

LESSON 3-3 Slopes of Lines

Then

- You used the properties of parallel lines to determine congruent angles.

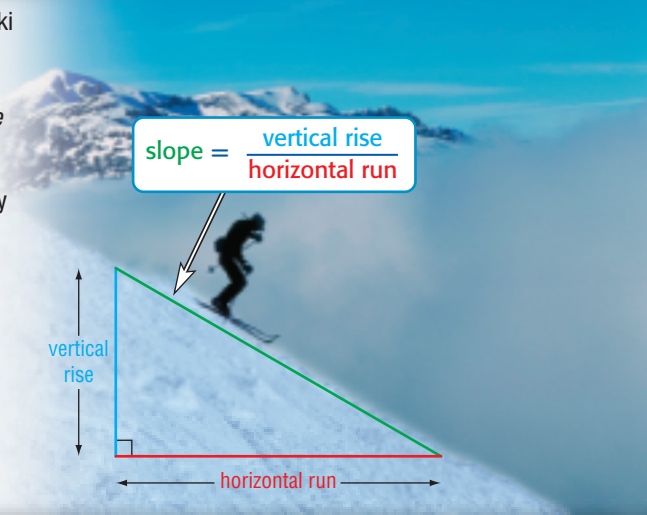
Now

- Find slopes of lines.
- Use slope to identify parallel and perpendicular lines.

Why?

- Ski resorts assign ratings to their ski trails according to their difficulty. A primary factor in determining this rating is a trail's steepness or *slope gradient*. A trail with a 6% or $\frac{6}{100}$ grade falls 6 feet vertically for every 100 feet traveled horizontally.

The easiest trails, labeled ●, have slopes ranging from 6% to 25%, while more difficult trails, labeled ◆ or ◆◆, have slopes of 40% or greater.



New Vocabulary
slope
rate of change

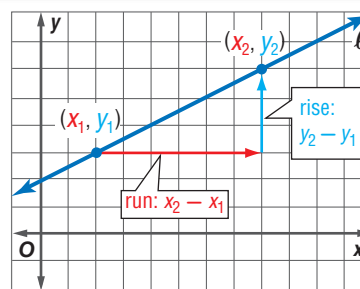
1 Slope of a Line The steepness or slope of a hill is described by the ratio of the hill's vertical rise to its horizontal run. In algebra, you learned that the slope of a line in the coordinate plane can be calculated using any two points on the line.

Key Concept Slope of a Line

In a coordinate plane, the **slope** of a line is the ratio of the change along the y -axis to the change along the x -axis between any two points on the line.

The slope m of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2.$$



$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Common Core State Standards

Content Standards

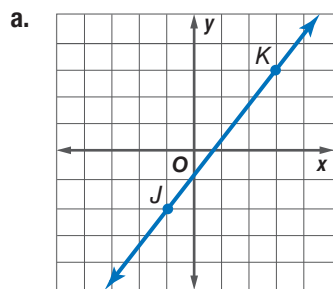
G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Mathematical Practices

- Model with mathematics.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

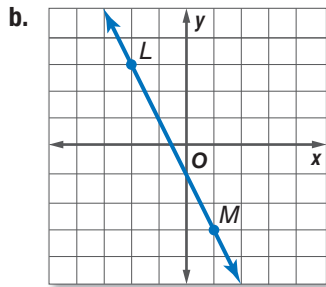
Example 1 Find the Slope of a Line

Find the slope of each line.



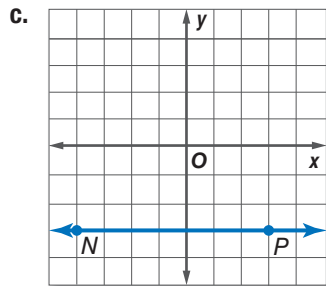
Substitute $(-1, -2)$ for (x_1, y_1) and $(3, 3)$ for (x_2, y_2) .

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope Formula} \\ &= \frac{3 - (-2)}{3 - (-1)} && \text{Substitution} \\ &= \frac{5}{4} && \text{Simplify.} \end{aligned}$$



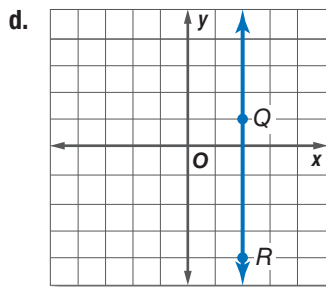
Substitute $(-2, 3)$ for (x_1, y_1) and $(1, -3)$ for (x_2, y_2) .

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope Formula} \\
 &= \frac{-3 - 3}{1 - (-2)} && \text{Substitution} \\
 &= -2 && \text{Simplify.}
 \end{aligned}$$



Substitute $(-4, -3)$ for (x_1, y_1) and $(3, -3)$ for (x_2, y_2) .

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope Formula} \\
 &= \frac{-3 - (-3)}{3 - (-4)} && \text{Substitution} \\
 &= \frac{0}{7} \text{ or } 0 && \text{Simplify.}
 \end{aligned}$$



Substitute $(2, 1)$ for (x_1, y_1) and $(2, -4)$ for (x_2, y_2) .

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope Formula} \\
 &= \frac{-4 - 1}{2 - 2} && \text{Substitution} \\
 &= \frac{-5}{0} && \text{Simplify.}
 \end{aligned}$$

This slope is **undefined**.

StudyTip

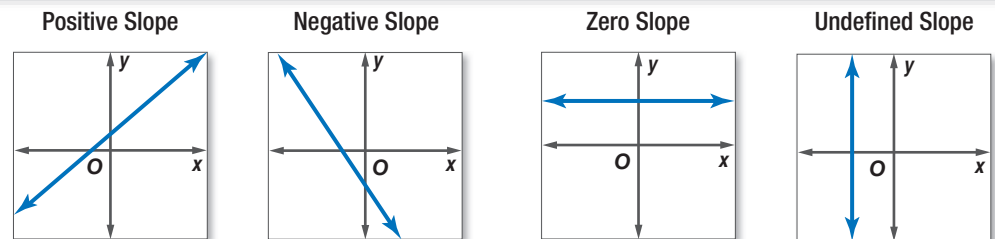
Dividing by 0 The slope $\frac{-5}{0}$ is undefined because there is no number that you can multiply by 0 and get -5 . Since this is true for any number, all numbers divided by 0 will have an undefined slope. All vertical lines have undefined slopes.

GuidedPractice

- 1A. the line containing $(6, -2)$ and $(-3, -5)$ 1B. the line containing $(8, -3)$ and $(-6, -2)$
 1C. the line containing $(4, 2)$ and $(4, -3)$ 1D. the line containing $(-3, 3)$ and $(4, 3)$

Example 1 illustrates the four different types of slopes.

ConceptSummary Classifying Slopes



Slope can be interpreted as a **rate of change**, describing how a quantity y changes in relation to quantity x . The slope of a line can also be used to identify the coordinates of any point on the line.



Real-World Example 2 Use Slope as Rate of Change

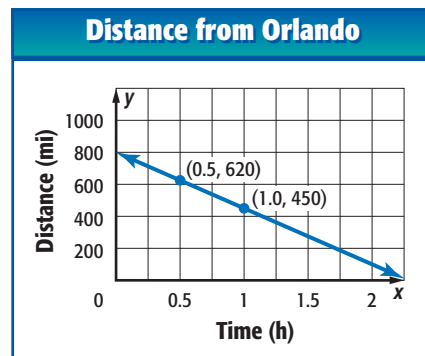
TRAVEL A pilot flies a plane from Columbus, Ohio, to Orlando, Florida. After 0.5 hour, the plane reaches its cruising altitude and is 620 miles from Orlando. Half an hour later, the plane is 450 miles from Orlando. How far was the plane from Orlando 1.25 hours after takeoff?

Understand Use the data given to graph the line that models the distance from Orlando y in miles as a function of time x in hours.

Assume that speed is constant. Plot the points $(0.5, 620)$ and $(1.0, 450)$, and draw a line through them.

You want to find the distance from Orlando after 1.25 hours.

From the graph we can estimate that after 1.25 hours, the distance was a little less than 400 miles.



Plan Find the slope of the line graphed. Use this rate of change in the plane's distance from Orlando per hour to find the distance from Orlando after 1.25 hours.

Solve Use the Slope Formula to find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(450 - 620) \text{ miles}}{(1.0 - 0.5) \text{ hours}} = \frac{-170 \text{ miles}}{0.5 \text{ hour}} \text{ or } -\frac{340 \text{ miles}}{1 \text{ hour}}$$

The plane traveled at an average speed of 340 miles per hour. The negative sign indicates a *decrease* in distance over time.

Use the slope of the line and one known point on the line to calculate the distance y when the time x is 1.25.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope Formula} \\ -340 &= \frac{y_2 - 620}{1.25 - 0.5} && m = -340, x_1 = 0.5, y_1 = 620, \text{ and } x_2 = 1.25 \\ -340 &= \frac{y_2 - 620}{0.75} && \text{Simplify.} \\ -255 &= y_2 - 620 && \text{Multiply each side by 0.75.} \\ 365 &= y_2 && \text{Add 620 to each side.} \end{aligned}$$

Thus, the distance from Orlando after 1.25 hours is 365 miles.

Check Since 365 is close to the estimate, our answer is reasonable. ✓

Guided Practice

2. **DOWNLOADS** In 2006, 500 million songs were legally downloaded from the Internet. In 2004, 200 million songs were legally downloaded.
 - A. Use the data given to graph the line that models the number of songs legally downloaded y as a function of time x in years.
 - B. Find the slope of the line, and interpret its meaning.
 - C. If this trend continues at the same rate, how many songs will be legally downloaded in 2020?



Real-World Career

Flight Attendants Flight attendants check tickets, assist passengers with boarding and carry-ons, and provide an overview of emergency equipment and procedures. A high school diploma is required, but airlines increasingly favor bi- or multi-lingual candidates with college degrees.



2 Parallel and Perpendicular Lines

You can use the slopes of two lines to determine whether the lines are parallel or perpendicular. Lines with the same slope are parallel.

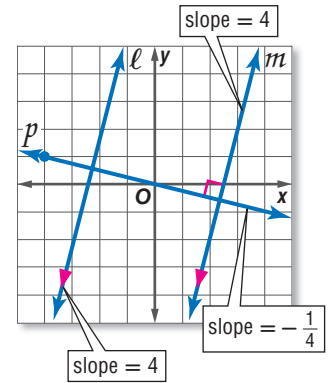
Postulates Parallel and Perpendicular Lines

3.2 Slopes of Parallel Lines Two nonvertical lines have the same slope if and only if they are parallel. All vertical lines are parallel.

Example Parallel lines ℓ and m have the same slope, 4.

3.3 Slopes of Perpendicular Lines Two nonvertical lines are perpendicular if and only if the product of their slopes is -1 . Vertical and horizontal lines are perpendicular.

Example line $m \perp$ line p
product of slopes = $4 \cdot -\frac{1}{4}$ or -1



Example 3 Determine Line Relationships

Determine whether \overleftrightarrow{AB} and \overleftrightarrow{CD} are *parallel*, *perpendicular*, or *neither* for $A(1, 1)$, $B(-1, -5)$, $C(3, 2)$, and $D(6, 1)$. Graph each line to verify your answer.

Step 1 Find the slope of each line.

$$\text{slope of } \overleftrightarrow{AB} = \frac{-5-1}{-1-1} = \frac{-6}{-2} \text{ or } 3 \qquad \text{slope of } \overleftrightarrow{CD} = \frac{1-2}{6-3} \text{ or } \frac{-1}{3}$$

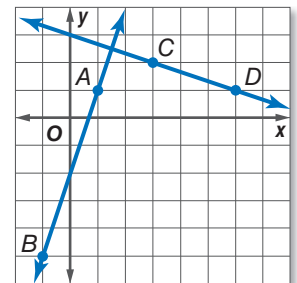
Step 2 Determine the relationship, if any, between the lines.

The two lines do not have the same slope, so they are *not* parallel. To determine if the lines are perpendicular, find the product of their slopes.

$$3\left(-\frac{1}{3}\right) = -1 \quad \text{Product of slopes for } \overleftrightarrow{AB} \text{ and } \overleftrightarrow{CD}$$

Since the product of their slopes is -1 , \overleftrightarrow{AB} is perpendicular to \overleftrightarrow{CD} .

CHECK When graphed, the two lines appear to intersect and form four right angles. ✓



StudyTip

Slopes of Perpendiculars

If a line ℓ has a slope of $\frac{a}{b}$, then the slope of a line perpendicular to line ℓ is the opposite reciprocal, $-\frac{b}{a}$, since $\frac{a}{b}\left(-\frac{b}{a}\right) = -1$.

GuidedPractice

Determine whether \overleftrightarrow{AB} and \overleftrightarrow{CD} are *parallel*, *perpendicular*, or *neither*. Graph each line to verify your answer.

3A. $A(14, 13)$, $B(-11, 0)$, $C(-3, 7)$, $D(-4, -5)$

3B. $A(3, 6)$, $B(-9, 2)$, $C(5, 4)$, $D(2, 3)$



Example 4 Use Slope to Graph a Line

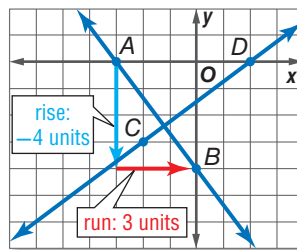
Graph the line that contains $A(-3, 0)$ and is perpendicular to \overleftrightarrow{CD} with $C(-2, -3)$ and $D(2, 0)$.

The slope of \overleftrightarrow{CD} is $\frac{0 - (-3)}{2 - (-2)}$ or $\frac{3}{4}$.

Since $\frac{3}{4}(\frac{4}{-3}) = -1$, the slope of the line

perpendicular to \overleftrightarrow{CD} through A is $-\frac{4}{3}$ or $\frac{-4}{3}$.

To graph the line, start at point A . Move down 4 units and then right 3 units. Label the point B and draw \overleftrightarrow{AB} .



Guided Practice

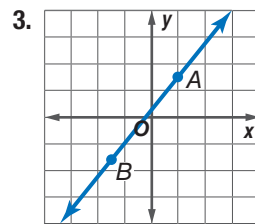
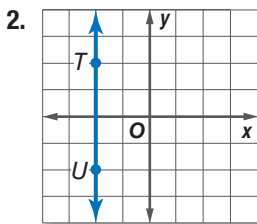
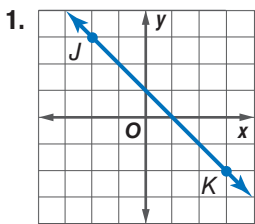
- Graph the line that contains $P(0, 1)$ and is perpendicular to \overleftrightarrow{QR} with $Q(-6, -2)$ and $R(0, -6)$.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

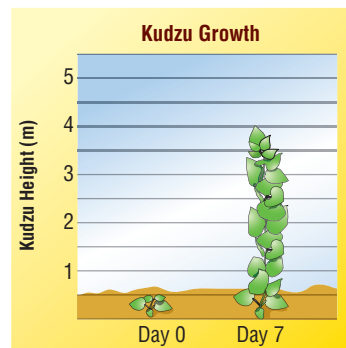


Example 1 Find the slope of each line.



- Example 2** 4. **BOTANY** Kudzu is a fast-growing vine found in the southeastern United States. An initial measurement of the length of a kudzu vine was 0.5 meter. Seven days later the plant was 4 meters long.

- Graph the line that models the length of the plant over time.
- What is the slope of your graph? What does it represent?
- Assuming that the growth rate of the plant continues, how long will the plant be after 15 days?



Example 3 Determine whether \overleftrightarrow{WX} and \overleftrightarrow{YZ} are *parallel*, *perpendicular*, or *neither*. Graph each line to verify your answer.

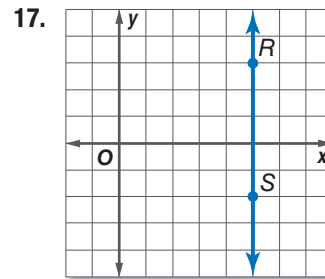
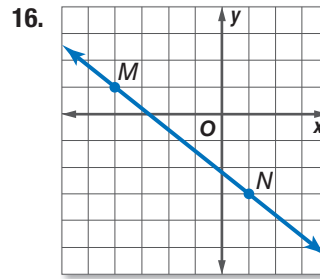
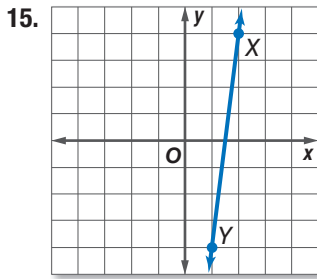
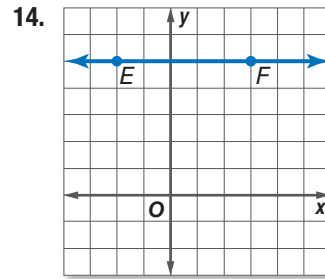
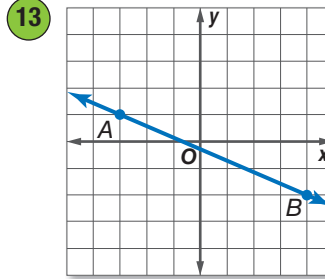
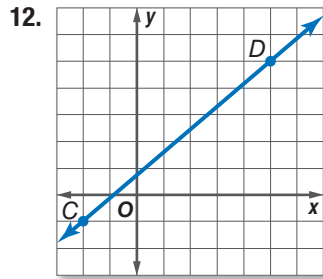
- $W(2, 4), X(4, 5), Y(4, 1), Z(8, -7)$
- $W(1, 3), X(-2, -5), Y(-6, -2), Z(8, 3)$
- $W(-7, 6), X(-6, 9), Y(6, 3), Z(3, -6)$
- $W(1, -3), X(0, 2), Y(-2, 0), Z(8, 2)$

Example 4 Graph the line that satisfies each condition.

- passes through $A(3, -4)$, parallel to \overleftrightarrow{BC} with $B(2, 4)$ and $C(5, 6)$
- slope = 3, passes through $A(-1, 4)$
- passes through $P(7, 3)$, perpendicular to \overleftrightarrow{LM} with $L(-2, -3)$ and $M(-1, 5)$



Example 1 Find the slope of each line.



Determine the slope of the line that contains the given points.

- | | |
|----------------------------|-------------------------------|
| 18. $C(3, 1), D(-2, 1)$ | 19. $E(5, -1), F(2, -4)$ |
| 20. $G(-4, 3), H(-4, 7)$ | 21. $J(7, -3), K(-8, -3)$ |
| 22. $L(8, -3), M(-4, -12)$ | 23. $P(-3, -5), Q(-3, -1)$ |
| 24. $R(2, -6), S(-6, 5)$ | 25. $T(-6, -11), V(-12, -10)$ |

Example 2

26. **CCSS MODELING** In 2004, 8 million Americans over the age of 7 participated in mountain biking, and in 2006, 8.5 million participated.
- Create a graph to show the number of participants in mountain biking based on the change in participation from 2004 to 2006.
 - Based on the data, what is the growth per year of the sport?
 - If participation continues at the same rate, what will be the participation in 2013 to the nearest 10,000?
27. **FINANCIAL LITERACY** Suppose an MP3 player cost \$499 in 2003 and \$249.99 in 2009.
- Graph a trend line to predict the price of the MP3 player for 2003 through 2009.
 - Based on the data, how much does the price drop per year?
 - If the trend continues, what will be the cost of an MP3 player in 2013?

Example 3

Determine whether \overleftrightarrow{AB} and \overleftrightarrow{CD} are *parallel*, *perpendicular*, or *neither*. Graph each line to verify your answer.

- | | |
|--|---|
| 28. $A(1, 5), B(4, 4), C(9, -10), D(-6, -5)$ | 29. $A(-6, -9), B(8, 19), C(0, -4), D(2, 0)$ |
| 30. $A(4, 2), B(-3, 1), C(6, 0), D(-10, 8)$ | 31. $A(8, -2), B(4, -1), C(3, 11), D(-2, -9)$ |
| 32. $A(8, 4), B(4, 3), C(4, -9), D(2, -1)$ | 33. $A(4, -2), B(-2, -8), C(4, 6), D(8, 5)$ |



Example 4

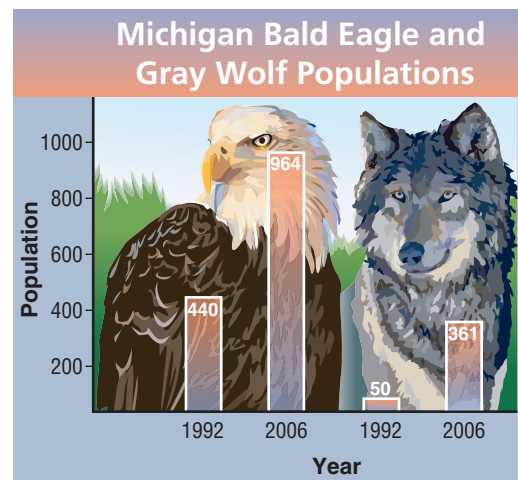
Graph the line that satisfies each condition.

34. passes through $A(2, -5)$, parallel to \overleftrightarrow{BC} with $B(1, 3)$ and $C(4, 5)$
 35. slope = -2 , passes through $H(-2, -4)$
 36. passes through $K(3, 7)$, perpendicular to \overleftrightarrow{LM} with $L(-1, -2)$ and $M(-4, 8)$
 37. passes through $X(1, -4)$, parallel to \overleftrightarrow{YZ} with $Y(5, 2)$ and $Z(-3, -5)$
 38. slope = $\frac{2}{3}$, passes through $J(-5, 4)$
 39. passes through $D(-5, -6)$, perpendicular to \overleftrightarrow{FG} with $F(-2, -9)$ and $G(1, -5)$
40. **STADIUMS** Before it was demolished, the RCA Dome was home to the Indianapolis Colts. The attendance in 2001 was 450,746, and the attendance in 2005 was 457,373.
- a. What is the approximate rate of change in attendance from 2001 to 2005?
 - b. If this rate of change continues, predict the attendance for 2012.
 - c. Will the attendance continue to increase indefinitely? Explain.
 - d. The Colts have now built a new, larger stadium. Do you think their decision was reasonable? Why or why not?

Determine which line passing through the given points has a steeper slope.

- | | |
|---|---|
| 41. Line 1: $(0, 5)$ and $(6, 1)$
Line 2: $(-4, 10)$ and $(8, -5)$ | 42. Line 1: $(0, -4)$ and $(2, 2)$
Line 2: $(0, -4)$ and $(4, 5)$ |
| 43. Line 1: $(-9, -4)$ and $(7, 0)$
Line 2: $(0, 1)$ and $(7, 4)$ | 44. Line 1: $(-6, 7)$ and $(9, -3)$
Line 2: $(-9, 9)$ and $(3, 5)$ |

45. **CCSS MODELING** Michigan provides habitat for two endangered species, the bald eagle and the gray wolf. The graph shows the Michigan population of each species in 1992 and 2006.
- a. Which species experienced a greater rate of change in population?
 - b. Make a line graph showing the growth of both populations.
 - c. If both species continue to grow at their respective rates, what will the population of each species be in 2012?



Find the value of x or y that satisfies the given conditions. Then graph the line.

46. The line containing $(4, -1)$ and $(x, -6)$ has a slope of $-\frac{5}{2}$.
 47. The line containing $(-4, 9)$ and $(4, 3)$ is parallel to the line containing $(-8, 1)$ and $(4, y)$.
 48. The line containing $(8, 7)$ and $(7, -6)$ is perpendicular to the line containing $(2, 4)$ and $(x, 3)$.
 49. The line containing $(1, -3)$ and $(3, y)$ is parallel to the line containing $(5, -6)$ and $(9, y)$.
50. **SCHOOLS** In 2000, Jefferson High School had 1125 students. By 2006, the student body had increased to 1425 students. When Fairview High School was built in 2001, it had 1275 students. How many students did Fairview High School have in 2006 if the student body grew at the same rate as Jefferson High School?



- 51 MUSIC** Maggie and Mikayla want to go to the music store near Maggie's house after school. They can walk 3.5 miles per hour and ride their bikes 10 miles per hour.
- Create a table to show how far Maggie and Mikayla can travel walking and riding their bikes. Include distances for 0, 1, 2, 3, and 4 hours.
 - Create a graph to show how far Maggie and Mikayla can travel based on time for both walking and riding their bikes. Be sure to label the axes of your graph.
 - What does the slope represent in your graph?
 - Maggie's mom says they can only go if they can make it to the music store and back in less than two hours. If they want to spend at least 30 minutes in the music store and it is four miles away, can they make it? Should they walk or ride their bikes? Explain your reasoning.

H.O.T. Problems Use Higher-Order Thinking Skills

- 52. WRITE A QUESTION** A classmate says that all lines have positive or negative slope. Write a question that would challenge his conjecture.
- 53. ERROR ANALYSIS** Terrell and Hale calculated the slope of the line passing through the points $Q(3, 5)$ and $R(-2, 2)$. Is either of them correct? Explain your reasoning.

$$\begin{array}{l} \text{Terrell} \\ m = \frac{5-2}{3-(-2)} \\ = \frac{3}{5} \end{array}$$

$$\begin{array}{l} \text{Hale} \\ m = \frac{5-2}{-2-3} \\ = -\frac{3}{5} \end{array}$$

- 54. CCSS REASONING** Draw a square $ABCD$ with opposite vertices at $A(2, -4)$ and $C(10, 4)$.
- Find the other two vertices of the square and label them B and D .
 - Show that $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$.
 - Show that the measure of each angle inside the square is equal to 90.

- 55. WRITING IN MATH** Describe the slopes of the Sears Tower and the Leaning Tower of Pisa.

- 56. CHALLENGE** In this lesson you learned that $m = \frac{y_2 - y_1}{x_2 - x_1}$. Use an algebraic proof to show that the slope can also be calculated using the equation $m = \frac{y_1 - y_2}{x_1 - x_2}$.



Sears Tower



Leaning Tower of Pisa

- 57. WRITING IN MATH** Find two additional points that lie along the same line as $X(3, -1)$ and $Y(-1, 7)$. Generalize a method you can use to find additional points on the line from any given point.



Standardized Test Practice

58. What is the slope of a line perpendicular to the line through the points $(-1, 6)$ and $(3, -4)$?

- A $m = -\frac{5}{2}$
- B $m = -1$
- C $m = -\frac{2}{5}$
- D $m = \frac{2}{5}$

59. **SHORT RESPONSE** A set of 25 cards is randomly placed face down on a table. 15 cards have only the letter A written on the face, and 10 cards have only the letter B. Patrick turned over 1 card. What is the probability of this card having the letter B written on its face?

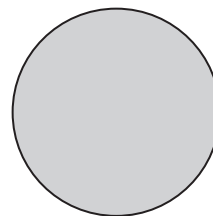
60. **ALGEBRA** Jamie is collecting money to buy an \$81 gift for her teacher. She has already contributed \$24. She will collect \$3 from each contributing student. How many other students must contribute?

- F 3 students
- G 9 students
- H 12 students
- J 19 students

61. **SAT/ACT** The area of a circle is 20π square centimeters. What is its circumference?

- A $\sqrt{5}\pi$ cm
- B $2\sqrt{5}\pi$ cm
- C $4\sqrt{5}\pi$ cm
- D 20π cm
- E 40π cm

$$A = 20\pi \text{ cm}^2$$

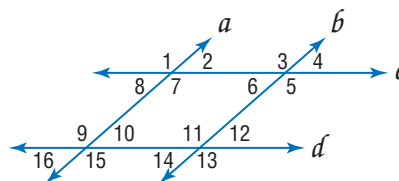


Spiral Review

In the figure, $a \parallel b$, $c \parallel d$, and $m\angle 4 = 57$.

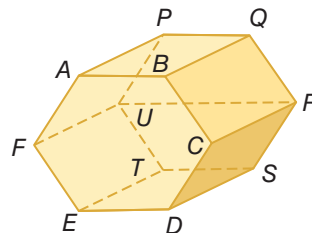
Find the measure of each angle. (Lesson 3-2)

- 62. $\angle 5$
- 63. $\angle 1$
- 64. $\angle 8$
- 65. $\angle 10$



Refer to the diagram at the right. (Lesson 3-1)

- 66. Name all segments parallel to \overline{TU} .
- 67. Name all planes intersecting plane BCR .
- 68. Name all segments skew to \overline{DE} .



Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning. (Lesson 2-4)

- 69. **Given:** $\angle B$ and $\angle C$ are vertical angles.
Conclusion: $\angle B \cong \angle C$
- 70. **Given:** $\angle W \cong \angle Y$
Conclusion: $\angle W$ and $\angle Y$ are vertical angles.
- 71. **CONSTRUCTION** There are four buildings on the Mansfield High School Campus, no three of which stand in a straight line. How many sidewalks need to be built so that each building is directly connected to every other building? (Lesson 1-1)

Skills Review

Solve for y .

- 72. $3x + y = 5$
- 73. $4x + 2y = 6$
- 74. $4y - 3x = 5$

