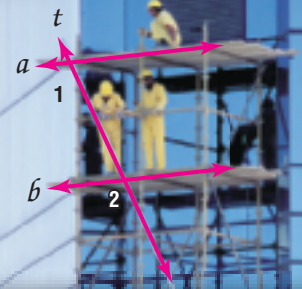# Angles and Parallel Lines

::Then
- You named angle pairs formed by parallel lines and transversals.

::Now
1. Use theorems to determine the relationships between specific pairs of angles.
2. Use algebra to find angle measurements.

::Why?
- Construction and maintenance workers often use an access scaffold. This structure provides support and access to elevated areas. The transversal *t* shown provides structural support to the two parallel working areas.

1 Parallel Lines and Angle Pairs

In the photo, line *t* is a transversal of lines *a* and *b*, and $\angle 1$ and $\angle 2$ are corresponding angles. Since lines *a* and *b* are parallel, there is a special relationship between corresponding angle pairs.

Postulate 3.1 Corresponding Angles Postulate

If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

Examples $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$, $\angle 5 \cong \angle 7$, $\angle 6 \cong \angle 8$
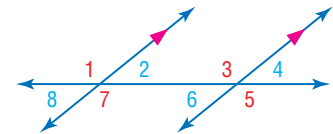
Example 1 Use Corresponding Angles Postulate

In the figure, $m\angle 5 = 72$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

a. $\angle 4$

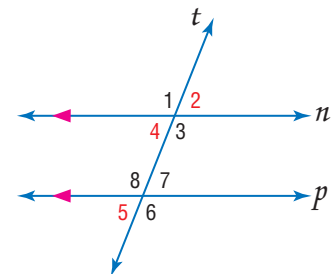|	
$\angle 4 \cong \angle 5$	Corresponding Angles Postulate
$m\angle 4 = m\angle 5$	Definition of congruent angles
$m\angle 4 = 72$	Substitution

b. $\angle 2$

$\angle 2 \cong \angle 4$	Vertical Angles Theorem
$\angle 4 \cong \angle 5$	Corresponding Angles Postulate
$\angle 2 \cong \angle 5$	Transitive Property of Congruence
$m\angle 2 = m\angle 5$	Definition of congruent angles
$m\angle 2 = 72$	Substitution

Guided Practice

In the figure, suppose that $m\angle 8 = 105$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

1A. $\angle 1$　　　　　**1B.** $\angle 2$　　　　　**1C.** $\angle 3$

In Example 1, $\angle 2$ and $\angle 5$ are congruent alternate exterior angles. This and other examples suggest the following theorems about the other angle pairs formed by two parallel lines cut by a transversal.

StudyTip

Angle Relationships These theorems generalize the relationships between specific pairs of angles. If you get confused about the relationships, you can verify them with the methods you used in Example 1, using only corresponding, vertical, and supplementary angles.

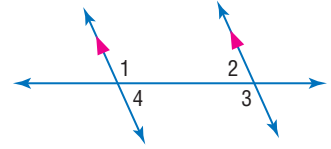## Theorems Parallel Lines and Angle Pairs

3.1 Alternate Interior Angles Theorem If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.

**Examples** $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$
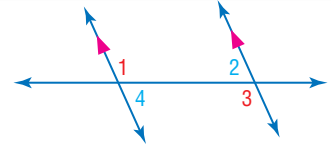
3.2 Consecutive Interior Angles Theorem If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.

Examples $\angle 1$ and $\angle 2$ are supplementary.
$\angle 3$ and $\angle 4$ are supplementary.

3.3 Alternate Exterior Angles Theorem If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.

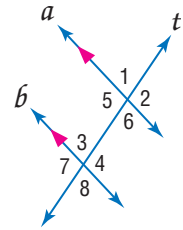**Examples** $\angle 5 \cong \angle 7$ and $\angle 6 \cong \angle 8$
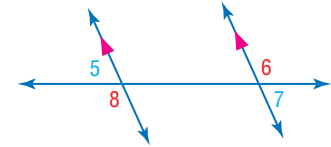
Since postulates are accepted without proof, you can use the Corresponding Angles Postulate to prove each of the theorems above.

Proof Alternate Interior Angles Theorem

Given: $a \parallel b$
t is a transversal of a and b.

Prove: $\angle 4 \cong \angle 5, \angle 3 \cong \angle 6$

Paragraph Proof: We are given that $a \parallel b$ with a transversal t. By the Corresponding Angles Postulate, corresponding angles are congruent. So, $\angle 2 \cong \angle 4$ and $\angle 6 \cong \angle 8$. Also, $\angle 5 \cong \angle 2$ and $\angle 8 \cong \angle 3$ because vertical angles are congruent. Therefore, $\angle 5 \cong \angle 4$ and $\angle 3 \cong \angle 6$ since congruence of angles is transitive.

Real-World Example 2 Use Theorems about Parallel Lines

COMMUNITY PLANNING Redding Lane and Creek Road are parallel streets that intersect Park Road along the west side of Wendell Park. If $m\angle 1 = 118$, find $m\angle 2$.

$\angle 2 \cong \angle 1$ Alternate Interior Angles Postulate

$m\angle 2 = m\angle 1$ Definition of congruent angles

$m\angle 2 = 118$ Substitution

Real-WorldLink

Some cities require that streets in newly planned subdivisions intersect at no less than a 60° angle.
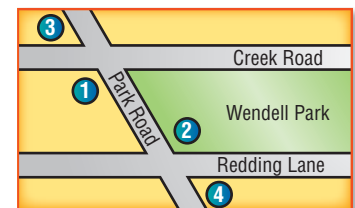
GuidedPractice

COMMUNITY PLANNING Refer to the diagram above to find each angle measure. Tell which postulate(s) or theorem(s) you used.

2A. If $m\angle 1 = 100$, find $m\angle 4$.

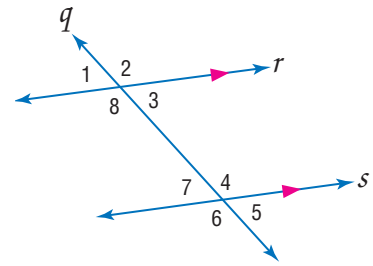**2B.** If $m\angle 3 = 70$, find $m\angle 4$.

2 **Algebra and Angle Measures** The special relationships between the angles formed by two parallel lines and a transversal can be used to find unknown values.

Example 3 Find Values of Variables

ALGEBRA Use the figure at the right to find the indicated variable. Explain your reasoning.

a. If $m\angle 4 = 2x - 17$ and $m\angle 1 = 85$, find x.

$\angle 3 \cong \angle 1$	Vertical Angles Theorem
$m\angle 3 = m\angle 1$	Definition of congruent angles
$m\angle 3 = 85$	Substitution

Since lines r and s are parallel, $\angle 4$ and $\angle 3$ are supplementary by the Consecutive Interior Angles Theorem.

$m\angle 3 + m\angle 4 = 180$	Definition of supplementary angles
$85 + 2x - 17 = 180$	Substitution
$2x + 68 = 180$	Simplify.
$2x = 112$	Subtract 68 from each side.
$x = 56$	Divide each side by 2.

b. Find y if $m\angle 3 = 4y + 30$ and $m\angle 7 = 7y + 6$.

$\angle 3 \cong \angle 7$	Alternate Interior Angles Theorem
$m\angle 3 = m\angle 7$	Definition of congruent angles
$4y + 30 = 7y + 6$	Substitution
$30 = 3y + 6$	Subtract 4y from each side.
$24 = 3y$	Subtract 6 from each side.
$8 = y$	Divide each side by 3.

▶ **Guided**Practice

3A. If $m\angle 2 = 4x + 7$ and $m\angle 7 = 5x - 13$, find x.

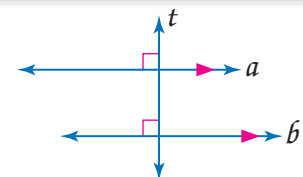**3B.** Find y if $m\angle 5 = 68$ and $m\angle 3 = 3y - 2$.
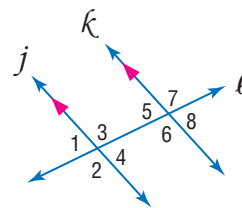
A special relationship exists when the transversal of two parallel lines is a perpendicular line.

Theorem 3.4 Perpendicular Transversal Theorem

In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.

Examples If line $a \parallel$ line b and line $a \perp$ line t, then line $b \perp$ line t.
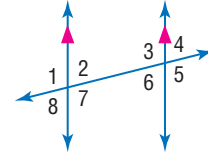
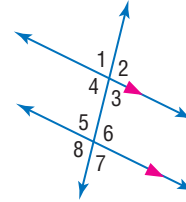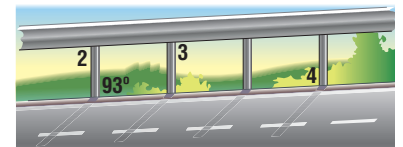You will prove Theorem 3.4 in Exercise 37.

Example 1   In the figure, $m\angle 1 = 94$. Find the measure of each angle.
Tell which postulate(s) or theorem(s) you used.

1. $\angle 3$ **2.** $\angle 5$ **3.** $\angle 4$

Example 2   In the figure, $m\angle 4 = 101$. Find the measure of each angle.
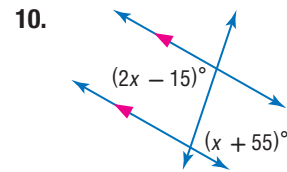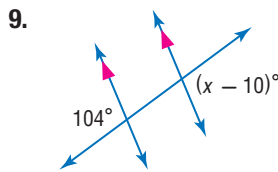Tell which postulate(s) or theorem(s) you used.
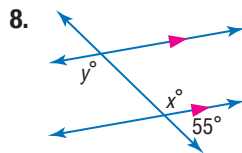
4. $\angle 6$ **5.** $\angle 7$ **6.** $\angle 5$

7. ROADS In the diagram, the guard rail is parallel to the
surface of the roadway and the vertical supports are
parallel to each other. Find the measures of angles 2,
3, and 4.

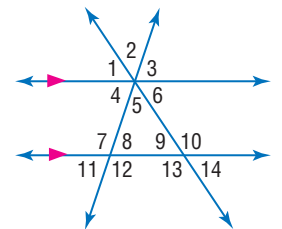**Example 3**   Find the value of the variable(s) in each figure. Explain your reasoning.

8.
$y°$
$x°$
$55°$

9.
$104°$
$(x - 10)°$

10.
$(2x - 15)°$
$(x + 55)°$

**Examples 1–2** In the figure, $m\angle 11 = 62$ and $m\angle 14 = 38$. Find the measure of
each angle. Tell which postulate(s) or theorem(s) you used.

11. $\angle 4$ **12.** $\angle 3$ **13.** $\angle 12$

14. $\angle 8$ **15.** $\angle 6$ **16.** $\angle 2$

17. $\angle 10$ **18.** $\angle 5$ **19.** $\angle 1$

Example 3   **CCSS MODELING** A solar dish collects energy by directing radiation from the Sun to a
receiver located at the focal point of the dish. Assume that the radiation rays are parallel.
Determine the relationship between each pair of angles, and explain your reasoning.
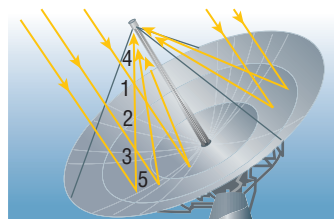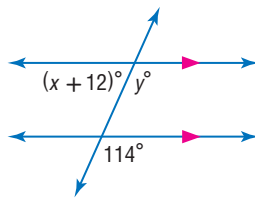
20. $\angle 1$ and $\angle 2$ **21** $\angle 1$ and $\angle 3$ **22.** $\angle 4$ and $\angle 5$ **23.** $\angle 3$ and $\angle 4$
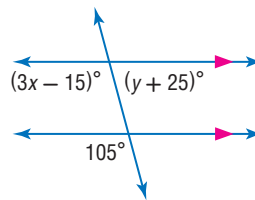
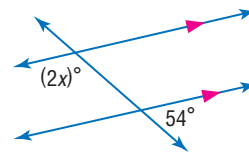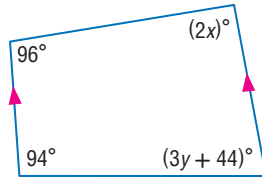**Find the value of the variable(s) in each figure. Explain your reasoning.**
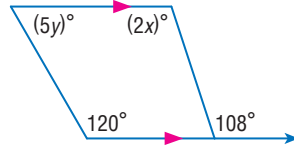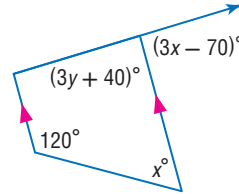
24.

$(x+12)°$ / $y°$

$114°$

25

$(3x-15)°$ $(y+25)°$

$105°$

26.

$(2x)°$

$54°$

27.

$96°$

$(2x)°$

$94°$ $(3y+44)°$

28.

$(5y)°$ $(2x)°$

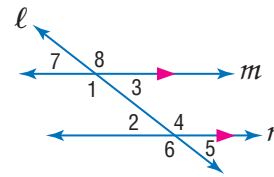$120°$ $108°$

29.

$(3x-70)°$

$(3y+40)°$

$120°$

$x°$

30. PROOF Copy and complete the proof of Theorem 3.2.

Given: $m \parallel n$; ℓ is a transversal.

Prove: ∠1 and ∠2 are supplementary;
∠3 and ∠4 are supplementary.

ℓ

7 8
1 3 m

2 4
6 5 n

Proof:

Statements	Reasons
a. ___?___	**a.** Given
b. ∠1 and ∠3 form a linear pair;	**b.** ___?___
∠2 and ∠4 form a linear pair.	**c.** If two angles form a linear pair,
c. ___?___	then they are supplementary.
d. ∠1 ≅ ∠4, ∠2 ≅ ∠3	**d.** ___?___
e. $m∠1 = m∠4$, $m∠2 = m∠3$	**e.** Definition of Congruence
f. ___?___	**f.** ___?___

STORAGE When industrial shelving needs to be accessible from either side, additional support is provided on the side by transverse members. Determine the relationship between each pair of angles and explain your reasoning.
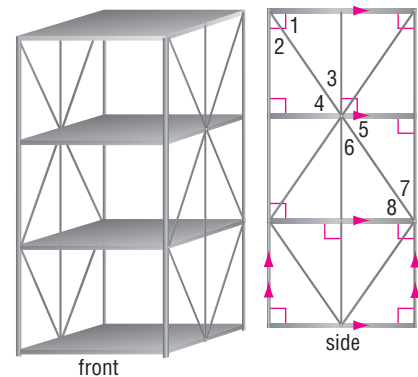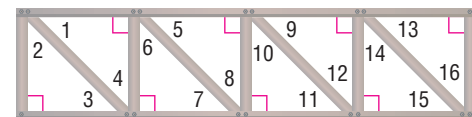
31. ∠1 and ∠8

32. ∠1 and ∠5

33. ∠3 and ∠6

34. ∠1 and ∠2

front

1
2
3
4
5
6
7
8

side

35. CCSS ARGUMENTS Write a two-column proof of the Alternate Exterior Angles Theorem. (Theorem 3.3)

36. BRIDGES Refer to the diagram of the double decker Michigan Avenue Bridge in Chicago, Illinois, at the right. The two levels of the bridge, and its diagonal braces, are parallel.

1 5 9 13
2 6 10 14
3 4 7 8 11 12 15 16

a. How are the measures of the odd-numbered angles related? Explain.

b. How are the measures of the even-numbered angles related? Explain.
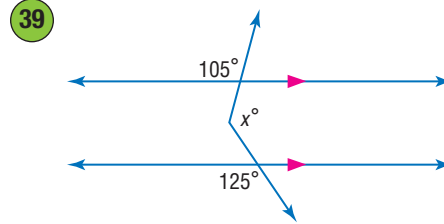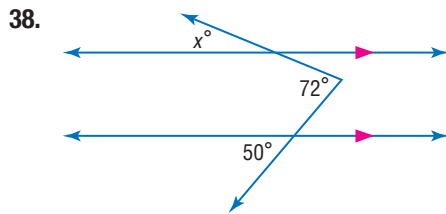
c. How are any pair of angles in which one is odd and the other is even related?

d. What geometric term(s) can be used to relate the two roadways contained by the bridge?

37. PROOF In a plane, prove that if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other. (Theorem 3.4)

CCSS TOOLS Find *x*. (*Hint:* Draw an auxiliary line.)

38.

39

40. PROBABILITY Suppose you were to pick any two angles in the figure below.

   a. How many possible angle pairings are there? Explain.

   b. Describe the possible relationships between the measures of the angles in each pair. Explain.

   c. Describe the likelihood of randomly selecting a pair of congruent angles. Explain your reasoning.
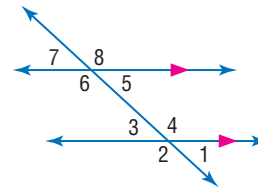
41. MULTIPLE REPRESENTATIONS In this problem, you will investigate the relationship between same-side exterior angles.

   a. Geometry Draw five pairs of parallel lines, *m* and *n*, *a* and *b*, *r* and *s*, *j* and *k*, and *x* and *y*, cut by a transversal *t*, and measure the four angles on one side of *t*.

   b. Tabular Record your data in a table.

   c. Verbal Make a conjecture about the relationship between the pair of angles formed on the exterior of parallel lines and on the same side of the transversal.

   d. Logical What type of reasoning did you use to form your conjecture? Explain.

   e. Proof Write a proof of your conjecture.

## H.O.T. Problems   Use Higher-Order Thinking Skills

42. WRITING IN MATH If line *a* is parallel to line *b* and $\angle 1 \cong \angle 2$, describe the relationship between lines *b* and *c*. Explain your reasoning.
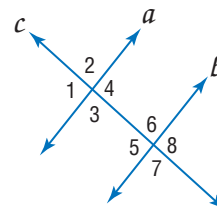
43. WRITING IN MATH Compare and contrast the Alternate Interior Angles Theorem and the Consecutive Interior Angles Theorem.

44. OPEN ENDED Draw a pair of parallel lines cut by a transversal and measure the two exterior angles on the same side of the transversal. Include the measures on your drawing. Based on the pattern you have seen for naming other pairs of angles, what do you think the name of the pair you measured would be?
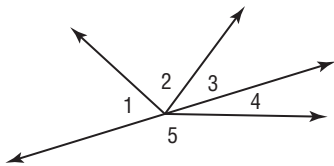
45. CHALLENGE Find *x* and *y*.
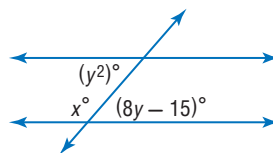
46. REASONING Determine the minimum number of angle measures you would have to know to find the measures of all the angles formed by two parallel lines cut by a transversal. Explain.
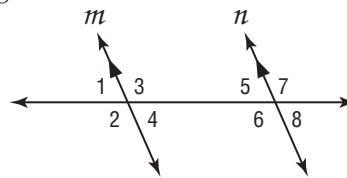
47. Suppose ∠4 and ∠5 form a linear pair. If $m\angle1 = 2x$, $m\angle2 = 3x - 20$, and $m\angle3 = x - 4$, what is $m\angle3$?

A 26° **C** 30°

B 28° **D** 32°

48. SAT/ACT A farmer raises chickens and pigs. If his animals have a total of 120 heads and a total of 300 feet, how many chickens does the farmer have?

F 60 **H** 80

G 70 **J** 90

**49. SHORT RESPONSE** If $m \parallel n$, then which of the following statements must be true?

   I. ∠3 and ∠6 are Alternate Interior Angles.

   II. ∠4 and ∠6 are Consecutive Interior Angles.

   III. ∠1 and ∠7 are Alternate Exterior Angles.

50. ALGEBRA If $-2 + x = -6$, then $-17 - x = ?$

A −13 **D** 13

B −4 **E** 21

C 9

**51. AVIATION** Airplanes are assigned an altitude level based on the direction they are flying. If one airplane is flying northwest at 34,000 feet and another airplane is flying east at 25,000 feet, describe the type of lines formed by the paths of the airplanes. Explain your reasoning. (Lesson 3-1)

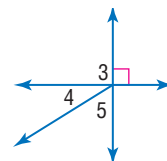**Use the given statement to find the measure of each numbered angle.** (Lesson 2-8)

52. ∠1 and ∠2 form a linear pair and $m\angle2 = 67$.
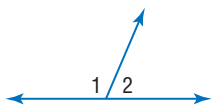
53. ∠6 and ∠8 are; complementary $m\angle8 = 47$.
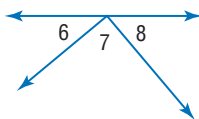
54. $m\angle4 = 32$

55. TRAINS A train company wants to provide routes to New York City, Dallas, Chicago, Los Angeles, San Francisco, and Washington, D.C. An engineer draws lines between each pair of cities on a map. No three of the cities are collinear. How many lines did the engineer draw? (Lesson 2-5)

Simplify each expression.

56. $\frac{6-5}{4-2}$

57. $\frac{-5-2}{4-7}$

58. $\frac{-11-4}{12-(-9)}$

59. $\frac{16-12}{15-11}$

60. $\frac{10-22}{8-17}$

61. $\frac{8-17}{12-(-3)}$