LESSON 4-1

Classifying Triangles

Then
- You measured and classified angles.

Now
1. Identify and classify triangles by angle measures.
2. Identify and classify triangles by side measures.

Why?
- Radio transmission towers are designed to support antennas for broadcasting radio or television signals. The structure of the tower shown reveals a pattern of triangular braces.

NewVocabulary
- acute triangle
- equiangular triangle
- obtuse triangle
- right triangle
- equilateral triangle
- isosceles triangle
- scalene triangle

Common Core State Standards

Content Standards

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

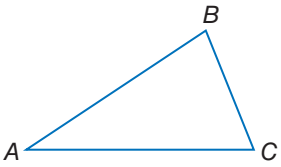
Mathematical Practices

2 Reason abstractly and quantitatively.

6 Attend to precision.

1 Classify Triangles by Angles

Recall that a triangle is a three-sided polygon. Triangle ABC, written $\triangle ABC$, has parts that are named using A, B, and C.



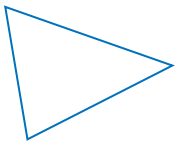
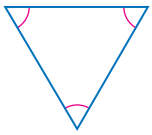
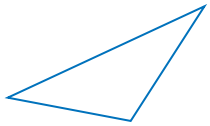
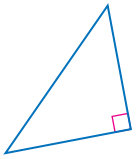
The sides of $\triangle ABC$ are $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.

The vertices are points A, B, and C.

The angles are $\angle BAC$ or $\angle A$, $\angle ABC$ or $\angle B$, and $\angle BCA$ or $\angle C$.

Triangles can be classified in two ways—by their angles or by their sides. All triangles have at least two acute angles, but the third angle is used to classify the triangle.

KeyConcept Classifications of Triangles by Angles

acute triangle	equiangular triangle	obtuse triangle	right triangle
			
3 acute angles	3 congruent acute angles	1 obtuse angle	1 right angle

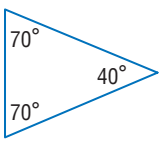
An equiangular triangle is a special kind of acute triangle.

When classifying triangles, be as specific as possible. While a triangle with three congruent acute angles is an acute triangle, it is more specific to classify it as an equiangular triangle.

Example 1 Classify Triangles by Angles

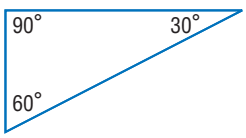
Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

a.



The triangle has three acute angles that are not all equal. It is an acute triangle.

b.



One angle of the triangle measures 90, so it is a right angle. Since the triangle has a right angle, it is a right triangle.

ReviewVocabulary

acute angle an angle with a degree measure less than 90

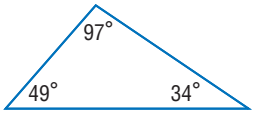
right angle an angle with a degree measure of 90

obtuse angle an angle with a degree measure greater than 90

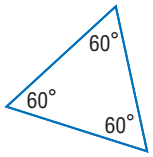
GuidedPractice

Classify each triangle as *acute, equiangular, obtuse,* or *right*.

1A.



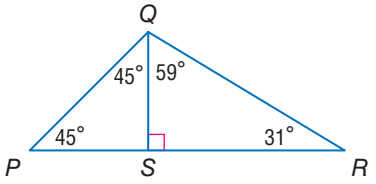
1B.



Example 2 Classify Triangles by Angles Within Figures

Classify $\triangle PQR$ as *acute, equiangular, obtuse,* or *right*. Explain your reasoning.

Point S is in the interior of $\angle PQR$, so by the Angle Addition Postulate, $m\angle PQR = m\angle PQS + m\angle SQR$. By substitution, $m\angle PQR = 45 + 59$ or 104.



Since $\triangle PQR$ has one obtuse angle, it is an obtuse triangle.

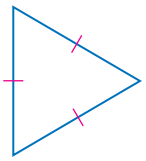
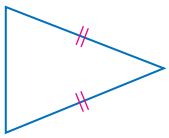
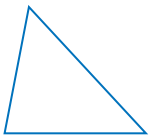
GuidedPractice

2. Use the diagram to classify $\triangle PQS$ as *acute, equiangular, obtuse* or *right*. Explain your reasoning.

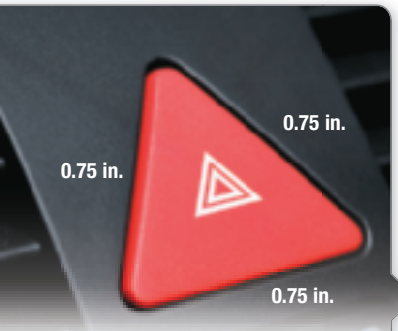
2 Classify Triangles by Sides

Triangles can also be classified according to the number of congruent sides they have. To indicate that sides of a triangle are congruent, an equal number of hash marks is drawn on the corresponding sides.

KeyConcept Classifications of Triangles by Sides

equilateral triangle	isosceles triangle	scalene triangle
		
3 congruent sides	at least 2 congruent sides	no congruent sides

An equilateral triangle is a special kind of isosceles triangle.



Real-WorldLink

In many cars, hazard lights are activated by pushing a small button located near the steering column. The switch is usually an icon shaped like an equilateral triangle.

Source: General Motors

Real-World Example 3 Classify Triangles by Sides

MUSIC Classify the sound box of the Russian lute below as *equilateral, isosceles,* or *scalene*.

Two sides have the same measure, 16 inches, so the triangle has two congruent sides. The triangle is isosceles.

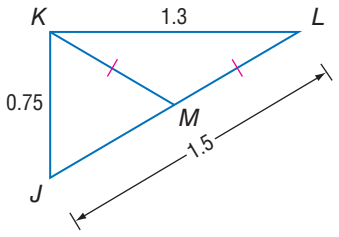
GuidedPractice

3. DRIVING SAFETY Classify the button in the picture at the left by its sides.

(l)fStop/SuperStock, (r)Stockbyte/Getty Images

Example 4 Classify Triangles by Sides Within Figures

If point M is the midpoint of $\overline{JL}$, classify $\triangle JKM$ as *equilateral*, *isosceles*, or *scalene*. Explain your reasoning.



By the definition of midpoint, $JM = ML$.

$JM + ML = JL$	Segment Addition Postulate
$ML + ML = 1.5$	Substitution
$2ML = 1.5$	Simplify.
$ML = 0.75$	Divide each side by 2.

$JM = ML$ or 0.75. Since $\overline{KM} \cong \overline{ML}$, $KM = ML$ or 0.75.

Since $KJ = JM = KM = 0.75$, the triangle has three sides with the same measure. Therefore, the triangle has three congruent sides, so it is equilateral.

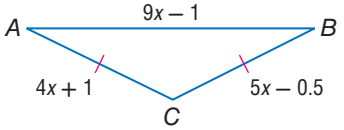
GuidedPractice

4. Classify $\triangle KML$ as *equilateral*, *isosceles*, or *scalene*. Explain your reasoning.

You can also use the properties of isosceles and equilateral triangles to find missing values.

Example 5 Finding Missing Values

ALGEBRA Find the measures of the sides of isosceles triangle ABC.



Step 1 Find x.

$AC = CB$	Given
$4x + 1 = 5x - 0.5$	Substitution
$1 = x - 0.5$	Subtract $4x$ from each side.
$1.5 = x$	Add 0.5 to each side.

Step 2 Substitute to find the length of each side.

$AC = 4x + 1$	Given
$= 4(1.5) + 1$ or 7	$x = 1.5$
$CB = AC$	Given
$= 7$	$AC = 7$
$AB = 9x - 1$	Given
$= 9(1.5) - 1$	$x = 1.5$
$= 12.5$	Simplify.

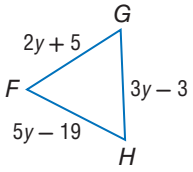
StudyTip

CCSS Perseverance In Example 5, to check your answer, test to see if $CB = AC$ when 1.5 is substituted for x in the expression for CB, $5x - 0.5$.

$CB = 5x - 0.5$
$= 5(1.5) - 0.5$ or 7 ✔

GuidedPractice

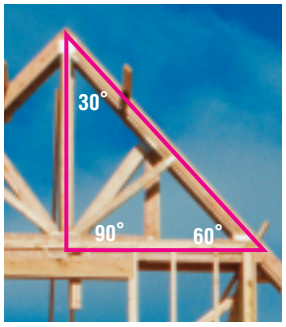
5. Find the measures of the sides of equilateral triangle FGH.

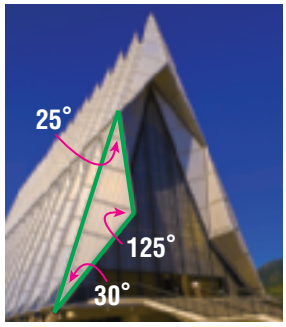


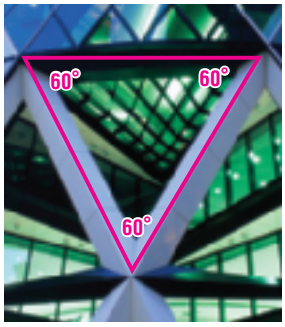
Check Your Understanding

= Step-by-Step Solutions begin on page R14.

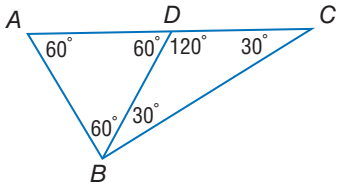
Example 1 **ARCHITECTURE** Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

1. 

2. 

3. 

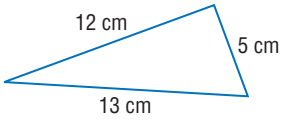
Example 2 Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*. Explain your reasoning.



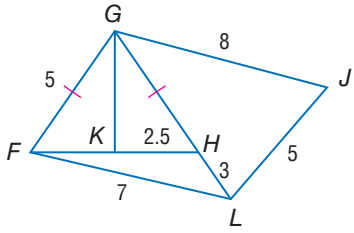
4. $\triangle ABD$
5. $\triangle BDC$
6. $\triangle ABC$

Example 3 CCSS **PRECISION** Classify each triangle as *equilateral*, *isosceles*, or *scalene*.

7. 

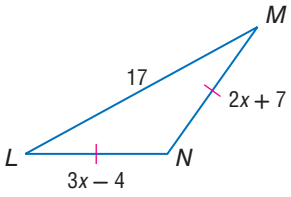
8. 

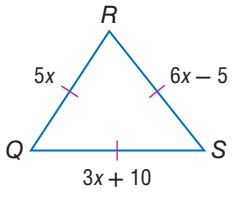
Example 4 If point K is the midpoint of $\overline{FH}$, classify each triangle in the figure at the right as *equilateral*, *isosceles*, or *scalene*.



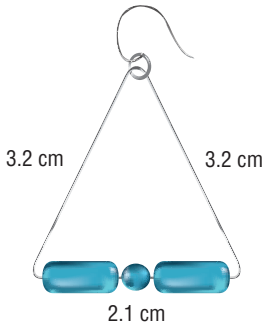
9. $\triangle FGH$
10. $\triangle GJL$
11. $\triangle FHL$

Example 5 **ALGEBRA** Find x and the measures of the unknown sides of each triangle.

12. 

13. 

14. **JEWELRY** Suppose you are bending stainless steel wire to make the earring shown. The triangular portion of the earring is an isosceles triangle. If 1.5 centimeters are needed to make the hook portion of the earring, how many earrings can be made from 45 centimeters of wire? Explain your reasoning.

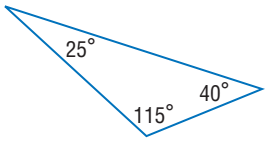


Practice and Problem Solving

Extra Practice is on page R4.

Example 1 Classify each triangle as *acute, equiangular, obtuse,* or *right*.

15.



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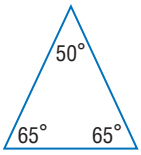

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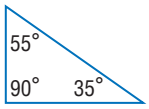

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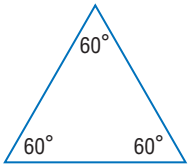

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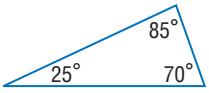
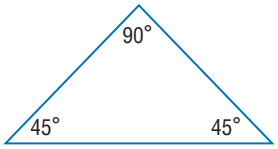


20.



Example 2 CCSS **PRECISION** Classify each triangle as *acute, equiangular, obtuse,* or *right*.

21. $\triangle UYZ$

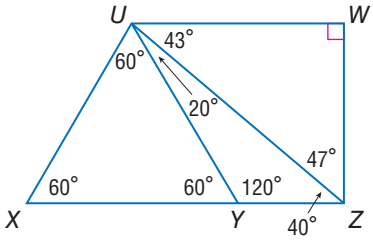
22. $\triangle BCD$

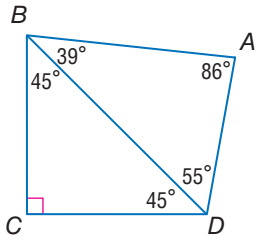
23. $\triangle ADB$

24. $\triangle UXZ$

25. $\triangle UWZ$

26. $\triangle UXY$



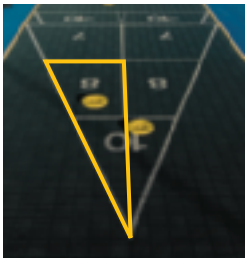


Example 3 Classify each triangle as *equilateral, isosceles,* or *scalene*.

27.

28.

29.



Example 4 If point C is the midpoint of $\overline{BD}$ and point E is the midpoint of $\overline{DF}$, classify each triangle as *equilateral, isosceles,* or *scalene*.

30. $\triangle ABC$

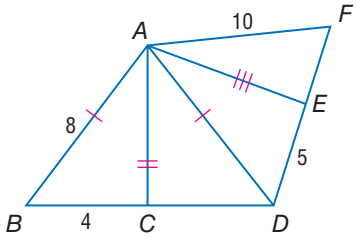
31. $\triangle AEF$

32. $\triangle ADF$

33. $\triangle ACD$

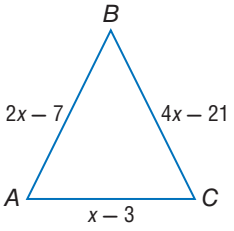
34. $\triangle AED$

35. $\triangle ABD$

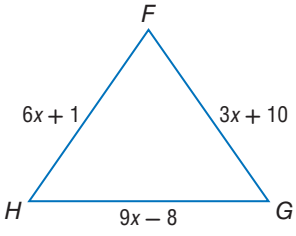


Example 5

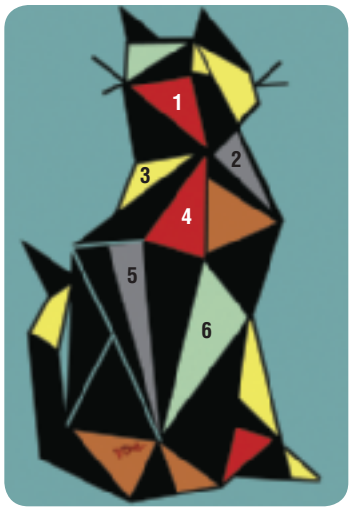
36. **ALGEBRA** Find x and the length of each side if $\triangle ABC$ is an isosceles triangle with $\overline{AB} \cong \overline{BC}$.



37 **ALGEBRA** Find x and the length of each side if $\triangle FGH$ is an equilateral triangle.



38. GRAPHIC ART Refer to the illustration shown. Classify each numbered triangle in *Kat* by its angles and by its sides. Use the corner of a sheet of notebook paper to classify angle measures and a ruler to measure sides.



Kat, 2002, by Diana Ong, computer graphic

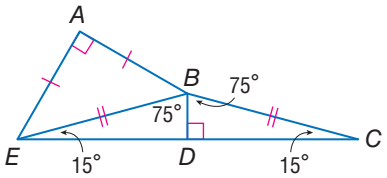
39. KALEIDOSCOPE Josh is building a kaleidoscope using PVC pipe, cardboard, bits of colored paper, and a 12-inch square mirror tile. The mirror tile is to be cut into strips and arranged to form an open prism with a base like that of an equilateral triangle. Make a sketch of the prism, giving its dimensions. Explain your reasoning.

CCSS PRECISION **Classify each triangle in the figure by its angles and sides.**

40. $\triangle ABE$

41. $\triangle EBC$

42. $\triangle BDC$



COORDINATE GEOMETRY **Find the measures of the sides of $\triangle XYZ$ and classify each triangle by its sides.**

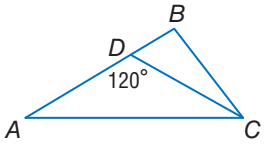
43. $X(-5, 9)$, $Y(2, 1)$, $Z(-8, 3)$

44. $X(7, 6)$, $Y(5, 1)$, $Z(9, 1)$

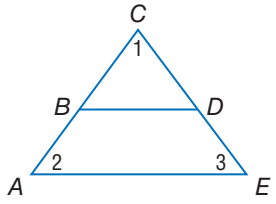
45. $X(3, -2)$, $Y(1, -4)$, $Z(3, -4)$

46. $X(-4, -2)$, $Y(-3, 7)$, $Z(4, -2)$

47. PROOF Write a paragraph proof to prove that $\triangle DBC$ is an acute triangle if $m\angle ADC = 120$ and $\triangle ABC$ is acute.



48. PROOF Write a two-column proof to prove that $\triangle BCD$ is equiangular if $\triangle ACE$ is equiangular and $\overline{BD} \parallel \overline{AE}$.



ALGEBRA **For each triangle, find x and the measure of each side.**

49. $\triangle FGH$ is an equilateral triangle with $FG = 3x - 10$, $GH = 2x + 5$, and $HF = x + 20$.

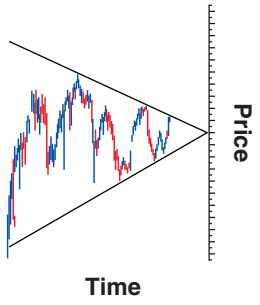
50. $\triangle JKL$ is isosceles with $\overline{JK} \cong \overline{KL}$, $JK = 4x - 1$, $KL = 2x + 5$, and $LJ = 2x - 1$.

51. $\triangle MNP$ is isosceles with $\overline{MN} \cong \overline{NP}$. MN is two less than five times x, NP is seven more than two times x, and PM is two more than three times x.

52. $\triangle RST$ is equilateral. RS is three more than four times x, ST is seven more than two times x, and TR is one more than five times x.

53. CONSTRUCTION Construct an equilateral triangle. Verify your construction using measurement and justify it using mathematics. (*Hint:* Use the construction for copying a segment.)

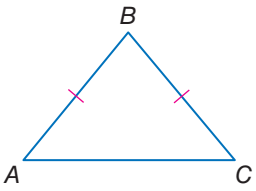
54. STOCKS Technical analysts use charts to identify patterns that can suggest future activity in stock prices. Symmetrical triangle charts are most useful when the fluctuation in the price of a stock is decreasing over time.



a. Classify by its sides and angles the triangle formed if a vertical line is drawn at any point on the graph.

b. How would the price have to fluctuate in order for the data to form an obtuse triangle? Draw an example to support your reasoning.

55 MULTIPLE REPRESENTATIONS In the diagram, the vertex *opposite* side $\overline{BC}$ is $\angle A$.



a. Geometric Draw four isosceles triangles, including one acute, one right, and one obtuse isosceles triangle. Label the vertices opposite the congruent sides as A and C. Label the remaining vertex B. Then measure the angles of each triangle and label each angle with its measure.

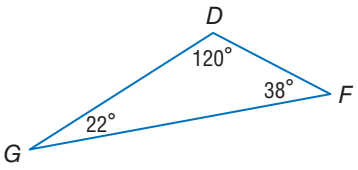
b. Tabular Measure all the angles of each triangle. Organize the measures for each triangle into a table. Include a column in your table to record the sum of these measures.

c. Verbal Make a conjecture about the measures of the angles that are opposite the congruent sides of an isosceles triangle. Then make a conjecture about the sum of the measures of the angles of an isosceles triangle.

d. Algebraic If x is the measure of one of the angles opposite one of the congruent sides in an isosceles triangle, write expressions for the measures of each of the other two angles in the triangle. Explain.

H.O.T. Problems Use Higher-Order Thinking Skills

56. ERROR ANALYSIS Elaina says that $\triangle DFG$ is obtuse. Ines disagrees, explaining that the triangle has more acute angles than obtuse angles so it must be acute. Is either of them correct? Explain your reasoning.



CCSS PRECISION Determine whether the statements below are *sometimes*, *always*, or *never* true. Explain your reasoning.

57. Equiangular triangles are also right triangles.

58. Equilateral triangles are isosceles.

59. Right triangles are equilateral.

60. CHALLENGE An equilateral triangle has sides that measure $5x + 3$ units and $7x - 5$ units. What is the perimeter of the triangle? Explain.

OPEN ENDED Draw an example of each type of triangle below using a protractor and a ruler. Label the sides and angles of each triangle with their measures. If not possible, explain why not.

61. scalene right

62. isosceles obtuse

63. equilateral obtuse

64. WRITING IN MATH Explain why classifying an equiangular triangle as an *acute* equiangular triangle is unnecessary.

Standardized Test Practice

65. Which type of triangle can serve as a counterexample to the conjecture below?

> If two angles of a triangle are acute, then the measure of the third angle must be greater than or equal to 90.

A equilateral **C** right
B obtuse **D** scalene

66. ALGEBRA A baseball glove originally cost $84.50. Kenji bought it at 40% off. How much was deducted from the original price?

F $50.70 **H** $33.80
G $44.50 **J** $32.62

67. GRIDDED RESPONSE Jorge is training for a 20-mile race. Jorge runs 7 miles on Monday, Tuesday, and Friday, and 12 miles on Wednesday and Saturday. After 6 weeks of training, Jorge will have run the equivalent of how many races?

68. SAT/ACT What is the slope of the line determined by the equation $2x + y = 5$?

A $-\frac{5}{2}$ **D** 2
B -2 **E** $\frac{5}{2}$
C -1

Spiral Review

Find the distance between each pair of parallel lines with the given equations. (Lesson 3-6)

69. $x = -2$, $x = 5$

70. $y = -6$, $y = 1$

71. $y = 2x + 3$, $y = 2x - 7$

72. $y = x + 2$, $y = x - 4$

73. FOOTBALL When striping the practice football field, Mr. Hawkins first painted the sidelines. Next he marked off 10-yard increments on one sideline. He then constructed lines perpendicular to the sidelines at each 10-yard mark. Why does this guarantee that the 10-yard lines will be parallel? (Lesson 3-5)

Identify the hypothesis and conclusion of each conditional statement. (Lesson 2-3)

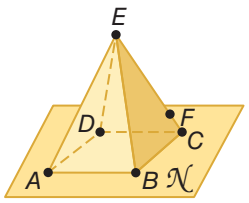
74. If three points lie on a line, then they are collinear.

75. If you are a teenager, then you are at least 13 years old.

76. If $2x + 6 = 10$, then $x = 2$.

77. If you have a driver's license, then you are at least 16 years old.

Refer to the figure at the right. (Lesson 1-1)



78. How many planes appear in this figure?

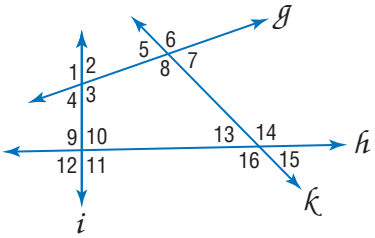
79. Name the intersection of plane AEB with plane $\mathcal{N}$.

80. Name three points that are collinear.

81. Are points D, E, C, and B coplanar?

Skills Review

Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior angles*.



82. $\angle 5$ and $\angle 3$

83. $\angle 9$ and $\angle 4$

84. $\angle 11$ and $\angle 13$

85. $\angle 1$ and $\angle 11$