LESSON 4-3

Congruent Triangles

Then

- You identified and used congruent angles.

Now

1. Name and use corresponding parts of congruent polygons.
2. Prove triangles congruent using the definition of congruence.

Why?

- As an antitheft device, many manufacturers make car stereos with removable faceplates. The shape and size of the faceplate and of the space where it fits must be exactly the same for the faceplate to properly attach to the car's dashboard.

New Vocabulary

congruent
congruent polygons
corresponding parts

Common Core State Standards

Content Standards

G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Mathematical Practices

6 Attend to precision.

3 Construct viable arguments and critique the reasoning of others.

1 Congruence and Corresponding Parts

If two geometric figures have exactly the same shape and size, they are **congruent**.

Congruent	Not Congruent
	
While positioned differently, Figures 1, 2, and 3 are exactly the same shape and size.	Figures 4 and 5 are exactly the same shape but not the same size. Figures 5 and 6 are the same size but not exactly the same shape.

In two **congruent polygons**, all of the parts of one polygon are congruent to the **corresponding parts** or matching parts of the other polygon. These corresponding parts include *corresponding angles* and *corresponding sides*.

KeyConcept Definition of Congruent Polygons

Words Two polygons are congruent if and only if their corresponding parts are congruent.

Example

Corresponding Angles

$\angle A \cong \angle H$ $\angle B \cong \angle J$ $\angle C \cong \angle K$

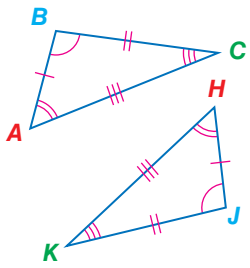
Corresponding Sides

$\overline{AB} \cong \overline{HJ}$ $\overline{BC} \cong \overline{JK}$ $\overline{AC} \cong \overline{HK}$

Congruence Statement

$\triangle ABC \cong \triangle HJK$

Model



Other congruence statements for the triangles above exist. Valid congruence statements for congruent polygons list corresponding vertices in the same order.

Valid Statement

$\triangle BCA \cong \triangle JKH$

Not a Valid Statement

$\triangle ABC \cong \triangle HKJ$

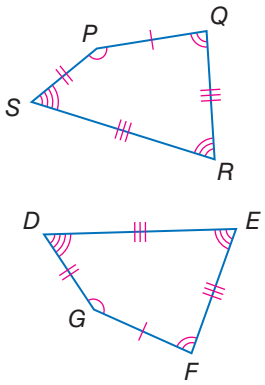
Math History Link

Johann Carl Friedrich Gauss (1777–1855) Gauss developed the congruence symbol to show that two sides of an equation were the same even if they weren't equal. He made many advances in math and physics, including a proof of the fundamental theorem of algebra.

Source: The Granger Collection, New York

Example 1 Identify Corresponding Congruent Parts

Show that the polygons are congruent by identifying all the congruent corresponding parts. Then write a congruence statement.



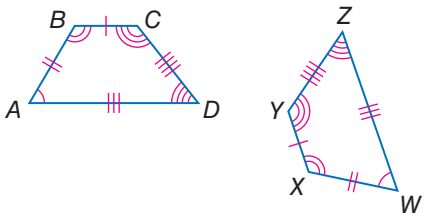
Angles: $\angle P \cong \angle G$, $\angle Q \cong \angle F$, $\angle R \cong \angle E$, $\angle S \cong \angle D$

Sides: $\overline{PQ} \cong \overline{GF}$, $\overline{QR} \cong \overline{FE}$, $\overline{RS} \cong \overline{ED}$, $\overline{SP} \cong \overline{DG}$

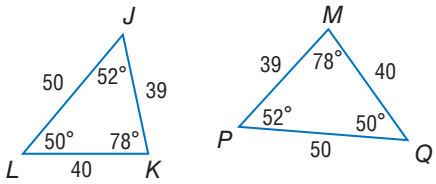
All corresponding parts of the two polygons are congruent. Therefore, polygon $PQRS \cong$ polygon $GFED$.

GuidedPractice

1A.



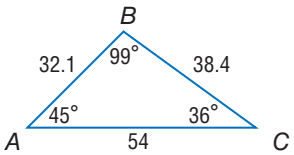
1B.

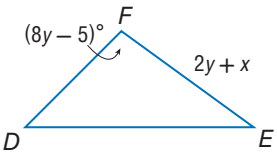


The phrase "if and only if" in the congruent polygon definition means that both the conditional and its converse are true. So, if two polygons are congruent, then their corresponding parts are congruent. For triangles, we say *Corresponding parts of congruent triangles are congruent*, or CPCTC.

Example 2 Use Corresponding Parts of Congruent Triangles

In the diagram, $\triangle ABC \cong \triangle DFE$. Find the values of x and y.





$\angle F \cong \angle B$	CPCTC
$m\angle F = m\angle B$	Definition of congruence
$8y - 5 = 99$	Substitution
$8y = 104$	Add 5 to each side.
$y = 13$	Divide each side by 8.
$\overline{FE} \cong \overline{BC}$	CPCTC
$FE = BC$	Definition of congruence
$2y + x = 38.4$	Substitution
$2(13) + x = 38.4$	Substitution
$26 + x = 38.4$	Simplify.
$x = 12.4$	Subtract 26 from each side.

StudyTip

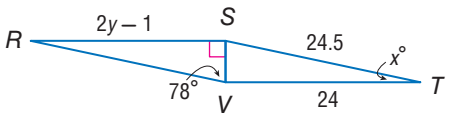
Using a Congruence Statement You can use a congruence statement to help you correctly identify corresponding sides.

$\triangle ABC \cong \triangle DFE$

$\overline{BC} \cong \overline{FE}$

GuidedPractice

2. In the diagram, $\triangle RSV \cong \triangle TVS$. Find the values of x and y.



akg-images

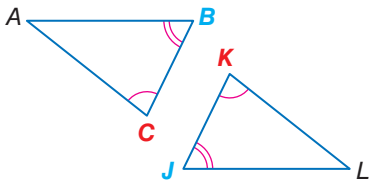
2 Prove Triangles Congruent

The Triangle Angle-Sum Theorem you learned in Lesson 4-2 leads to another theorem about the angles in two triangles.

Theorem 4.3 Third Angles Theorem

Words: If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.

Example: If $\angle C \cong \angle K$ and $\angle B \cong \angle J$, then $\angle A \cong \angle L$.



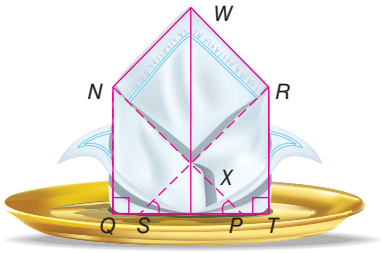
You will prove this theorem in Exercise 21.

Real-WorldLink

Using some basic skills with napkin folding can add an elegant touch to any party. Many of the folds use triangles.

Real-World Example 3 Use the Third Angles Theorem

PARTY PLANNING **The planners of the Senior Banquet decide to fold the dinner napkins using the Triangle Pocket Fold so that they can place a small gift in the pocket. If $\angle NPQ \cong \angle RST$, and $m\angle NPQ = 40$, find $m\angle SRT$.**



$\angle NPQ \cong \angle RST$, and since all right angles are congruent, $\angle NQP \cong \angle RTS$. So by the Third Angles Theorem, $\angle QNP \cong \angle SRT$. By the definition of congruence, $m\angle QNP = m\angle TRS$.

$m\angle QNP + m\angle NPQ = 90$	The acute angles of a right triangle are complementary.
$m\angle QNP + 40 = 90$	Substitution
$m\angle QNP = 50$	Subtract 40 from each side.

By substitution, $m\angle SRT = m\angle QNP$ or 50.

GuidedPractice

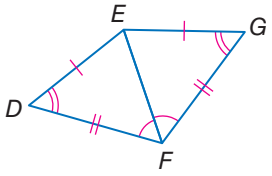
3. In the diagram above, if $\angle WNX \cong \angle WRX$, $\overline{WX}$ bisects $\angle NXR$, $m\angle WNX = 88$, and $m\angle NXW = 49$, find $m\angle NWR$. Explain your reasoning.

Example 4 Prove That Two Triangles are Congruent

Write a two-column proof.

Given: $\overline{DE} \cong \overline{GE}$, $\overline{DF} \cong \overline{GF}$, $\angle D \cong \angle G$, $\angle DFE \cong \angle GFE$

Prove: $\triangle DEF \cong \triangle GEF$



Proof:

Statements	Reasons
1. $\overline{DE} \cong \overline{GE}$, $\overline{DF} \cong \overline{GF}$	**1.** Given
2. $\overline{EF} \cong \overline{EF}$	**2.** Reflexive Property of Congruence
3. $\angle D \cong \angle G$, $\angle DFE \cong \angle GFE$	**3.** Given
4. $\angle DEF \cong \angle GEF$	**4.** Third Angles Theorem
5. $\triangle DEF \cong \triangle GEF$	**5.** Definition of Congruent Polygons

StudyTip

Reflexive Property

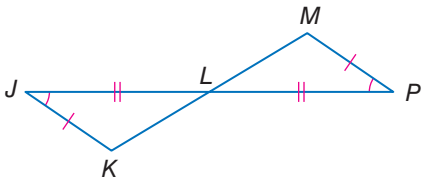
When two triangles share a common side, use the Reflexive Property of Congruence to establish that the common side is congruent to itself.

GuidedPractice

4. Write a two column proof.

Given: $\angle J \cong \angle P$, $\overline{JK} \cong \overline{PM}$, $\overline{JL} \cong \overline{PL}$, and L bisects $\overline{KM}$.

Prove: $\triangle JLK \cong \triangle PLM$



Like congruence of segments and angles, congruence of triangles is reflexive, symmetric, and transitive.

Theorem 4.4 Properties of Triangle Conguruence

Reflexive Property of Triangle Congruence

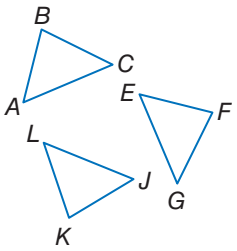
$\triangle ABC \cong \triangle ABC$

Symmetric Property of Triangle Congruence

If $\triangle ABC \cong \triangle EFG$, then $\triangle EFG \cong \triangle ABC$.

Transitive Property of Triangle Congruence

If $\triangle ABC \cong \triangle EFG$ and $\triangle EFG \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



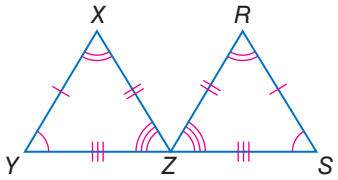
You will prove the reflexive, symmetric, and transitive parts of Theorem 4.4 in Exercises 27, 22, and 26, respectively.

Check Your Understanding

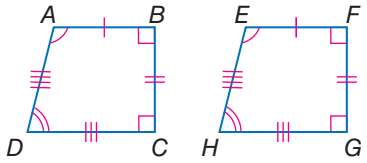
= Step-by-Step Solutions begin on page R14.

Example 1 **Show that polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.**

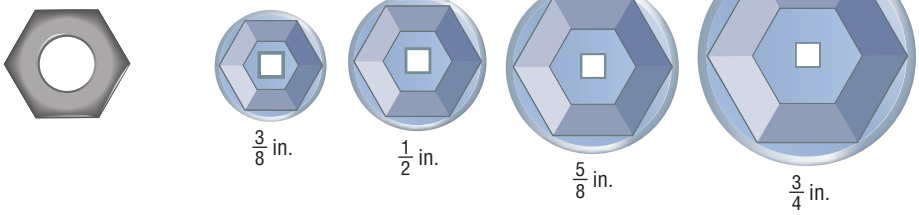
1.



2.



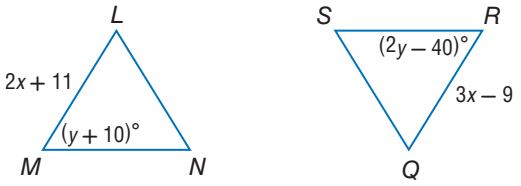
3. **TOOLS** Sareeta is changing the tire on her bike and the nut securing the tire looks like the one shown. Which of the sockets below should she use with her wrench to remove the tire? Explain your reasoning.



Example 2 **In the figure, $\triangle LMN \cong \triangle QRS$.**

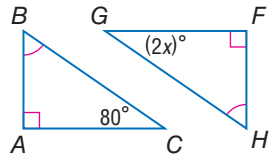
4. Find x.

5 Find y.

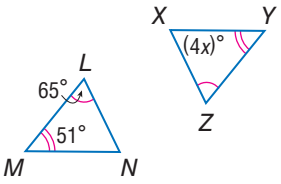


Example 3 **CCSS REGULARITY Find x. Explain your reasoning.**

6.



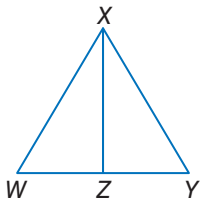
7.



Example 4 **8. PROOF** Write a paragraph proof.

Given: $\angle WXZ \cong \angle YXZ$, $\angle XZW \cong \angle XZY$, $\overline{WX} \cong \overline{YX}$, $\overline{WZ} \cong \overline{YZ}$

Prove: $\triangle WXZ \cong \triangle YXZ$

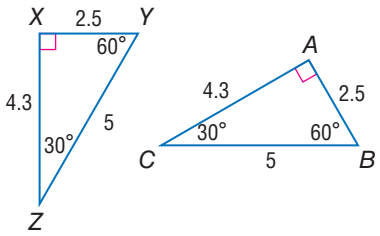


Practice and Problem Solving

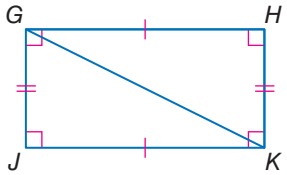
Extra Practice is on page R4.

Example 1 **Show that polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.**

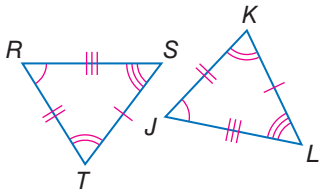
9.



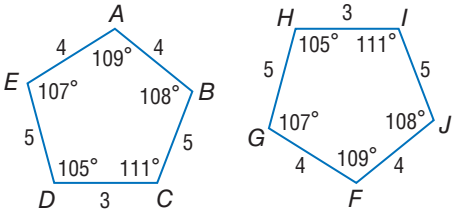
10.



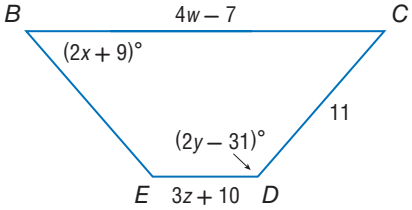
11.

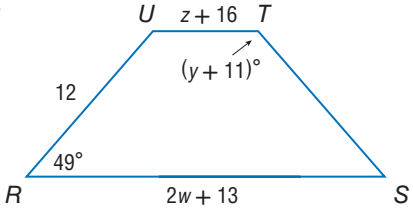


12.



Example 2 **Polygon $BCDE \cong$ polygon $RSTU$. Find each value.**





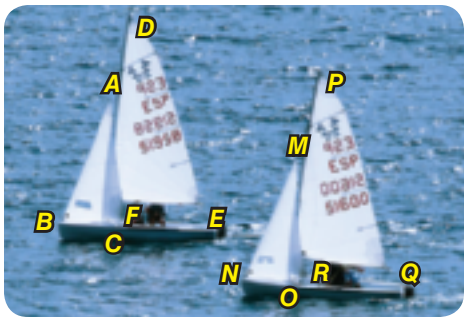
13. x **14.** y **15** z **16.** w

17. SAILING To ensure that sailboat races are fair, the boats and their sails are required to be the same size and shape.

a. Write a congruence statement relating the triangles in the photo.

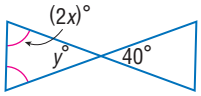
b. Name six pairs of congruent segments.

c. Name six pairs of congruent angles.

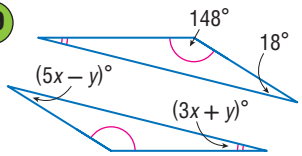


Example 3 **Find x and y.**

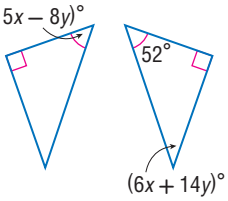
18.



19



20.



Example 4

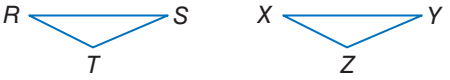
21. PROOF Write a two-column proof of Theorem 4.3.

22. PROOF Put the statements used to prove the statement below in the correct order. Provide the reasons for each statement.

Congruence of triangles is symmetric. (Theorem 4.4)

Given: $\triangle RST \cong \triangle XYZ$

Prove: $\triangle XYZ \cong \triangle RST$



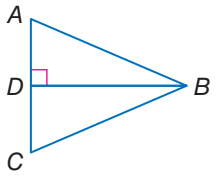
Proof:

$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$, $\overline{XY} \cong \overline{RS}$, $\overline{YZ} \cong \overline{ST}$, $\overline{XZ} \cong \overline{RT}$	$\angle R \cong \angle X$, $\angle S \cong \angle Y$, $\angle T \cong \angle Z$, $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$, $\overline{RT} \cong \overline{XZ}$	$\triangle RST \cong \triangle XYZ$	$\triangle XYZ \cong \triangle RST$
?	?	?	?

CCSS ARGUMENTS **Write a two-column proof.**

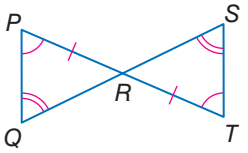
23. Given: $\overline{BD}$ bisects $\angle B$.
$\overline{BD} \perp \overline{AC}$

Prove: $\angle A \cong \angle C$

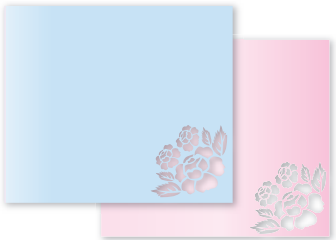


24. Given: $\angle P \cong \angle T$, $\angle S \cong \angle Q$
$\overline{TR} \cong \overline{PR}$, $\overline{RP} \cong \overline{RQ}$,
$\overline{RT} \cong \overline{RS}$
$\overline{PQ} \cong \overline{TS}$

Prove: $\triangle PRQ \cong \triangle TRS$



25. SCRAPBOOKING Lanie is using a flower-shaped corner decoration punch for a scrapbook she is working on. If she punches the corners of two pages as shown, what property guarantees that the punched designs are congruent? Explain.



PROOF **Write the specified type of proof of the indicated part of Theorem 4.4.**

26. Congruence of triangles is transitive. (paragraph proof)

27. Congruence of triangles is reflexive. (flow proof)

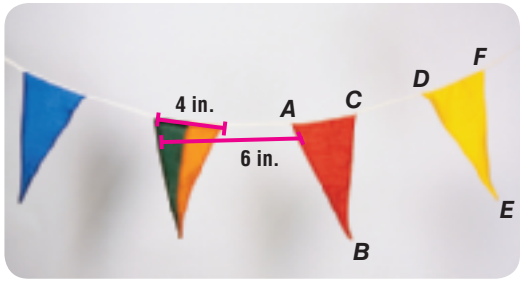
ALGEBRA **Draw and label a figure to represent the congruent triangles. Then find x and y.**

28. $\triangle ABC \cong \triangle DEF$, $AB = 7$, $BC = 9$, $AC = 11 + x$, $DF = 3x - 13$, and $DE = 2y - 5$

29. $\triangle LMN \cong \triangle RST$, $m\angle L = 49$, $m\angle M = 10y$, $m\angle S = 70$, and $m\angle T = 4x + 9$

30. $\triangle JKL \cong \triangle MNP$, $JK = 12$, $LJ = 5$, $PM = 2x - 3$, $m\angle L = 67$, $m\angle K = y + 4$ and $m\angle N = 2y - 15$

31 **PENNANTS** Scott is in charge of roping off an area of 100 square feet for the band to use during a pep rally. He is using a string of pennants that are congruent isosceles triangles.

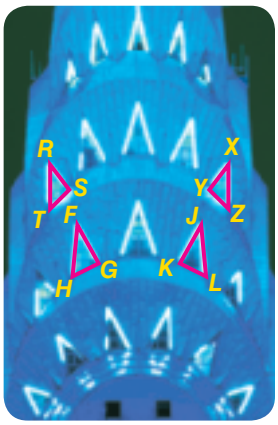


a. List seven pairs of congruent segments in the photo.

b. If the area he ropes off is a square, how long will the pennant string need to be?

c. How many pennants will be on the string?

32. CCSS **SENSE-MAKING** In the photo of New York City's Chrysler Building at the right, $\overline{TS} \cong \overline{ZY}$, $\overline{XY} \cong \overline{RS}$, $\overline{TR} \cong \overline{ZX}$, $\angle X \cong \angle R$, $\angle T \cong \angle Z$, $\angle Y \cong \angle S$, and $\triangle HGF \cong \triangle LKJ$.



a. Which triangle, if any, is congruent to $\triangle YXZ$? Explain your reasoning.

b. Which side(s) are congruent to $\overline{JL}$? Explain your reasoning.

c. Which angle(s) are congruent to $\angle G$? Explain your reasoning.

33. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the statement *The areas of congruent triangles are equal.*

a. Verbal Write a conditional statement to represent the relationship between the areas of a pair of congruent triangles.

b. Verbal Write the converse of your conditional statement. Is the converse *true* or *false*? Explain your reasoning.

c. Geometric If possible, draw two equilateral triangles that have the same area but are not congruent. If not possible, explain why not.

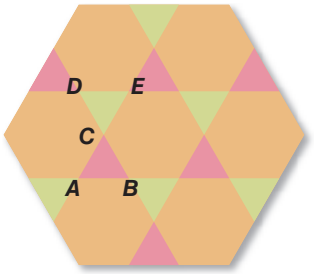
d. Geometric If possible, draw two rectangles that have the same area but are not congruent. If not possible, explain why not.

e. Geometric If possible, draw two squares that have the same area but are not congruent. If not possible, explain why not.

f. Verbal For which polygons will the following conditional and its converse both be true? Explain your reasoning.

If a pair of _____ are congruent, then they have the same area.

34. PATTERNS The pattern shown is created using regular polygons.



a. What two polygons are used to create the pattern?

b. Name a pair of congruent triangles.

c. Name a pair of corresponding angles.

d. If $CB = 2$ inches, what is AE? Explain.

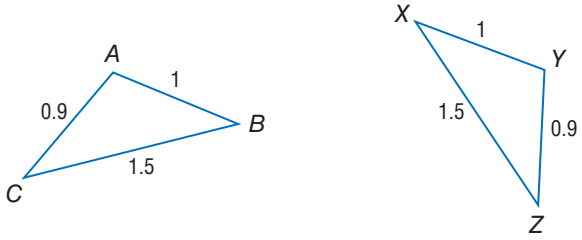
e. What is the measure of $\angle D$? Explain.

35. FITNESS A fitness instructor is starting a new aerobics class using fitness hoops. She wants to confirm that all of the hoops are the same size. What measure(s) can she use to prove that all of the hoops are congruent? Explain your reasoning.

H.O.T. Problems Use Higher-Order Thinking Skills

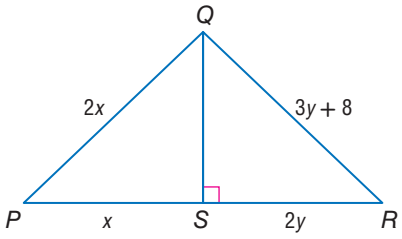
36. WRITING IN MATH Explain why the order of the vertices is important when naming congruent triangles. Give an example to support your answer.

37. ERROR ANALYSIS Jasmine and Will are evaluating the congruent figures below. Jasmine says that $\triangle CAB \cong \triangle ZYX$ and Will says that $\triangle ABC \cong \triangle YXZ$. Is either of them correct? Explain.



38. WRITE A QUESTION A classmate is using the Third Angles Theorem to show that if 2 corresponding pairs of the angles of two triangles are congruent, then the third pair is also congruent. Write a question to help him decide if he can use the same strategy for quadrilaterals.

39. CHALLENGE Find x and y if $\triangle PQS \cong \triangle RQS$.

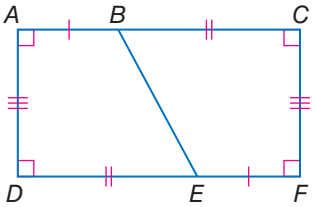


CCSS **ARGUMENTS** **Determine whether each statement is *true* or *false*. If false, give a counterexample. If true, explain your reasoning.**

40. Two triangles with two pairs of congruent corresponding angles and three pairs of congruent corresponding sides are congruent.

41. Two triangles with three pairs of corresponding congruent angles are congruent.

42. CHALLENGE Write a paragraph proof to prove polygon $ABED \cong$ polygon $FEBC$.

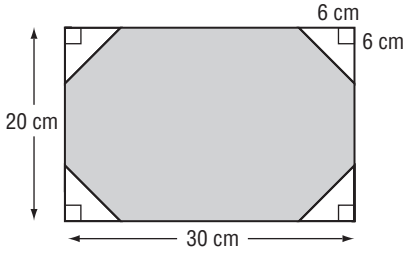


43. WRITING IN MATH Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

Equilateral triangles are congruent.

Standardized Test Practice

44. Barrington cut four congruent triangles off the corners of a rectangle to make an octagon as shown below. What is the area of the octagon?



A 456 cm^2
B 528 cm^2
C 552 cm^2
D 564 cm^2

45. GRIDDED RESPONSE Triangle ABC is congruent to $\triangle HIJ$. The vertices of $\triangle ABC$ are $A(-1, 2)$, $B(0, 3)$ and $C(2, -2)$. What is the measure of side $\overline{HJ}$?

46. ALGEBRA Which is a factor of $x^2 + 19x - 42$?

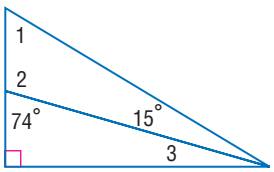
F $x + 14$
G $x + 2$
H $x - 2$
J $x - 14$

47. SAT/ACT Mitsu travels a certain distance at 30 miles per hour and returns the same route at 65 miles per hour. What is his average speed in miles per hour for the round trip?

A 32.5
B 35.0
C 41.0
D 47.5
E 55.3

Spiral Review

Find each measure in the triangle at the right. (Lesson 4-2)



48. $m\angle 2$
49. $m\angle 1$
50. $m\angle 3$

COORDINATE GEOMETRY **Find the measures of the sides of $\triangle JKL$ and classify each triangle by the measures of its sides.** (Lesson 4-1)

51. $J(-7, 10)$, $K(15, 0)$, $L(-2, -1)$
52. $J(9, 9)$, $K(12, 14)$, $L(14, 6)$
53. $J(4, 6)$, $K(4, 11)$, $L(9, 6)$
54. $J(16, 14)$, $K(7, 6)$, $L(-5, -14)$

Determine whether each statement is *always*, *sometimes*, or *never* true. (Lesson 1-5)

55. Two angles that form a linear pair are supplementary.

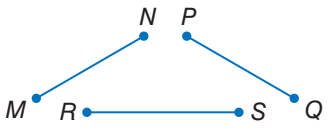
56. If two angles are supplementary, then one of the angles is obtuse.

57. CARPENTRY A carpenter must cut two pieces of wood at angles so that they fit together to form the corner of a picture frame. What type of angles must he use to make sure that a 90° corner results? (Lesson 1-5)

Skills Review

58. Copy and complete the proof.

Given: $\overline{MN} \cong \overline{PQ}$, $\overline{PQ} \cong \overline{RS}$
Prove: $\overline{MN} \cong \overline{RS}$
Proof:



Statements	Reasons
a. ___?___	**a.** Given
b. $MN = PQ$, $PQ = RS$	**b.** ___?___
c. ___?___	**c.** ___?___
d. $\overline{MN} \cong \overline{RS}$	**d.** Definition of congruent segments