LESSON 3-1

Parallel Lines and Transversals

Then

- You used angle and line segment relationships to prove theorems.

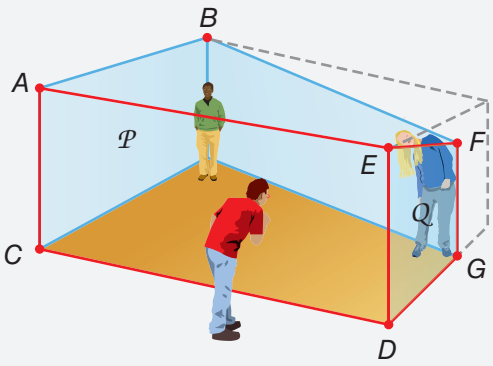
Now

1. Identify the relationships between two lines or two planes.
2. Name angle pairs formed by parallel lines and transversals.

Why?

- An Ames room creates the illusion that a person standing in the right corner is much larger than a person standing in the left corner.

From a front viewing hole the front and back walls appear parallel, when in fact they are slanted. The ceiling and floor appear horizontal, but are actually tilted.



New Vocabulary

- parallel lines
- skew lines
- parallel planes
- transversal
- interior angles
- exterior angles
- consecutive interior angles
- alternate interior angles
- alternate exterior angles
- corresponding angles

Common Core State Standards

Content Standards

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

3 Construct viable arguments and critique the reasoning of others.

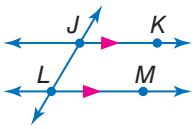
1 Relationships Between Lines and Planes

The construction of the Ames room above makes use of intersecting, parallel, and skew lines, as well as intersecting and parallel planes, to create an optical illusion.

KeyConcepts Parallel and Skew

Parallel lines are coplanar lines that do not intersect.

Example $\overleftrightarrow{JK} \parallel \overleftrightarrow{LM}$



Arrows are used to indicate that lines are parallel.

Skew lines are lines that do not intersect and are not coplanar.

Example Lines ℓ and m are skew.

Parallel planes are planes that do not intersect.

Example Planes $\mathcal{A}$ and $\mathcal{B}$ are parallel.

$\overleftrightarrow{JK} \parallel \overleftrightarrow{LM}$ is read as *line JK is parallel to line LM.*

If segments or rays are contained within lines that are parallel or skew, then the segments or rays are parallel or skew.

Real-World Example 1 Identify Parallel and Skew Relationships

Identify each of the following using the wedge of cheese below.

a. all segments parallel to $\overline{JP}$

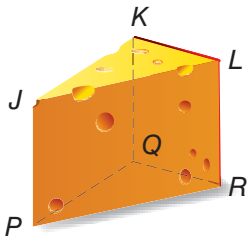
$\overline{KQ}$ and $\overline{LR}$

b. a segment skew to $\overline{KL}$

$\overline{JP}$, $\overline{PQ}$, or $\overline{PR}$

c. a plane parallel to plane PQR

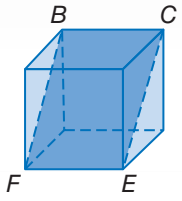
Plane JKL is the only plane parallel to plane PQR.



WatchOut!

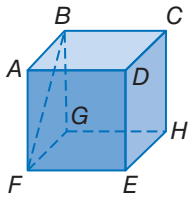
Parallel vs. Skew

In Check Your Progress 1A, $\overleftrightarrow{FE}$ is *not* skew to $\overleftrightarrow{BC}$. Instead, these lines are parallel in plane *BCF*.



GuidedPractice

Identify each of the following using the cube shown.



1A. all segments skew to $\overleftrightarrow{BC}$

1B. a segment parallel to $\overleftrightarrow{EH}$

1C. all planes parallel to plane *DCH*

2 Transversal Angle Pair Relationships

A line that intersects two or more coplanar lines at two different points is called a **transversal**. In the diagram below, line t is a transversal of lines q and r. Notice that line t forms a total of eight angles with lines q and r. These angles, and specific pairings of these angles, are given special names.

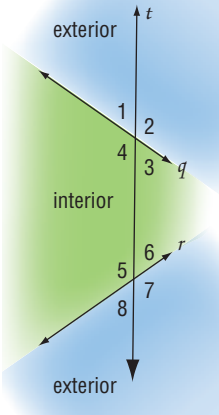
ReadingMath

Same-Side Interior Angles

Consecutive interior angles are also called *same-side interior angles*.

KeyConcept Transversal Angle Pair Relationships

Four **interior angles** lie in the region between lines q and r.	$\angle 3, \angle 4, \angle 5, \angle 6$
Four **exterior angles** lie in the two regions that are not between lines q and r.	$\angle 1, \angle 2, \angle 7, \angle 8$
Consecutive interior angles are interior angles that lie on the same side of transversal t.	$\angle 4$ and $\angle 5$, $\angle 3$ and $\angle 6$
Alternate interior angles are nonadjacent interior angles that lie on opposite sides of transversal t.	$\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$
Alternate exterior angles are nonadjacent exterior angles that lie on opposite sides of transversal t.	$\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 8$
Corresponding angles lie on the same side of transversal t and on the same side of lines q and r.	$\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$ $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$



Example 2 Classify Angle Pair Relationships

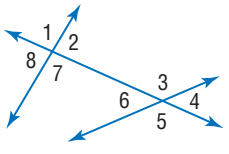
Refer to the figure below. Classify the relationship between each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.

a. $\angle 1$ and $\angle 5$
alternate exterior

b. $\angle 6$ and $\angle 7$
consecutive interior

c. $\angle 2$ and $\angle 4$
corresponding

d. $\angle 2$ and $\angle 6$
alternate interior



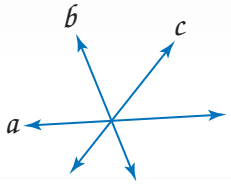
GuidedPractice

2A. $\angle 3$ and $\angle 7$ **2B.** $\angle 5$ and $\angle 7$ **2C.** $\angle 4$ and $\angle 8$ **2D.** $\angle 2$ and $\angle 3$

When more than one line can be considered a transversal, first identify the transversal for a given angle pair by locating the line that connects the vertices of the angles.

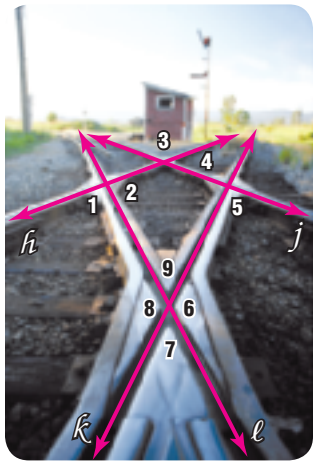
StudyTip

Nonexample In the figure below, line c is *not* a transversal of lines a and b, since line c intersects lines a and b in only one point.



Example 3 Identify Transversals and Classify Angle Pairs

Identify the transversal connecting each pair of angles in the photo. Then classify the relationship between each pair of angles.



a. ∠1 and ∠3

The transversal connecting ∠1 and ∠3 is line h. These are alternate exterior angles.

b. ∠5 and ∠6

The transversal connecting ∠5 and ∠6 is line k. These are consecutive interior angles.

c. ∠2 and ∠6

The transversal connecting ∠2 and ∠6 is line ℓ. These are corresponding angles.

GuidedPractice

3A. ∠3 and ∠5

3B. ∠2 and ∠8

3C. ∠5 and ∠7

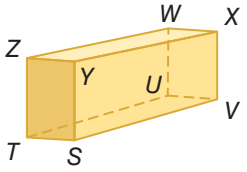
3D. ∠2 and ∠9

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1

Refer to the figure at the right to identify each of the following.

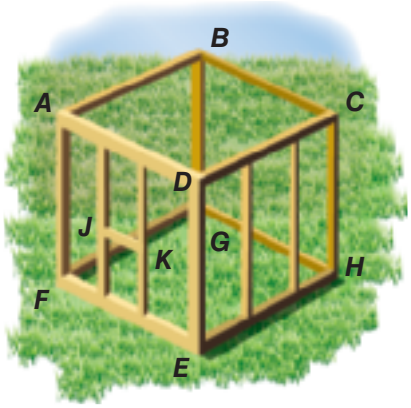


1. a plane parallel to plane ZWX

2. a segment skew to $\overline{TS}$ that contains point W

3. all segments parallel to $\overline{SV}$

4. CONSTRUCTION Use the diagram of the partially framed storage shed shown to identify each of the following.



a. Name three pairs of parallel planes.

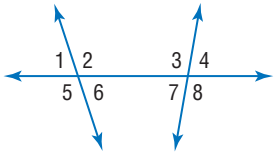
b. Name three segments parallel to $\overline{DE}$.

c. Name two segments parallel to $\overline{FE}$.

d. Name two pairs of skew segments.

Example 2

Classify the relationship between each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.



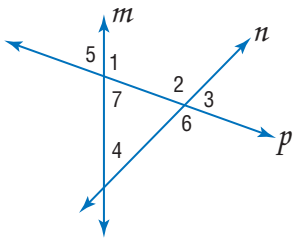
5 ∠1 and ∠8

6. ∠2 and ∠4

7. ∠3 and ∠6

8. ∠6 and ∠7

Example 3 **Identify the transversal connecting each pair of angles. Then classify the relationship between each pair of angles.**



9. $\angle 2$ and $\angle 4$

10. $\angle 5$ and $\angle 6$

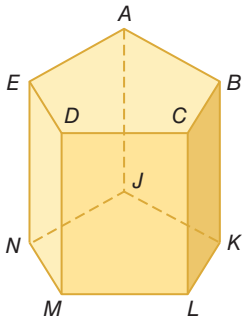
11. $\angle 4$ and $\angle 7$

12. $\angle 2$ and $\angle 7$

Practice and Problem Solving

Extra Practice is on page R3.

Example 1 **Refer to the figure to identify each of the following.**



13. all segments parallel to $\overline{DM}$

14. a plane parallel to plane ACD

15 a segment skew to $\overline{BC}$

16. all planes intersecting plane EDM

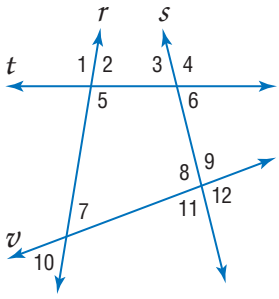
17. all segments skew to $\overline{AE}$

18. a segment parallel to $\overline{EN}$

19. a segment parallel to $\overline{AB}$ through point J

20. a segment skew to $\overline{CL}$ through point E

Examples 2–3 CCSS **PRECISION** **Identify the transversal connecting each pair of angles. Then classify the relationship between each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.**



21. $\angle 4$ and $\angle 9$

22. $\angle 5$ and $\angle 7$

23. $\angle 3$ and $\angle 5$

24. $\angle 10$ and $\angle 11$

25. $\angle 1$ and $\angle 6$

26. $\angle 6$ and $\angle 8$

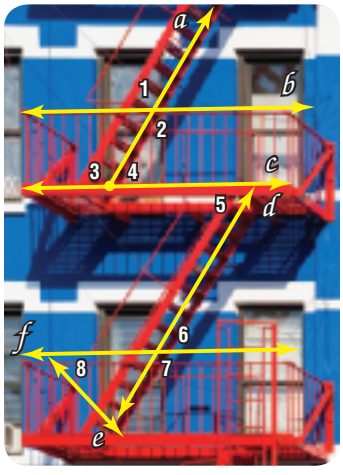
27. $\angle 2$ and $\angle 3$

28. $\angle 9$ and $\angle 10$

29. $\angle 4$ and $\angle 11$

30. $\angle 7$ and $\angle 11$

Example 3 **SAFETY** **Identify the transversal connecting each pair of angles in the photo of a fire escape shown. Then classify the relationship between each pair of angles.**



31. $\angle 1$ and $\angle 2$

32. $\angle 2$ and $\angle 4$

33. $\angle 4$ and $\angle 5$

34. $\angle 6$ and $\angle 7$

35. $\angle 7$ and $\angle 8$

36. $\angle 2$ and $\angle 3$

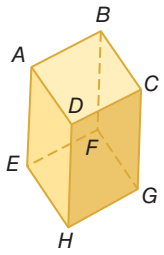
37. **POWER** Power lines are not allowed to intersect.

a. What must be the relationship between power lines p and m? Explain your reasoning.

b. What is the relationship between line q and lines p and m?

(t)Image Source/Getty Images, (b)Robert Llewellyn/CORBIS

Describe the relationship between each pair of segments as *parallel*, *skew*, or *intersecting*.



38. $\overline{FG}$ and $\overline{BC}$

39. $\overline{AB}$ and $\overline{CG}$

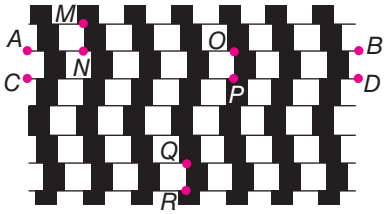
40. $\overline{DH}$ and $\overline{HG}$

41. $\overline{DH}$ and $\overline{BF}$

42. $\overline{EF}$ and $\overline{BC}$

43. $\overline{CD}$ and $\overline{AD}$

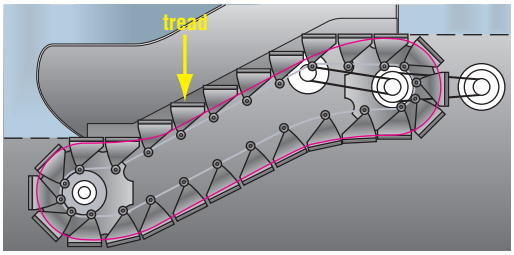
44. CCSS **SENSE-MAKING** The illusion at the right is created using squares and straight lines.



a. How are $\overline{AB}$ and $\overline{CD}$ related? Justify your reasoning.

b. How are $\overline{MN}$ and $\overline{QR}$ related? $\overline{AB}$, $\overline{CD}$, and $\overline{OP}$?

45 **ESCALATORS** Escalators consist of steps on a continuous loop that is driven by a motor. At the top and bottom of the platform, the steps collapse to provide a level surface for entrance and exit.



a. What is the relationship between the treads of the ascending stairs?

b. What is the relationship between the treads of the two steps at the top of the incline?

c. How do the treads of the steps on the incline of the escalator relate to the treads of the steps on the bottom of the escalator?

H.O.T. Problems Use Higher-Order Thinking Skills

46. **OPEN ENDED** Plane $\mathcal{P}$ contains lines a and b. Line c intersects plane $\mathcal{P}$ at point J. Lines a and b are parallel, lines a and c are skew, and lines b and c are not skew. Draw a figure based upon this description.

47. **CHALLENGE** Suppose points A, B, and C lie in plane $\mathcal{P}$, and points D, E, and F lie in plane Q. Line m contains points D and F and does not intersect plane $\mathcal{P}$. Line n contains points A and E.

a. Draw a diagram to represent the situation.

b. What is the relationship between planes $\mathcal{P}$ and Q?

c. What is the relationship between lines m and n?

REASONING **Plane $\mathcal{X}$ and plane $\mathcal{Y}$ are parallel and plane $\mathcal{Z}$ intersects plane $\mathcal{X}$. Line $\overleftrightarrow{AB}$ is in plane $\mathcal{X}$, line $\overleftrightarrow{CD}$ is in plane $\mathcal{Y}$, and line $\overleftrightarrow{EF}$ is in plane $\mathcal{Z}$. Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.**

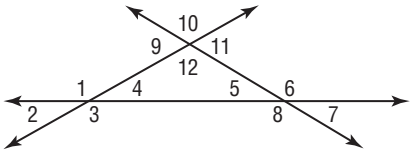
48. $\overleftrightarrow{AB}$ is skew to $\overleftrightarrow{CD}$.

49. $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{EF}$.

50. **WRITING IN MATH** Can a pair of planes be described as skew? Explain.

Standardized Test Practice

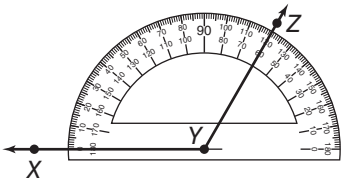
51. Which of the following angle pairs are alternate exterior angles?



A $\angle 1$ and $\angle 5$ **C** $\angle 2$ and $\angle 10$

B $\angle 2$ and $\angle 6$ **D** $\angle 5$ and $\angle 9$

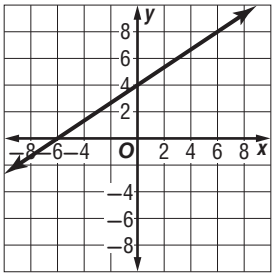
52. What is the measure of $\angle XYZ$?



F 30° **H** 120°

G 60° **J** 150°

53. SHORT RESPONSE Name the coordinates of the points representing the x- and y-intercepts of the graph shown below.



54. SAT/ACT Of the following, the one that is *not* equivalent to 485 is:

A $(3 \times 100) + (4 \times 10) + 145$

B $(3 \times 100) + (18 \times 10) + 5$

C $(4 \times 100) + (8 \times 10) + 15$

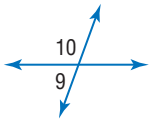
D $(4 \times 100) + (6 \times 10) + 25$

E $(4 \times 100) + (5 \times 10) + 35$

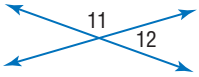
Spiral Review

Find the measure of each numbered angle. (Lesson 2-8)

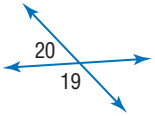
55. $m\angle 9 = 2x - 4$, $m\angle 10 = 2x + 4$



56. $m\angle 11 = 4x$, $m\angle 12 = 2x - 6$



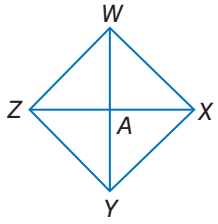
57. $m\angle 19 = 100 + 20x$, $m\angle 20 = 20x$



58. PROOF Prove the following. (Lesson 2-7)

Given: $\overline{WY} \cong \overline{ZX}$
A is the midpoint of $\overline{WY}$.
A is the midpoint of $\overline{ZX}$.

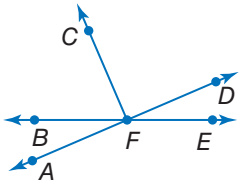
Prove: $\overline{WA} \cong \overline{ZA}$



ALGEBRA **Use the figure at the right.** (Lesson 1-5)

59. If $m\angle CFD = 12a + 45$, find a so that $\overrightarrow{FC} \perp \overrightarrow{FD}$.

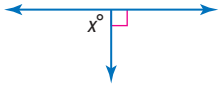
60. If $m\angle AFB = 8x - 6$ and $m\angle BFC = 14x + 8$, find the value of x so that $\angle AFC$ is a right angle.



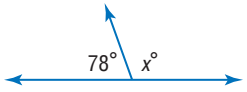
Skills Review

Find x.

61.



62.



63.