LESSON 3-5

Proving Lines Parallel

Then

- You used slopes to identify parallel and perpendicular lines.

Now

1. Recognize angle pairs that occur with parallel lines.
2. Prove that two lines are parallel.

Why?

- When you see a roller coaster track, the two sides of the track are always the same distance apart, even though the track curves and turns. The tracks are carefully constructed to be parallel at all points so that the car is secure on the track.

Common Core State Standards

Content Standards

G.CO.9 Prove theorems about lines and angles.

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

Mathematical Practices

1 Make sense of problems and persevere in solving them.

3 Construct viable arguments and critique the reasoning of others.

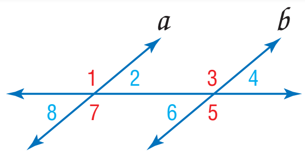
1 Identify Parallel Lines

The two sides of the track of a roller coaster are parallel, and all of the supports along the track are also parallel. Each of the angles formed between the track and the supports are corresponding angles. We have learned that corresponding angles are congruent when lines are parallel. The converse of this relationship is also true.

Postulate 3.4 Converse of Corresponding Angles Postulate

If two lines are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.

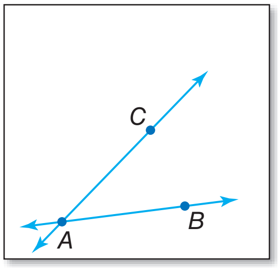
Examples If $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$, $\angle 5 \cong \angle 7$, $\angle 6 \cong \angle 8$, then $a \parallel b$.



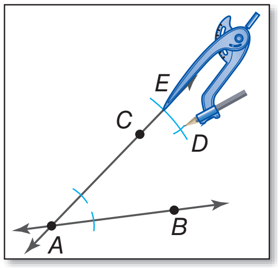
The Converse of the Corresponding Angles Postulate can be used to construct parallel lines.

Construction Parallel Line Through a Point Not on the Line

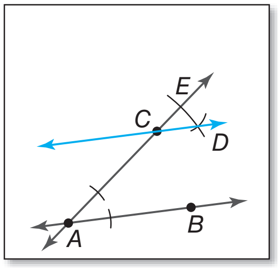
Step 1 Use a straightedge to draw $\overleftrightarrow{AB}$. Draw a point C that is not on $\overleftrightarrow{AB}$. Draw $\overleftrightarrow{CA}$.



Step 2 Copy $\angle CAB$ so that C is the vertex of the new angle. Label the intersection points D and E.



Step 3 Draw CD. Because $\angle ECD \cong \angle CAB$ by construction and they are corresponding angles, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

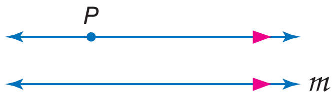


The construction establishes that there is *at least* one line through C that is parallel to $\overleftrightarrow{AB}$. The following postulate guarantees that this line is the *only* one.

> **StudyTip**
> **Euclid's Postulates** The father of modern geometry, Euclid (c. 300 B.C.) realized that only a few postulates were needed to prove the theorems in his day. Postulate 3.5 is one of Euclid's five original postulates. Postulate 2.1 and Theorem 2.10 also reflect two of Euclid's postulates.

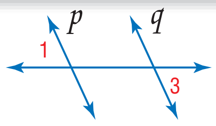
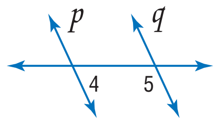
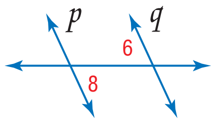
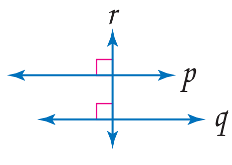
Postulate 3.5 Parallel Postulate

If given a line and a point not on the line, then there exists exactly one line through the point that is parallel to the given line.



Parallel lines that are cut by a transversal create several pairs of congruent angles. These special angle pairs can also be used to prove that a pair of lines are parallel.

Theorems Proving Lines Parallel

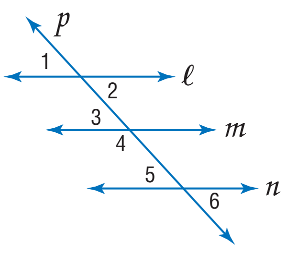
3.5 Alternate Exterior Angles Converse If two lines in a plane are cut by a transversal so that a pair of alternate exterior angles is congruent, then the two lines are parallel.	 If $\angle 1 \cong \angle 3$, then $p \parallel q$.
3.6 Consecutive Interior Angles Converse If two lines in a plane are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the lines are parallel.	 If $m\angle 4 + m\angle 5 = 180$, then $p \parallel q$.
3.7 Alternate Interior Angles Converse If two lines in a plane are cut by a transversal so that a pair of alternate interior angles is congruent, then the lines are parallel.	 If $\angle 6 \cong \angle 8$, then $p \parallel q$.
3.8 Perpendicular Transversal Converse In a plane, if two lines are perpendicular to the same line, then they are parallel.	 If $p \perp r$ and $q \perp r$, then $p \parallel q$.

You will prove Theorems 3.5, 3.6, 3.7, and 3.8 in Exercises 6, 23, 31, and 30, respectively.

PT

Example 1 Identify Parallel Lines

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



a. $\angle 1 \cong \angle 6$

$\angle 1$ and $\angle 6$ are alternate exterior angles of lines ℓ and n.

Since $\angle 1 \cong \angle 6$, $\ell \parallel n$ by the Converse of the Alternate Exterior Angles Theorem.

b. $\angle 2 \cong \angle 3$

$\angle 2$ and $\angle 3$ are alternate interior angles of lines ℓ and m.

Since $\angle 2 \cong \angle 3$, $\ell \parallel m$ by the Converse of the Alternate Interior Angles Theorem.

Guided Practice

1A. $\angle 2 \cong \angle 8$

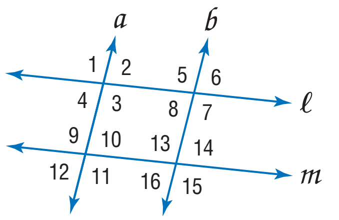
1B. $\angle 3 \cong \angle 11$

1C. $\angle 12 \cong \angle 14$

1D. $\angle 1 \cong \angle 15$

1E. $m\angle 8 + m\angle 13 = 180$

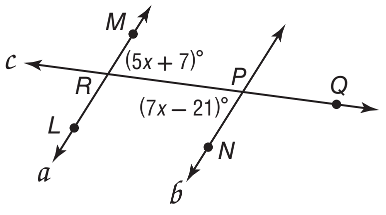
1F. $\angle 8 \cong \angle 6$



Angle relationships can be used to solve problems involving unknown values.

Standardized Test Example 2 Use Angle Relationships

OPEN ENDED **Find $m\angle MRQ$ so that $a \parallel b$. Show your work.**



Read the Test Item

From the figure, you know that $m\angle MRQ = 5x + 7$ and $m\angle RPN = 7x - 21$. You are asked to find the measure of $\angle MRQ$.

Solve the Test Item

$\angle MRQ$ and $\angle RPN$ are alternate interior angles. For lines a and b to be parallel, alternate interior angles must be congruent, so $\angle MRQ \cong \angle RPN$. By the definition of congruence, $m\angle MRQ = m\angle RPN$. Substitute the given angle measures into this equation and solve for x.

$m\angle MRQ = m\angle RPN$	Alternate interior angles
$5x + 7 = 7x - 21$	Substitution
$7 = 2x - 21$	Subtract $5x$ from each side.
$28 = 2x$	Add 21 to each side.
$14 = x$	Divide each side by 2.

Now, use the value of x to find $\angle MRQ$.

$m\angle MRQ = 5x + 7$	Substitution
$= 5(14) + 7$	$x = 14$
$= 77$	Simplify.

CHECK Check your answer by using the value of x to find $m\angle RPN$.

$m\angle RP = 7x - 21$

$= 7(14) - 21$ or 77 ✓

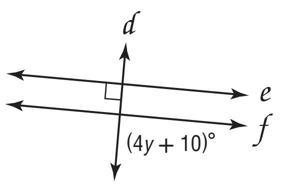
Since $m\angle MRQ = m\angle RPN$, $\angle MRQ \cong \angle RPN$ and $a \parallel b$. ✓

Study Tip

Finding What Is Asked For Be sure to reread test questions carefully to be sure you are answering the question that was asked. In Example 2, a common error would be to stop after you have found the value of x and say that the solution of the problem is 14.

Guided Practice

2. Find y so that $e \parallel f$. Show your work.



StudyTip

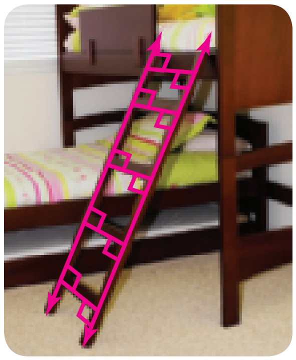
Proving Lines Parallel When two parallel lines are cut by a transversal, the angle pairs formed are either congruent or supplementary. When a pair of lines forms angles that do not meet this criterion, the lines cannot possibly be parallel.

2 Prove Lines Parallel

The angle pair relationships formed by a transversal can be used to prove that two lines are parallel.

Real-World Example 3 Prove Lines Parallel

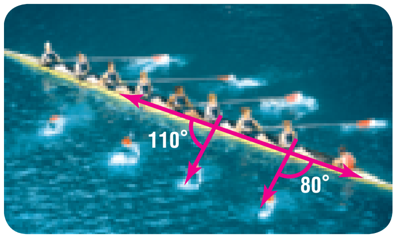
HOME FURNISHINGS In the ladder shown, each rung is perpendicular to the two rails. Is it possible to prove that the two rails are parallel and that all of the rungs are parallel? If so, explain how. If not, explain why not.



Since both rails are perpendicular to each rung, the rails are parallel by the Perpendicular Transversal Converse. Since any pair of rungs is perpendicular to the rails, they are also parallel.

GuidedPractice

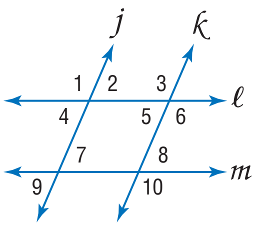
3. **ROWING** In order to move in a straight line with maximum efficiency, rower's oars should be parallel. Refer to the photo at the right. Is it possible to prove that any of the oars are parallel? If so, explain how. If not, explain why not.



Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.**



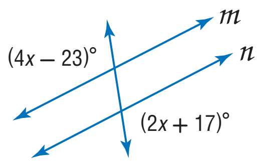
1. $\angle 1 \cong \angle 3$

2. $\angle 2 \cong \angle 5$

3. $\angle 3 \cong \angle 10$

4. $m\angle 6 + m\angle 8 = 180$

Example 2 **5.** **SHORT RESPONSE** Find x so that $m \parallel n$. Show your work.

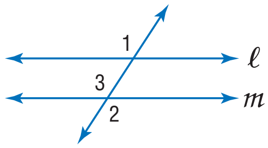


Example 3 **6.** **PROOF** Copy and complete the proof of Theorem 3.5.

Given: $\angle 1 \cong \angle 2$

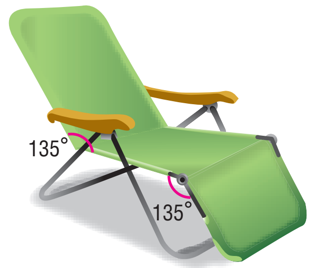
Prove: $\ell \parallel m$

Proof:



Statements	Reasons
a. $\angle 1 \cong \angle 2$	**a.** Given
b. $\angle 2 \cong \angle 3$	**b.** ___?___
c. $\angle 1 \cong \angle 3$	**c.** Transitive Property
d. ___?___	**d.** ___?___

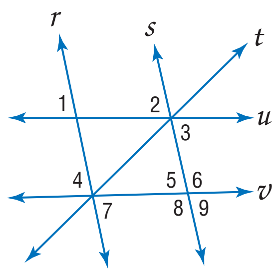
7. **RECREATION** Is it possible to prove that the backrest and footrest of the lounging beach chair are parallel? If so, explain how. If not, explain why not.



Practice and Problem Solving

Extra Practice is on page R3.

Example 1 **Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.**



8. $\angle 1 \cong \angle 2$

9. $\angle 2 \cong \angle 9$

10. $\angle 5 \cong \angle 7$

11. $m\angle 7 + m\angle 8 = 180$

12. $m\angle 3 + m\angle 6 = 180$

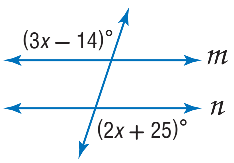
13. $\angle 3 \cong \angle 5$

14. $\angle 3 \cong \angle 7$

15. $\angle 4 \cong \angle 5$

Example 2 **Find x so that $m \parallel n$. Identify the postulate or theorem you used.**

16.



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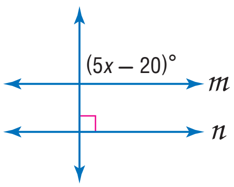

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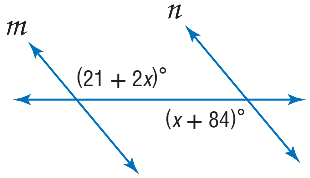

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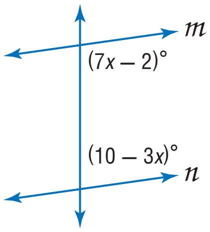

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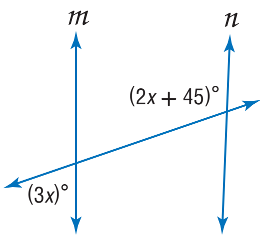

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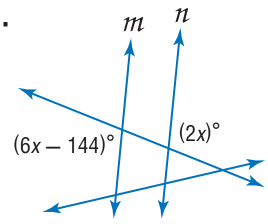
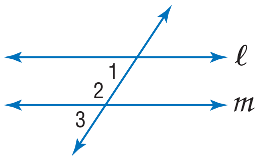


22. CCSS **SENSE-MAKING** Wooden picture frames are often constructed using a miter box or miter saw. These tools allow you to cut at an angle of a given size. If each of the four pieces of framing material is cut at a 45° angle, will the sides of the frame be parallel? Explain your reasoning.

Example 3 **23.** **PROOF** Copy and complete the proof of Theorem 3.6.

Given: $\angle 1$ and $\angle 2$ are supplementary.

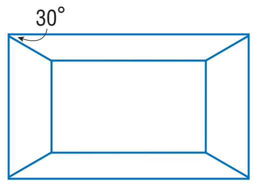
Prove: $\ell \parallel m$



Proof:

Statements	Reasons
a. ___?___	**a.** Given
b. $\angle 2$ and $\angle 3$ form a linear pair.	**b.** ___?___
c. ___?___	**c.** ___?___
d. $\angle 1 \cong \angle 3$	**d.** ___?___
e. $\ell \parallel m$	**e.** ___?___

24. **CRAFTS** Jacqui is making a stained glass piece. She cuts the top and bottom pieces at a 30° angle. If the corners are right angles, explain how Jacqui knows that each pair of opposite sides are parallel.

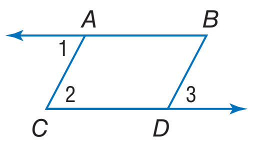


PROOF **Write a two-column proof for each of the following.**

25. **Given:** $\angle 1 \cong \angle 3$

$\overline{AC} \parallel \overline{BD}$

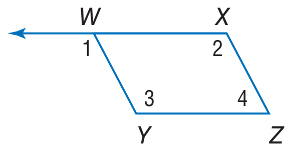
Prove: $\overline{AB} \parallel \overline{CD}$



26. **Given:** $\overline{WX} \parallel \overline{YZ}$

$\angle 2 \cong \angle 3$

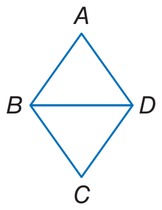
Prove: $\overline{WY} \parallel \overline{XZ}$



27. **Given:** $\angle ABC \cong \angle ADC$

$m\angle A + m\angle ABC = 180$

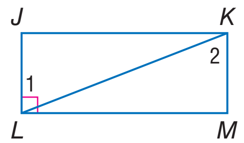
Prove: $\overline{AB} \parallel \overline{CD}$



28. **Given:** $\angle 1 \cong \angle 2$

$\overline{LJ} \perp \overline{ML}$

Prove: $\overline{KM} \perp \overline{ML}$

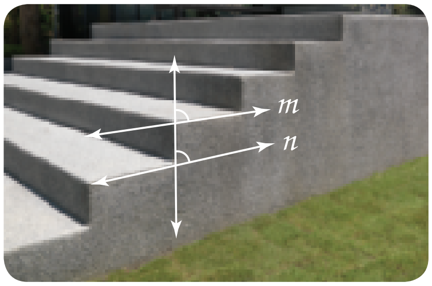


29 **MAILBOXES** Mail slots are used to make the organization and distribution of mail easier. In the mail slots shown, each slot is perpendicular to each of the sides. Explain why you can conclude that the slots are parallel.

30. **PROOF** Write a paragraph proof of Theorem 3.8.

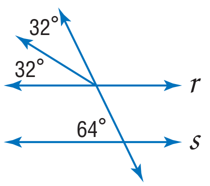
31. **PROOF** Write a two-column proof of Theorem 3.7.

32. CCSS **REASONING** Based upon the information given in the photo of the staircase at the right, what is the relationship between each step? Explain your answer.



Determine whether lines r and s are parallel. Justify your answer.

33.



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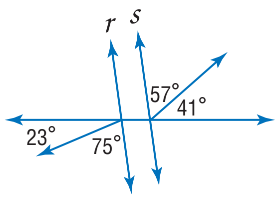

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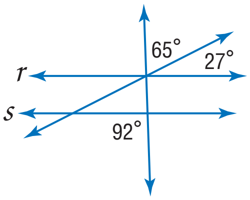
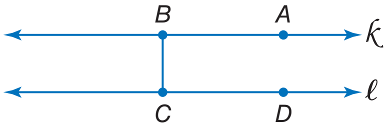


36. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the shortest distance between two parallel lines.

   a. **Geometric** Draw three sets of parallel lines k and ℓ, s and t, and x and y. For each set, draw the shortest segment $\overline{BC}$ and label points A and D as shown below.



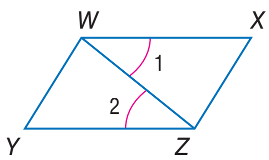
   b. **Tabular** Copy the table below, measure $\angle ABC$ and $\angle BCD$, and complete the table.

Set of Parallel Lines	$m\angle ABC$	$m\angle BCD$
k and ℓ		
s and t		
x and y		

   c. **Verbal** Make a conjecture about the angle the shortest segment forms with both parallel lines.

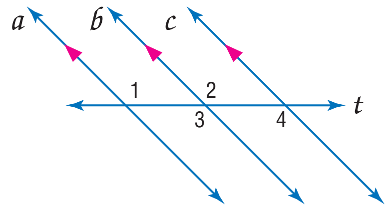
H.O.T. Problems Use Higher-Order Thinking Skills

37. **ERROR ANALYSIS** Sumi and Daniela are determining which lines are parallel in the figure at the right. Sumi says that since $\angle 1 \cong \angle 2$, $\overline{WY} \parallel \overline{XZ}$. Daniela disagrees and says that since $\angle 1 \cong \angle 2$, $\overline{WX} \parallel \overline{YZ}$. Is either of them correct? Explain.



38. **CCSS REASONING** Is Theorem 3.8 still true if the two lines are not coplanar? Draw a figure to justify your answer.

39. **CHALLENGE** Use the figure at the right to prove that two lines parallel to a third line are parallel to each other.



40. **OPEN ENDED** Draw a triangle ABC.

   a. Construct the line parallel to $\overline{BC}$ through point A.

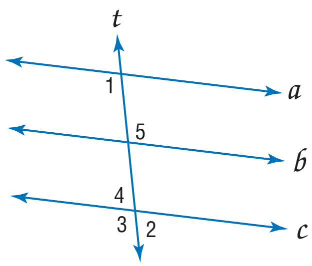
   b. Use measurement to justify that the line you constructed is parallel to $\overline{BC}$.

   c. Use mathematics to justify this construction.

41. **CHALLENGE** Refer to the figure at the right.

   a. If $m\angle 1 + m\angle 2 = 180$, prove that $a \parallel c$.

   b. Given that $a \parallel c$, if $m\angle 1 + m\angle 3 = 180$, prove that $t \perp c$.

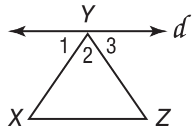


42. **WRITING IN MATH** Summarize the five methods used in this lesson to prove that two lines are parallel.

43. **WRITING IN MATH** Can a pair of angles be supplementary and congruent? Explain your reasoning.

Standardized Test Practice

44. Which of the following facts would be sufficient to prove that line *d* is parallel to $\overline{XZ}$?



A $\angle 1 \cong \angle 3$ **C** $\angle 1 \cong \angle Z$

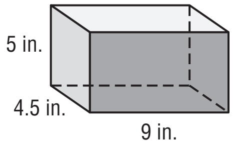
B $\angle 3 \cong \angle Z$ **D** $\angle 2 \cong \angle X$

45. ALGEBRA The expression $\sqrt{52} + \sqrt{117}$ is equivalent to

F 13 **H** $6\sqrt{13}$

G $5\sqrt{13}$ **J** $13\sqrt{13}$

46. What is the approximate surface area of the figure?



A 101.3 in^2 **C** 202.5 in^2

B 108 in^2 **D** 216 in^2

47. SAT/ACT If $x^2 = 25$ and $y^2 = 9$, what is the greatest possible value of $(x - y)^2$?

F 4 **J** 64

G 16 **K** 70

H 58

Spiral Review

Write an equation in slope-intercept form of the line having the given slope and *y*-intercept. (Lesson 3-4)

48. m: 2.5, (0, 0.5) **49.** m: $\frac{4}{5}$, (0, −9) **50.** m: $-\frac{7}{8}$, $\left(0, -\frac{5}{6}\right)$

51. ROAD TRIP Anne is driving 400 miles to visit Niagara Falls. She manages to travel the first 100 miles of her trip in two hours. If she continues at this rate, how long will it take her to drive the remaining distance? (Lesson 3-3)

Find a counterexample to show that each conjecture is false. (Lesson 2-1)

52. Given: $\angle 1$ and $\angle 2$ are complementary angles.

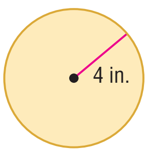
Conjecture: $\angle 1$ and $\angle 2$ form a right angle.

53. Given: points *W*, *X*, *Y*, and *Z*

Conjecture: *W*, *X*, *Y*, and *Z* are noncollinear.

Find the perimeter or circumference and area of each figure. Round to the nearest tenth. (Lesson 1-6)

54.



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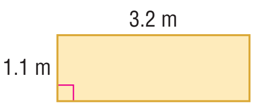
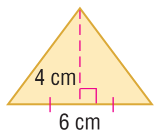


56.



Skills Review

57. Find *x* and *y* so that $\overline{BE}$ and $\overline{AD}$ are perpendicular.