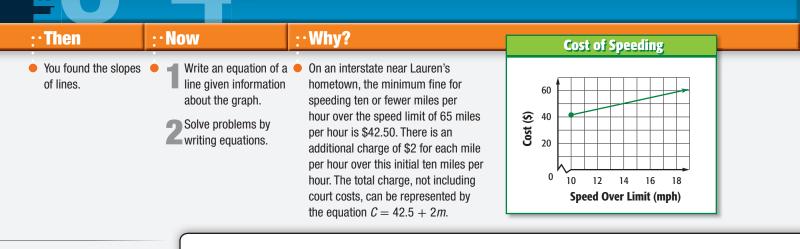
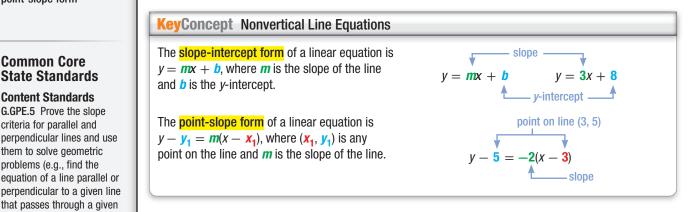
# **Equations of Lines**



**Write Equations of Lines** You may remember from algebra that an equation of a nonvertical line can be written in different but equivalent forms.



When given the slope and either the *y*-intercept or a point on a line, you can use these forms to write the equation of the line.

#### **Example 1** Slope and *y*-intercept

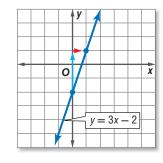


Write an equation in slope-intercept form of the line with slope 3 and y-intercept of -2. Then graph the line.

y = mx + b	Slope-intercept form
y = 3x + (-2)	m = 3, b = -2
y = 3x - 2	Simplify.

Plot a point at the *y*-intercept, -2. Use the slope of

3 or  $\frac{3}{1}$  to find another point 3 units up and 1 unit to the right of the *y*-intercept. Then draw the line through these two points.



## **Guided**Practice

1. Write an equation in slope-intercept form of the line with slope  $-\frac{1}{2}$  and *y*-intercept of 8. Then graph the line.

**NewVocabularv** 

slope-intercept form point-slope form

point).

**Mathematical Practices** 

4 Model with mathematics.8 Look for and express regularity in repeated

reasoning.

#### **Example 2** Slope and a Point on the Line



PT

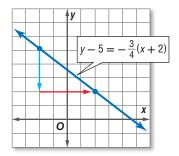
Write an equation in point-slope form of the line with slope  $-\frac{3}{4}$  that contains (-2, 5). Then graph the line.

$$y - y_1 = m(x - x_1)$$
Point-Slope form $y - 5 = -\frac{3}{4}[x - (-2)]$  $m = -\frac{3}{4}, (x_1, y_1) = (-2, 5)$  $y - 5 = -\frac{3}{4}(x + 2)$ Simplify.

Graph the given point (-2, 5). Use the slope  $-\frac{3}{4}$  or  $\frac{-3}{4}$  to find another point 3 units down and 4 units to the right. Then draw the line through these two points.

#### GuidedPractice

**2.** Write an equation in point-slope form of the line with slope 4 that contains (-3, -6). Then graph the line.



When the slope of a line is not given, use two points on the line to calculate the slope. Then use the point-slope or slope-intercept form to write an equation of the line.

#### **Example 3** Two Points

Write an equation of the line through each pair of points in slope-intercept form.

**a.** (0, 3) and (−2, −1)

**Step 1** Find the slope of the line through the points.

 $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{-2 - 0} = \frac{-4}{-2}$  or 2 Use the Slope Formula.

**Step 2** Write an equation of the line.

Slope-Intercept form m = 2; (0, 3) is the *y*-intercept.

**b.** (-7, 4) and (9, -4)

y = mx + b

y = 2x + 3

Step 1 
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 4}{9 - (-7)} = \frac{-8}{16} \text{ or } -\frac{1}{2}$$
  
Step 2  $y - y_1 = m(x - x_1)$   
 $y - 4 = -\frac{1}{2}[x - (-7)]$   
 $y - 4 = -\frac{1}{2}(x + 7)$   
 $y - 4 = -\frac{1}{2}x - \frac{7}{2}$   
 $y = -\frac{1}{2}x + \frac{1}{2}$ 

Use the Slope Formula.

Point-Slope form

$$m = -\frac{1}{2}, (x_1, y_1) = (-7, 4)$$

Simplify.

Distribute.

Add 4 to each side: 
$$\frac{7}{2} + 4 = -\frac{7}{2} + \frac{8}{2}$$
$$= \frac{1}{2}$$

#### GuidedPractice

**3A.** (-2, 4) and (8, 10)

**3B.** (-1, 3) and (7, 3)

## WatchOut!

Substituting Negative Coordinates When substituting negative coordinates, use parentheses to avoid making errors with the signs.

**CCSS** Perseverance In

Example 3b, you could also use the slope-intercept form and one point to find the *y*-intercept and write the equation.

$$y = mx + b$$
  

$$4 = -\frac{1}{2}(-7) + b$$
  

$$4 = \frac{7}{2} + b$$
  

$$4 - \frac{7}{2} = b$$
  

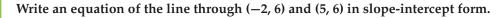
$$b = \frac{1}{2}$$
  
So,  $y = -\frac{1}{2}x + \frac{1}{2}$ .

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Math HistoryLink **Gaspard Monge** (1746-1818) Monge presented the point-slope form of an equation of a line in a paper published in 1784.

## **Example 4** Horizontal Line



line.

Step 1
 
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 6}{5 - (-2)} = \frac{0}{7}$$
 or 0
 This is a horizontal

 Step 2
  $y - y_1 = m(x - x_1)$ 
 Point-Slope form

  $y - 6 = 0[x - (-2)]$ 
 $m = -\frac{1}{2}, (x_1, y_1) = (-2, 6)$ 
 $y - 6 = 0$ 
 Simplify.

  $y = 6$ 
 Add 6 to each side.

## **Guided**Practice

E

**4.** Write an equation of the line through (5, 0) and (-1, 0) in slope-intercept form.

The equations of horizontal and vertical lines involve only one variable.

KeyConcepts Horizontal and Vertical Line Equations	
The equation of a horizontal line is $y = b$ , where b is the y-intercept of the line.	
Example $y = -3$	x = -2
The equation of a vertical line is $x = a$ , where $a$ is the $x$ -intercept of the line.	
Example $x = -2$	

Parallel lines that are not vertical have equal slopes. Two nonvertical lines are perpendicular if the product of their slope is -1. Vertical and horizontal lines are always perpendicular to one another.

**Example 5** Write Equations of Parallel or Perpendicular Lines

Write an equation in slope-intercept form for a line perpendicular to y = -3x + 2 containing (4, 0).

The slope of y = -3x + 2 is -3, so the slope of a line perpendicular to it is  $\frac{1}{3}$ .

$$y = mx + b$$
  
Slope-Intercept form  
$$0 = \frac{1}{3}(4) + b$$
$$m = \frac{1}{3} \text{ and } (x, y) = (4, 0)$$
$$0 = \frac{4}{3} + b$$
Simplify.

$$-\frac{4}{3} = b$$
 Subtract  $\frac{4}{3}$  from each side.

So, the equation is  $y = \frac{1}{3}x + (-\frac{4}{3})$  or  $y = \frac{1}{3}x - 1\frac{1}{3}$ .

## **Guided**Practice

**5.** Write an equation in slope-intercept form for a line parallel to  $y = -\frac{3}{4}x + 3$ containing (-3, 6).

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## **Reading**Math

Linear The word *linear* indicates a line. The graph of a linear equation is a line.

**2** Write Equations to Solve Problems Many real-world situations can be modeled using a linear equation.

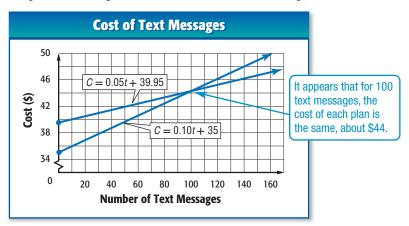
## Seal-World Example 6 Write Linear Equations

**FINANCIAL LITERACY** Benito's current wireless phone plan, Plan X, costs \$39.95 per month for unlimited calls and \$0.05 per text message. He is considering switching to Plan Y, which costs \$35 per month for unlimited calls plus \$0.10 for each text message. Which plan offers him the better rate?

- **Understand** Plan X costs \$39.95 per month plus \$0.05 per text message. Plan Y costs \$35 per month plus \$0.10 per text message. You want to compare the two plans to determine when the cost of one plan is less than the other.
  - **Plan** Write an equation to model the total monthly cost *C* of each plan for *t* text messages sent or received. Then graph the equations in order to compare the two plans.
  - **Solve** The rates of increase, or slopes *m*, in the total costs are 0.05 for Plan X and 0.10 for Plan Y. When the number of text messages is 0, the total charge is just the monthly fee. So, the *y*-intercept *b* is 39.95 for Plan X and 35 for Plan Y.

Plan X		Plan Y
C = mt + b	Slope-intercept form	C = mt + b
C = 0.05t + 39.95	Substitute for <i>m</i> and <i>b</i> .	C = 0.10t + 35

Graph the two equations on the same coordinate plane.



From the graph, it appears that if Benito sends or receives less than about 100 text messages, Plan Y offers the lower rate. For more than 100 messages, Plan X is lower.

**Check** Check your estimate. For 100 text messages, Plan X costs 0.05(100) + 39.95 or 44.95, and Plan Y costs 0.1(100) + 35 or 45. Adjusting our estimate, we find that when the number of messages is 99, both plans cost \$44.90. ✓

## GuidedPractice

**6.** Suppose the rate for Plan Y was \$44 a month and \$0.02 per text message. Which plan would offer Benito the better rate? Justify your answer.

## **Problem-Solving**Tip

Draw a Graph In Example 6, although Plan Y has a lower monthly fee, the charge per text message is higher. This makes the plans more difficult to compare. A graph can often give you a better comparison of two linear situations.



PT

#### **Check Your Understanding**

**Example 1** Write an equation in slope-intercept form of the line having the given slope and *y*-intercept. Then graph the line.

## **ble 2** Write an equation in point-slope form of the line having the given slope that contains

- **Example 2** Write an equation in point-slope form of the line having the given slope that contains the given point. Then graph the line.
  - **4.** m = 5, (3, -2) **5.**  $m = \frac{1}{4}, (-2, -3)$  **6.** m = -4.25, (-4, 6)

#### **Examples 3–4** Write an equation of the line through each pair of points in slope-intercept form.



**Example 5** 

**1.** *m*: 4, *y*-intercept: -3



**10.** Write an equation in slope-intercept form for a line perpendicular to

**11.** Write an equation in slope-intercept form for a line parallel to y = 4x - 5



**2.**  $m: \frac{1}{2}$ , *y*-intercept: -1 **3.**  $m: -\frac{2}{3}$ , *y*-intercept: 5

- monthly cost
- Example 6 12. Some MODELING Kameko currently subscribes to Ace Music, an online music service, but she is considering switching to another online service, Orange Tunes. The plan for each online music service is shown.
  - **a.** Write an equation to represent the total monthly cost for each plan.
  - **b.** Graph the equations.

y = -2x + 6 containing (3, 2).

containing (-1, 5).

**c.** If Kameko downloads 15 songs per month, should she keep her current plan, or change to the other plan? Explain.

#### **Practice and Problem Solving**

**Example 1** Write an equation in slope-intercept form of the line having the given slope and *y*-intercept or points. Then graph the line.

<b>13.</b> <i>m</i> : -5, <i>y</i> -intercept: -2	<b>14.</b> <i>m</i> : −7, <i>b</i> : −4	<b>15.</b> <i>m</i> : 9, <i>b</i> : 2
<b>16.</b> <i>m</i> : 12, <i>y</i> -intercept: $\frac{4}{5}$	<b>17.</b> $m: -\frac{3}{4}$ , (0, 4)	<b>18.</b> $m: \frac{5}{11}, (0, -3)$

**Example 2** Write an equation in point-slope form of the line having the given slope that contains the given point. Then graph the line.

<b>19</b> $m = 2, (3, 11)$	<b>20.</b> $m = 4, (-4, 8)$	<b>21.</b> $m = -7, (1, 9)$
<b>22.</b> $m = \frac{5}{7}, (-2, -5)$	<b>23.</b> $m = -\frac{4}{5}$ , $(-3, -6)$	<b>24.</b> <i>m</i> = -2.4, (14, -12)

#### **Examples 3–4** Write an equation of the line through each pair of points in slope-intercept form.

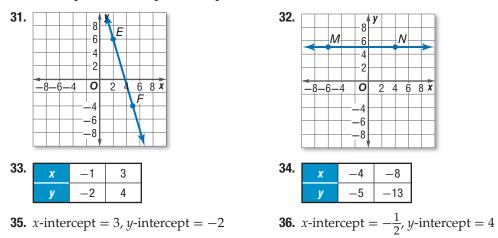
<b>25.</b> (-1, -4) and (3, -4)	<b>26.</b> (2, −1) and (2, 6)
<b>27.</b> (-3, -2) and (-3, 4)	<b>28.</b> (0, 5) and (3, 3)
<b>29.</b> (-12, -6) and (8, 9)	<b>30.</b> (2, 4) and (-4, -11)

= Step-by-Step Solutions begin on page R14.

 $\checkmark$ 



Write an equation in slope-intercept form for each line shown or described.



Example 5

Write an equation in slope-intercept form for each line described.

- **37.** passes through (-7, -4), perpendicular to  $y = \frac{1}{2}x + 9$
- **38.** passes through (-1, -10), parallel to y = 7
- **39.** passes through (6, 2), parallel to  $y = -\frac{2}{3}x + 1$
- **40.** passes through (-2, 2), perpendicular to y = -5x 8
- **Example 6** (41) PLANNING Karen is planning a graduation party for the senior class. She plans to rent a meeting room at the convention center that costs \$400. There is an additional fee of \$5.50 for each person who attends the party.
  - **a.** Write an equation to represent the cost *y* of the party if *x* people attend.
  - **b.** Graph the equation.
  - **c.** There are 285 people in Karen's class. If  $\frac{2}{3}$  of these people attend, how much will the party cost?
  - d. If the senior class has raised \$2000 for the party, how many people can attend?
  - 42. CSS MODELING Victor is saving his money to buy a new satellite radio for his car. He wants to save enough money for the radio and one year of satellite radio service before he makes the purchase. He started saving for the radio with \$50 that he got for his birthday. Since then, he has been adding \$15 every week after he cashes his paycheck.
    - **a.** Write an equation to represent Victor's savings *y* after *x* weeks.
    - **b.** Graph the equation.
    - **c.** How long will it take Victor to save \$150?
    - **d.** A satellite radio costs \$180. Satellite radio service costs \$10 per month. If Victor started saving two weeks ago, how much longer will it take him to save enough money? Explain.

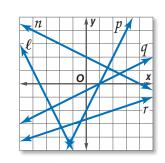
Name the line(s) on the graph shown that match each description.

- **43.** parallel to y = 2x 3
- **44.** perpendicular to  $y = \frac{1}{2}x + 7$

**45.** intersecting, but not perpendicular to  $y = \frac{1}{2}x - 5$ 

Determine whether the lines are *parallel*, *perpendicular*, or *neither*.

**46.** 
$$y = 2x + 4$$
,  $y = 2x - 10$   
**48.**  $y - 4 = 3(x + 5)$ ,  $y + 3 = -\frac{1}{3}(x + 1)$ 



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**50.** Write an equation in slope-intercept form for a line containing (4, 2) that is parallel to the line y - 2 = 3(x + 7).

51 Write an equation for a line containing (-8, 12) that is perpendicular to the line containing the points (3, 2) and (-7, 2).

- **52.** Write an equation in slope-intercept form for a line containing (5, 3) that is parallel to the line  $y + 11 = \frac{1}{2}(4x + 6)$ .
- **53. POTTERY** A community center offers pottery classes. A \$40 enrollment fee covers supplies and materials, including one bag of clay. Extra bags of clay cost \$15 each. Write an equation to represent the cost of the class and *x* bags of clay.
- **54.** Solution of a system of two linear equations is an ordered pair that is a solution of both equations. Consider lines *q*, *r*, *s*, and *t* with the equations given.

line q: y = 3x + 2 line r: y = 0.5x - 3 line s: 2y = x - 6 line t: y = 3x - 3

- **a.** Tabular Make a table of values for each equation for x = -3, -2, -1, 0, 1, 2, and 3. Which pairs of lines appear to represent a system of equations with one solution? no solution? infinitely many solutions? Use your tables to explain your reasoning.
- **b. Graphical** Graph the equations on the same coordinate plane. Describe the geometric relationship between each pair of lines, including points of intersection.
- **c. Analytical** How could you have determined your answers to part **a** using only the equations of the lines?
- **d. Verbal** Explain how to determine whether a given system of two linear equations has one solution, no solution, or infinitely many solutions using a table, a graph, or the equations of the lines.

#### H.O.T. Problems Use Higher-Order Thinking Skills

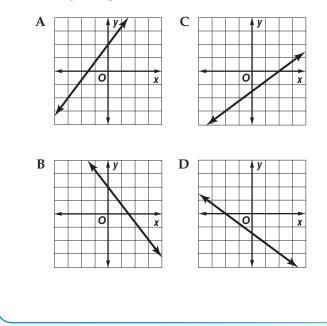
- **55. CHALLENGE** Find the value of *n* so that the line perpendicular to the line with the equation -2y + 4 = 6x + 8 passes through the points at (n, -4) and (2, -8).
- **56. REASONING** Determine whether the points at (-2, 2), (2, 5), and (6, 8) are collinear. Justify your answer.
- **57. OPEN ENDED** Write equations for two different pairs of perpendicular lines that intersect at the point at (-3, -7).
- **58. CRITIQUE** Mark and Josefina wrote an equation of a line with slope -5 that passes through the point (-2, 4). Is either of them correct? Explain your reasoning.

MarkJosefinay - 4 = -5(x - (-2))y - 4 = -5(x - (-2))y - 4 = -5(x + 2)y - 4 = -5(x + 2)y - 4 = -5x - 10y = -5x - 6

**59.** WRITING IN MATH When is it easier to use the point-slope form to write an equation of a line and when is it easier to use the slope-intercept form?

## **Standardized Test Practice**

60. Which graph best represents a line passing through the point (-2, -3)?



**61.** Which equation describes the line that passes through the point at (-2, 1) and is perpendicular to the line  $y = \frac{1}{3}x + 5$ ?



- 62. GRIDDED RESPONSE At Jefferson College, 80% of students have cell phones. Of the students who have cell phones, 70% have computers. What percent of the students at Jefferson College have both a cell phone and a computer?
- **63. SAT/ACT** Which expression is equivalent to  $4(x-6) - \frac{1}{2}(x^2+8)?$

**A**  $4x^2 + 4x - 28$  **D** 3x - 20**B**  $-\frac{1}{2}x^2 + 4x - 20$  **E**  $-\frac{1}{2}x^2 + 4x - 28$ C  $-\frac{1}{2}x^2 + 6x - 24$ 

#### **Spiral Review**

Determine the slope of the line that contains the given points. (Lesson 3-3) **64.** *J*(4, 3), *K*(5, −2)

**65.** X(0, 2), Y(-3, -4)

**66.** *A*(2, 5), *B*(5, 1)

Find x and y in each figure. (Lesson 3-2)



69. DRIVING Lacy's home is located at the midpoint between Newman's Gas Station and Gas-O-Rama. Newman's Gas Station is a quarter mile away from Lacy's home. How far away is Gas-O-Rama from Lacy's home? How far apart are the two gas stations? (Lesson 1-3)

## **Skills Review**

Determine the relationship between each pair of angles.

- **70.** ∠1 and ∠12
- **71.** ∠7 and ∠10
- **72.** ∠4 and ∠8
- **73.** ∠2 and ∠11

