD$.

  b. **Verbal** Make a conjecture as to the relationship between $\angle JKL$ and $\angle DBC$.

  c. **Logical** Prove your conjecture.

H.O.T. Problems Use Higher-Order Thinking Skills

31. **OPEN ENDED** Draw an angle WXZ such that $m\angle WXZ = 45$. Construct $\angle YXZ$ congruent to $\angle WXZ$. Make a conjecture as to the measure of $\angle WXY$, and then prove your conjecture.

32. **WRITING IN MATH** Write the steps that you would use to complete the proof below.

Given: $\overline{BC} \cong \overline{CD}$, $AB = \frac{1}{2}BD$

Prove: $\overline{AB} \cong \overline{CD}$

33. **CHALLENGE** In this lesson, one case of the Congruent Supplements Theorem was proven. In Exercise 6, you proved the same case for the Congruent Complements Theorem. Explain why there is another case for each of these theorems. Then write a proof of this second case for each theorem.

34. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

If one of the angles formed by two intersecting lines is acute, then the other three angles formed are also acute.

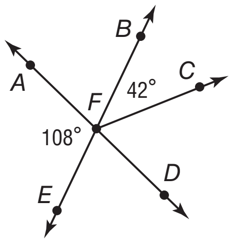
35. **WRITING IN MATH** Explain how you can use your protractor to quickly find the measure of the supplement of an angle.

Standardized Test Practice

36. GRIDDED RESPONSE What is the mode of this set of data?

4, 3, −2, 1 , 4, 0, 1, 4

37. Find the measure of $\angle CFD$.



A 66°
B 72°
C 108°
D 138°

38. ALGEBRA Simplify.

$$4(3x - 2)(2x + 4) + 3x^2 + 5x - 6$$

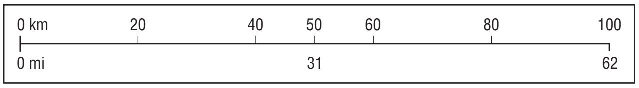
F $9x^2 + 3x - 14$
G $9x^2 + 13x - 14$
H $27x^2 + 37x - 38$
J $27x^2 + 27x - 26$

39. SAT/ACT On a coordinate grid where each unit represents 1 mile, Isabel's house is located at (3, 0) and a mall is located at (0, 4). What is the distance between Isabel's house and the mall?

A 3 miles
B 5 miles
C 12 miles
D 13 miles
E 25 miles

Spiral Review

40. MAPS On a U.S. map, there is a scale that lists kilometers on the top and miles on the bottom.



Suppose $\overline{AB}$ and $\overline{CD}$ are segments on this map. If $AB = 100$ kilometers and $CD = 62$ miles, is $\overline{AB} \cong \overline{CD}$? Explain. (Lesson 2-7)

State the property that justifies each statement. (Lesson 2-6)

41. If $y + 7 = 5$, then $y = -2$.

42. If $MN = PQ$, then $PQ = MN$.

43. If $a - b = x$ and $b = 3$, then $a - 3 = x$.

44. If $x(y + z) = 4$, then $xy + xz = 4$.

Determine the truth value of the following statement for each set of conditions.
If you have a fever, then you are sick. (Lesson 2-3)

45. You do not have a fever, and you are sick.

46. You have a fever, and you are not sick.

47. You do not have a fever, and you are not sick.

48. You have a fever, and you are sick.

Skills Review

Refer to the figure.

49. Name a line that contains point P.

50. Name the intersection of lines n and m.

51. Name a point not contained in lines ℓ, m, or n.

52. What is another name for line n?

53. Does line ℓ intersect line m or line n? Explain.