

LESSON 4-2

Angles of Triangles

Then

- You classified triangles by their side or angle measures.

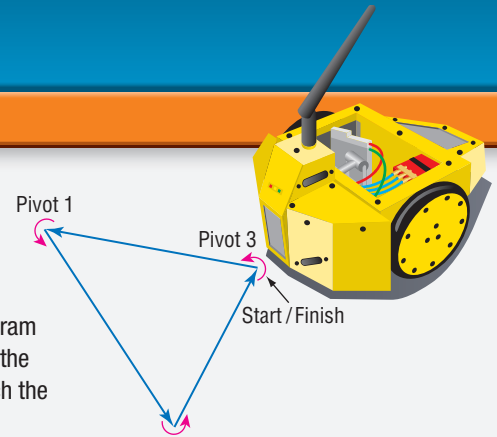
Now

- 1 Apply the Triangle Angle-Sum Theorem.
- 2 Apply Exterior Angle Theorem.

Why?

- Massachusetts Institute of Technology (MIT) sponsors the annual *Design 2.007* contest in which students design and build a robot.

One test of a robot's movements is to program it to move in a triangular path. The sum of the measures of the pivot angles through which the robot must turn will always be the same.



New Vocabulary

- auxiliary line
- exterior angle
- remote interior angles
- flow proof
- corollary



Common Core State Standards

Content Standards
G.CO.10 Prove theorems about triangles.

Mathematical Practices

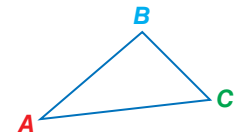
- 1 Make sense of problems and persevere in solving them.
- 3 Construct viable arguments and critique the reasoning of others.

1 Triangle Angle-Sum Theorem The Triangle Angle-Sum Theorem gives the relationship among the interior angle measures of any triangle.

Theorem 4.1 Triangle Angle-Sum Theorem

Words The sum of the measures of the angles of a triangle is 180.

Example $m\angle A + m\angle B + m\angle C = 180$



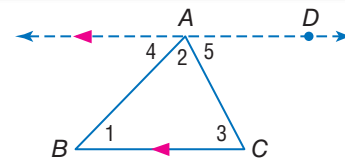
The proof of the Triangle Angle-Sum Theorem requires the use of an auxiliary line. An **auxiliary line** is an extra line or segment drawn in a figure to help analyze geometric relationships. As with any statement in a proof, you must justify any properties of an auxiliary line that you have drawn.

Proof Triangle Angle-Sum Theorem

Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$

Proof:



Statements	Reasons
1. $\triangle ABC$	1. Given
2. Draw \overleftrightarrow{AD} through A parallel to \overline{BC} .	2. Parallel Postulate
3. $\angle 4$ and $\angle BAD$ form a linear pair.	3. Def. of a linear pair
4. $\angle 4$ and $\angle BAD$ are supplementary.	4. If 2 \sphericalangle form a linear pair, they are supplementary.
5. $m\angle 4 + m\angle BAD = 180$	5. Def. of suppl. \sphericalangle
6. $m\angle BAD = m\angle 2 + m\angle 5$	6. Angle Addition Postulate
7. $m\angle 4 + m\angle 2 + m\angle 5 = 180$	7. Substitution
8. $\angle 4 \cong \angle 1$, $\angle 5 \cong \angle 3$	8. Alt. Int. \sphericalangle Theorem
9. $m\angle 4 = m\angle 1$, $m\angle 5 = m\angle 3$	9. Def. of $\cong \sphericalangle$
10. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	10. Substitution





Real-WorldLink

The pass-and-move soccer drill incorporates several fundamental aspects of passing. All passes in this drill are made in a triangle, which is the basis of all ball movement. Additionally, the players are forced to move immediately after passing the ball.

Problem-SolvingTip

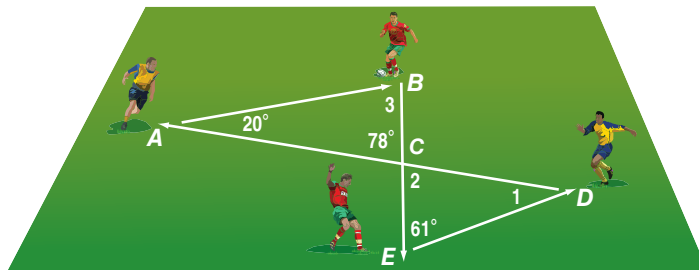
CCSS Sense-Making Often a complex problem can be more easily solved if you first break it into more manageable parts. In Example 1, before you can find $m\angle 1$, you must first find $m\angle 2$.

The Triangle Angle-Sum Theorem can be used to determine the measure of the third angle of a triangle when the other two angle measures are known.



Real-World Example 1 Use the Triangle Angle-Sum Theorem

SOCCER The diagram shows the path of the ball in a passing drill created by four friends. Find the measure of each numbered angle.



Understand Examine the information given in the diagram. You know the measures of two angles of one triangle and only one measure of another. You also know that $\angle ACB$ and $\angle 2$ are vertical angles.

Plan Find $m\angle 3$ using the Triangle Angle-Sum Theorem, because the measures of two angles of $\triangle ABC$ are known. Use the Vertical Angles Theorem to find $m\angle 2$. Then you will have enough information to find the measure of $\angle 1$ in $\triangle CDE$.

$$\begin{aligned} \text{Solve } m\angle 3 + m\angle BAC + m\angle ACB &= 180 && \text{Triangle Angle-Sum Theorem} \\ m\angle 3 + 20 + 78 &= 180 && \text{Substitution} \\ m\angle 3 + 98 &= 180 && \text{Simplify.} \\ m\angle 3 &= 82 && \text{Subtract 98 from each side.} \end{aligned}$$

$\angle ACB$ and $\angle 2$ are congruent vertical angles. So, $m\angle 2 = 78$.

Use $m\angle 2$ and $\angle CED$ of $\triangle CDE$ to find $m\angle 1$.

$$\begin{aligned} m\angle 1 + m\angle 2 + m\angle CED &= 180 && \text{Triangle Angle-Sum Theorem} \\ m\angle 1 + 78 + 61 &= 180 && \text{Substitution} \\ m\angle 1 + 139 &= 180 && \text{Simplify.} \\ m\angle 1 &= 41 && \text{Subtract 139 from each side.} \end{aligned}$$

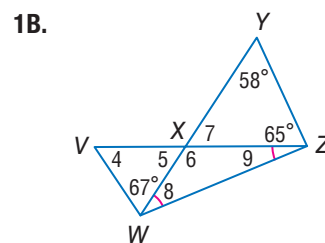
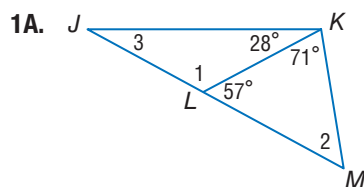
Check The sums of the measures of the angles of $\triangle ABC$ and $\triangle CDE$ should be 180.

$$\triangle ABC: m\angle 3 + m\angle BAC + m\angle ACB = 82 + 20 + 78 \text{ or } 180 \quad \checkmark$$

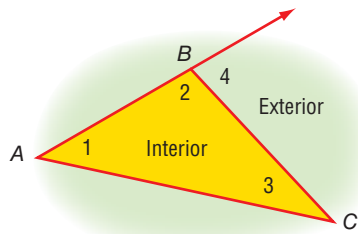
$$\triangle CDE: m\angle 1 + m\angle 2 + m\angle CED = 41 + 78 + 61 \text{ or } 180 \quad \checkmark$$

Guided Practice

Find the measures of each numbered angle.



2 Exterior Angle Theorem In addition to its three interior angles, a triangle can have **exterior angles** formed by one side of the triangle and the extension of an adjacent side. Each exterior angle of a triangle has two **remote interior angles** that are not adjacent to the exterior angle.

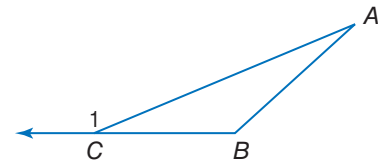


$\angle 4$ is an exterior angle of $\triangle ABC$. Its two remote interior angles are $\angle 1$ and $\angle 3$.

Theorem 4.2 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

Example $m\angle A + m\angle B = m\angle 1$



ReadingMath

Flowchart Proof A flow proof is sometimes called a *flowchart proof*.

A **flow proof** uses statements written in boxes and arrows to show the logical progression of an argument. The reason justifying each statement is written below the box. You can use a flow proof to prove the Exterior Angle Theorem.

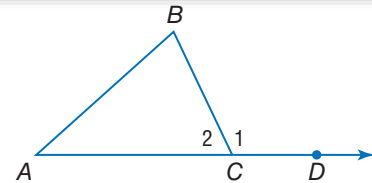
StudyTip

Flow Proofs Flow proofs can be written vertically or horizontally.

Proof Exterior Angle Theorem

Given: $\triangle ABC$

Prove: $m\angle A + m\angle B = m\angle 1$



Flow Proof:

<p>$\triangle ABC$ Given</p> <p>$m\angle A + m\angle B + m\angle 2 = 180$ Triangle Angle-Sum Theorem</p>	<p>$\angle 2$ and $\angle 1$ form a linear pair. Definition of a linear pair</p> <p>$\angle 2$ and $\angle 1$ are supplementary. If 2\angles form a linear pair, they are supplementary.</p> <p>$m\angle 2 + m\angle 1 = 180$ Definition of supplementary</p>
<p>$m\angle A + m\angle B + m\angle 2 = m\angle 2 + m\angle 1$ Substitution</p> <p>$m\angle A + m\angle B = m\angle 1$ Subtraction Property of Equality</p>	

The Exterior Angle Theorem can also be used to find missing measures.





Real-World Example 2 Use the Exterior Angle Theorem

FITNESS Find the measure of $\angle JKL$ in the Triangle Pose shown.

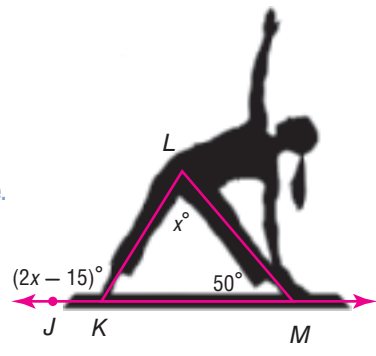
$$m\angle KLM + m\angle LMK = m\angle JKL \quad \text{Exterior Angle Theorem}$$

$$x + 50 = 2x - 15 \quad \text{Substitution}$$

$$50 = x - 15 \quad \text{Subtract } x \text{ from each side.}$$

$$65 = x \quad \text{Add 15 to each side.}$$

So, $m\angle JKL = 2(65) - 15$ or 115.

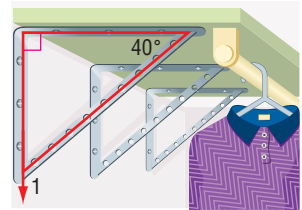


Real-World Career

Personal Trainer Personal trainers instruct and motivate individuals in exercise activities. They demonstrate various exercises and help clients improve their exercise techniques. Personal trainers must obtain certification in the fitness field.

Guided Practice

2. **CLOSET ORGANIZING** Tanya mounts the shelving bracket shown to the wall of her closet. What is the measure of $\angle 1$, the angle that the bracket makes with the wall?



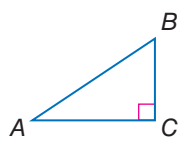
A **corollary** is a theorem with a proof that follows as a direct result of another theorem. As with a theorem, a corollary can be used as a reason in a proof. The corollaries below follow directly from the Triangle Angle-Sum Theorem.

Corollaries Triangle Angle-Sum Corollaries

4.1 The acute angles of a right triangle are complementary.

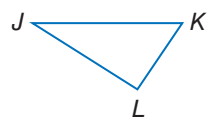
Abbreviation: *Acute \triangle of a rt. \triangle are comp.*

Example: If $\angle C$ is a right angle, then $\angle A$ and $\angle B$ are complementary.



4.2 There can be at most one right or obtuse angle in a triangle.

Example: If $\angle L$ is a right or an obtuse angle, then $\angle J$ and $\angle K$ must be acute angles.



You will prove Corollaries 4.1 and 4.2 in Exercises 34 and 35.



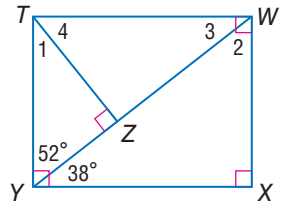
Example 3 Find Angle Measures in Right Triangles

Find the measures of each numbered angle.

$$m\angle 1 + m\angle TYZ = 90 \quad \text{Acute } \triangle \text{ of a rt. } \triangle \text{ are comp.}$$

$$m\angle 1 + 52 = 90 \quad \text{Substitution}$$

$$m\angle 1 = 38 \quad \text{Subtract 52 from each side.}$$



Guided Practice

- 3A. $\angle 2$
- 3B. $\angle 3$
- 3C. $\angle 4$

Digital Vision/Getty Images

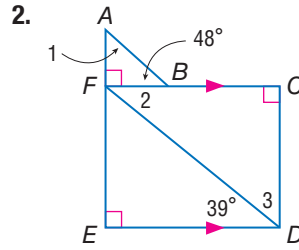
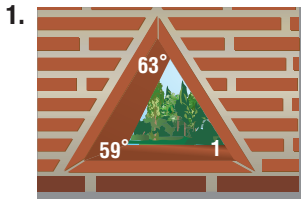
StudyTip

Check for Reasonableness When you are solving for the measure of one or more angles of a triangle, always check to make sure that the sum of the angle measures is 180.



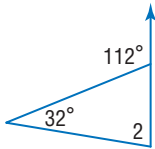


Example 1 Find the measures of each numbered angle.

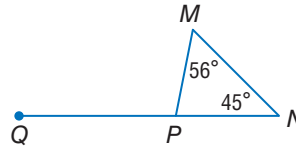


Example 2 Find each measure.

3. $m\angle 2$



4. $m\angle MPQ$



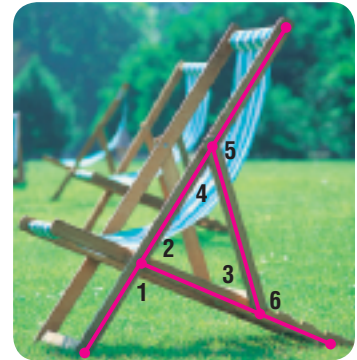
DECK CHAIRS The brace of this deck chair forms a triangle with the rest of the chair's frame as shown. If $m\angle 1 = 102$ and $m\angle 3 = 53$, find each measure.

5. $m\angle 4$

6. $m\angle 6$

7. $m\angle 2$

8. $m\angle 5$

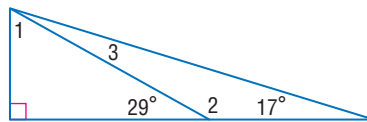


Example 3 **REGULARITY** Find each measure.

9. $m\angle 1$

10. $m\angle 3$

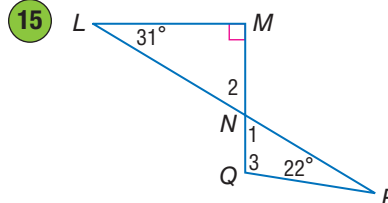
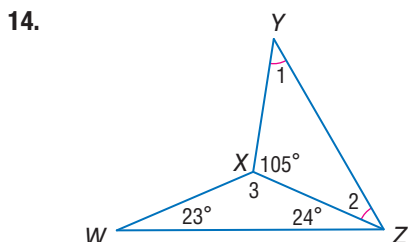
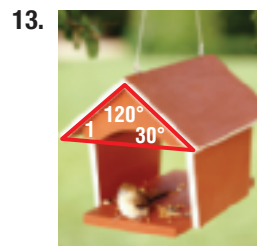
11. $m\angle 2$



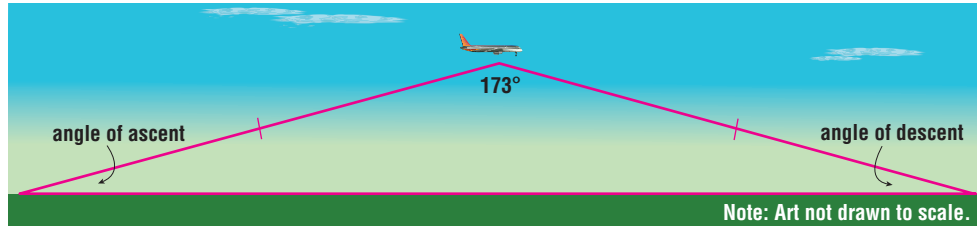
Practice and Problem Solving

Extra Practice is on page R4.

Example 1 Find the measure of each numbered angle.



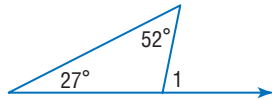
16. **AIRPLANES** The path of an airplane can be modeled using two sides of a triangle as shown. The distance covered during the plane's ascent is equal to the distance covered during its descent.



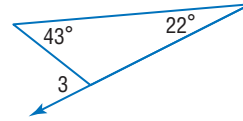
- Classify the model using its sides and angles.
- The angles of ascent and descent are congruent. Find their measures.

Example 2 Find each measure.

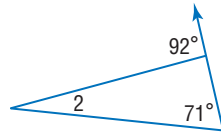
17. $m\angle 1$



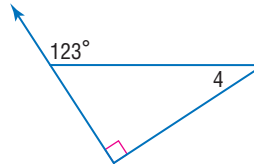
18. $m\angle 3$



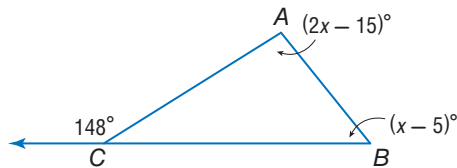
19. $m\angle 2$



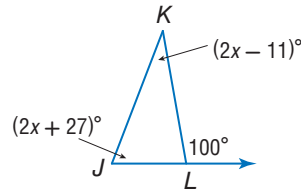
20. $m\angle 4$



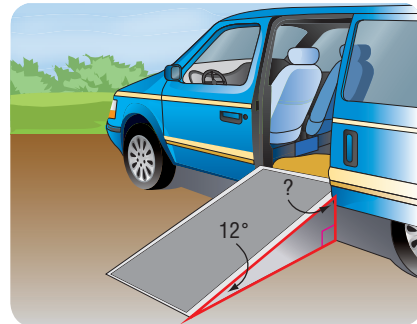
21. $m\angle ABC$



22. $m\angle JKL$

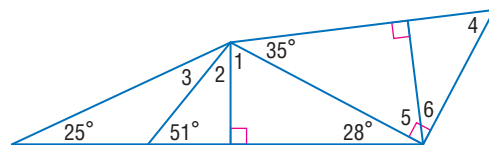


- Example 3** 23. **WHEELCHAIR RAMP** Suppose the wheelchair ramp shown makes a 12° angle with the ground. What is the measure of the angle the ramp makes with the van door?

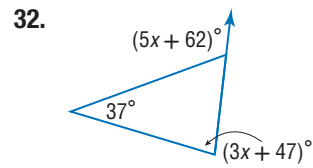
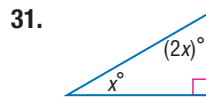
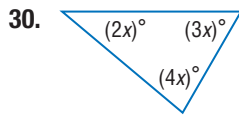


CCSS REGULARITY Find each measure.

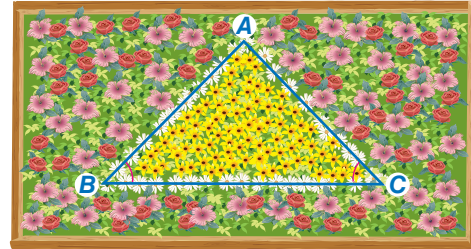
- | | |
|-----------------|-----------------|
| 24. $m\angle 1$ | 25. $m\angle 2$ |
| 26. $m\angle 3$ | 27. $m\angle 4$ |
| 28. $m\angle 5$ | 29. $m\angle 6$ |



ALGEBRA Find the value of x . Then find the measure of each angle.



33. **GARDENING** A landscaper is forming an isosceles triangle in a flowerbed using chrysanthemums. She wants $m\angle A$ to be three times the measure of $\angle B$ and $\angle C$. What should the measure of each angle be?

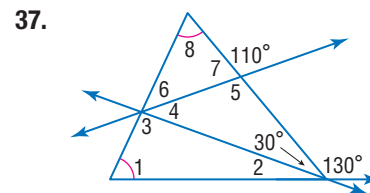
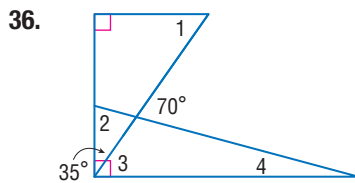


PROOF Write the specified type of proof.

34. flow proof of Corollary 4.1

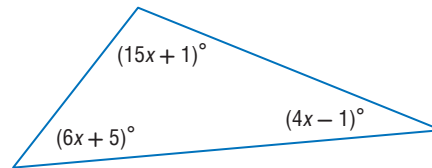
35. paragraph proof of Corollary 4.2

CCSS REGULARITY Find the measure of each numbered angle.



38. **ALGEBRA** Classify the triangle shown by its angles. Explain your reasoning.

39. **ALGEBRA** The measure of the larger acute angle in a right triangle is two degrees less than three times the measure of the smaller acute angle. Find the measure of each angle.



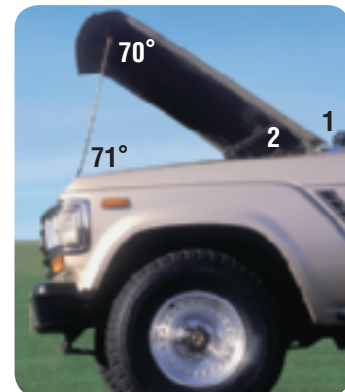
40. Determine whether the following statement is *true* or *false*. If false, give a counterexample. If true, give an argument to support your conclusion.

If the sum of two acute angles of a triangle is greater than 90, then the triangle is acute.

41. **ALGEBRA** In $\triangle XYZ$, $m\angle X = 157$, $m\angle Y = y$, and $m\angle Z = z$. Write an inequality to describe the possible measures of $\angle Z$. Explain your reasoning.

42. **CARS** Refer to the photo at the right.

- Find $m\angle 1$ and $m\angle 2$.
- If the support for the hood were shorter than the one shown, how would $m\angle 1$ change? Explain.
- If the support for the hood were shorter than the one shown, how would $m\angle 2$ change? Explain.

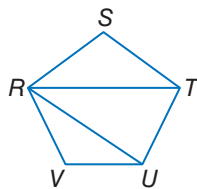


PROOF Write the specified type of proof.

43. two-column proof

Given: $RSTUV$ is a pentagon.

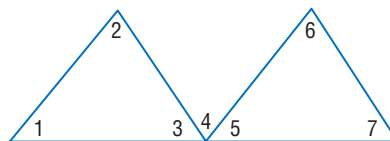
Prove: $m\angle S + m\angle STU + m\angle TUV + m\angle V + m\angle VRS = 540$



44. flow proof

Given: $\angle 3 \cong \angle 5$

Prove: $m\angle 1 + m\angle 2 = m\angle 6 + m\angle 7$



45. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the sum of the measures of the exterior angles of a triangle.

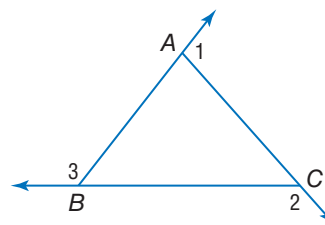
a. **Geometric** Draw five different triangles, extending the sides and labeling the angles as shown. Be sure to include at least one obtuse, one right, and one acute triangle.

b. **Tabular** Measure the exterior angles of each triangle. Record the measures for each triangle and the sum of these measures in a table.

c. **Verbal** Make a conjecture about the sum of the exterior angles of a triangle. State your conjecture using words.

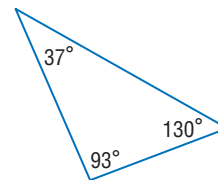
d. **Algebraic** State the conjecture you wrote in part c algebraically.

e. **Analytical** Write a paragraph proof of your conjecture.

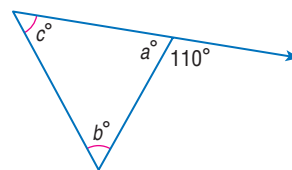


H.O.T. Problems Use Higher-Order Thinking Skills

46. **CCSS CRITIQUE** Curtis measured and labeled the angles of the triangle as shown. Arnoldo says that at least one of his measures is incorrect. Explain in at least two different ways how Arnoldo knows that this is true.

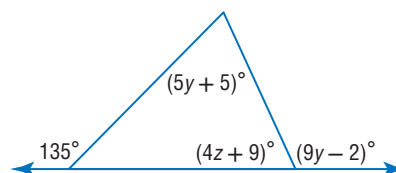


47. **WRITING IN MATH** Explain how you would find the missing measures in the figure shown.



48. **OPEN ENDED** Construct a right triangle and measure one of the acute angles. Find the measure of the second acute angle using calculation and explain your method. Confirm your result using a protractor.

49. **CHALLENGE** Find the values of y and z in the figure at the right.



50. **REASONING** If an exterior angle adjacent to $\angle A$ is acute, is $\triangle ABC$ acute, right, obtuse, or can its classification not be determined? Explain your reasoning.

51. **WRITING IN MATH** Explain why a triangle cannot have an obtuse, acute, and a right exterior angle.



Standardized Test Practice

52. PROBABILITY Mr. Glover owns a video store and wants to survey his customers to find what type of movies he should buy. Which of the following options would be the best way for Mr. Glover to get accurate survey results?

- A surveying customers who come in from 9 P.M. until 10 P.M.
- B surveying customers who come in on the weekend
- C surveying the male customers
- D surveying at different times of the week and day

53. SHORT RESPONSE Two angles of a triangle have measures of 35° and 80° . Find the values of the exterior angle measures of the triangle.

54. ALGEBRA Which equation is equivalent to $7x - 3(2 - 5x) = 8x$?

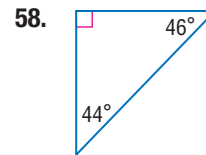
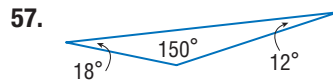
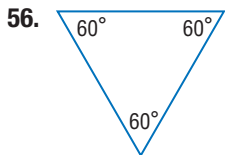
- F $2x - 6 = 8$
- G $22x - 6 = 8x$
- H $-8x - 6 = 8x$
- J $22x + 6 = 8x$

55. SAT/ACT Joey has 4 more video games than Solana and half as many as Melissa. If together they have 24 video games, how many does Melissa have?

- A 7
- B 9
- C 12
- D 13
- E 14

Spiral Review

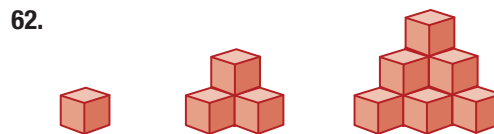
Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*. (Lesson 4-1)



COORDINATE GEOMETRY Find the distance from P to ℓ . (Lesson 3-6)

59. Line ℓ contains points $(0, -2)$ and $(1, 3)$. Point P has coordinates $(-4, 4)$.
60. Line ℓ contains points $(-3, 0)$ and $(3, 0)$. Point P has coordinates $(4, 3)$.

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence. (Lesson 2-1)



Skills Review

State the property that justifies each statement.

63. If $\frac{x}{2} = 7$, then $x = 14$.
64. If $x = 5$ and $b = 5$, then $x = b$.
65. If $XY - AB = WZ - AB$, then $XY = WZ$.
66. If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, $m\angle A = m\angle C$.
67. If $m\angle 1 + m\angle 2 = 90$ and $m\angle 2 = m\angle 3$, then $m\angle 1 + m\angle 3 = 90$.