LESSON 4-4

Proving Triangles Congruent–SSS, SAS

Then

- You proved triangles congruent using the definition of congruence.

Now

1. Use the SSS Postulate to test for triangle congruence.
2. Use the SAS Postulate to test for triangle congruence.

Why?

- An A-frame sandwich board is a convenient way to display information. Not only does it fold flat for easy storage, but with each sidearm locked into place, the frame is extremely sturdy. With the sidearms the same length and positioned the same distance from the top on either side, the open frame forms two congruent triangles.

NewVocabulary
included angle

Common Core State Standards

Content Standards

G.CO.10 Prove theorems about triangles.

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

1 Make sense of problems and persevere in solving them.

1 SSS Postulate

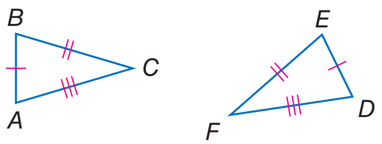
In Lesson 4-3, you proved that two triangles were congruent by showing that all six pairs of corresponding parts were congruent. It is possible to prove two triangles congruent using fewer pairs.

The sandwich board demonstrates that if two triangles have the same three side lengths, then they are congruent. This is expressed in the postulate below.

Postulate 4.1 Side-Side-Side (SSS) Congruence

If three sides of one triangle are congruent to three sides of a second triangle, then the triangles are congruent.

Example If Side $\overline{AB} \cong \overline{DE}$,
Side $\overline{BC} \cong \overline{EF}$, and
Side $\overline{AC} \cong \overline{DF}$,
then $\triangle ABC \cong \triangle DEF$.

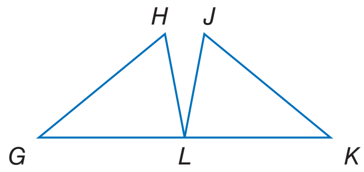


Example 1 Use SSS to Prove Triangles Congruent

Write a flow proof.

Given: $\overline{GH} \cong \overline{KJ}$, $\overline{HL} \cong \overline{JL}$, and L is the midpoint of $\overline{GK}$.

Prove: $\triangle GHL \cong \triangle KJL$



Flow Proof:

$\overline{GH} \cong \overline{KJ}$ — Given

$\overline{HL} \cong \overline{JL}$ — Given

L is the midpoint of $\overline{GK}$. — Given → $\overline{GL} \cong \overline{KL}$ — Midpoint Theorem

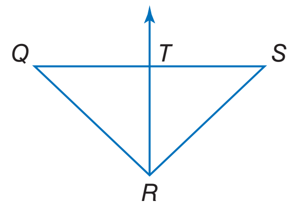
→ $\triangle GHL \cong \triangle KJL$ — SSS

GuidedPractice

1. Write a flow proof.

Given: $\triangle QRS$ is isosceles with $\overline{QR} \cong \overline{SR}$. $\overline{RT}$ bisects $\overline{QS}$ at point T.

Prove: $\triangle QRT \cong \triangle SRT$



Standardized Test Example 2 SSS on the Coordinate Plane

EXTENDED RESPONSE **Triangle ABC has vertices $A(1, 1)$, $B(0, 3)$, and $C(2, 5)$. Triangle EFG has vertices $E(1, -1)$, $F(2, -5)$, and $G(4, -4)$.**

a. Graph both triangles on the same coordinate plane.

b. Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning.

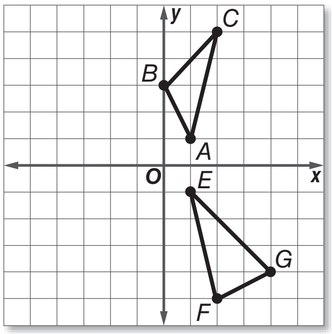
c. Write a logical argument using coordinate geometry to support the conjecture you made in part b.

Read the Test Item

You are asked to do three things in this problem. In part **a**, you are to graph $\triangle ABC$ and $\triangle EFG$ on the same coordinate plane. In part **b**, you should make a conjecture that $\triangle ABC \cong \triangle EFG$ or $\triangle ABC \ncong \triangle EFG$ based on your graph. Finally, in part **c**, you are asked to prove your conjecture.

Solve the Test Item

a.



b. From the graph, it appears that the triangles do not have the same shape, so we can conjecture that they are not congruent.

c. Use the Distance Formula to show that not all corresponding sides have the same measure.

$$AB = \sqrt{(0-1)^2 + (3-1)^2} = \sqrt{1+4} \text{ or } \sqrt{5}$$

$$EF = \sqrt{(2-1)^2 + [-5-(-1)]^2} = \sqrt{1+16} \text{ or } \sqrt{17}$$

$$BC = \sqrt{(2-0)^2 + (5-3)^2} = \sqrt{4+4} \text{ or } \sqrt{8}$$

$$FG = \sqrt{(4-2)^2 + [-4-(-5)]^2} = \sqrt{4+1} \text{ or } \sqrt{5}$$

$$AC = \sqrt{(2-1)^2 + (5-1)^2} = \sqrt{1+16} \text{ or } \sqrt{17}$$

$$EG = \sqrt{(4-1)^2 + [-4-(-1)]^2} = \sqrt{9+9} \text{ or } \sqrt{18}$$

While $AB = FG$ and $AC = EF$, $BC \neq EG$. Since SSS congruence is not met, $\triangle ABC \ncong \triangle EFG$.

Test-Taking Tip

CCSS Tools When you are solving problems using the coordinate plane, remember to use tools like the Distance, Midpoint, and Slope Formulas to solve problems and to check your solutions.

Reading Math

Symbols $\triangle ABC \ncong \triangle EFG$ is read as *triangle ABC is not congruent to triangle EFG.*

Guided Practice

2. Triangle JKL has vertices $J(2, 5)$, $K(1, 1)$, and $L(5, 2)$. Triangle NPQ has vertices $N(-3, 0)$, $P(-7, 1)$, and $Q(-4, 4)$.

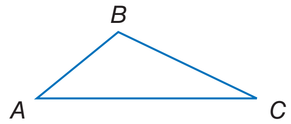
a. Graph both triangles on the same coordinate plane.

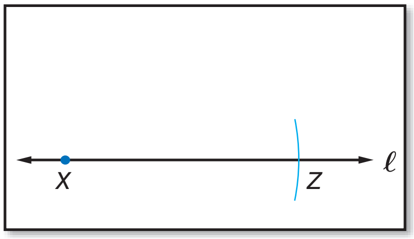
b. Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning.

c. Write a logical argument using coordinate geometry to support the conjecture you made in part **b**.

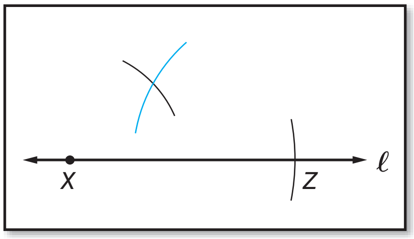
Construction Congruent Triangles Using Sides

Draw a triangle and label it $\triangle ABC$. Then use the SSS Postulate to construct $\triangle XYZ \cong \triangle ABC$.

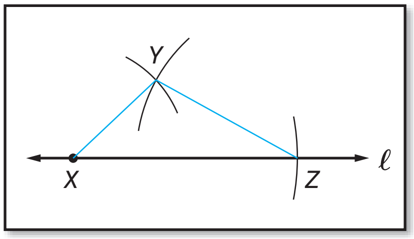




Step 1 Draw point X on a line ℓ. Then construct $\overline{XZ} \cong \overline{AC}$ on line ℓ



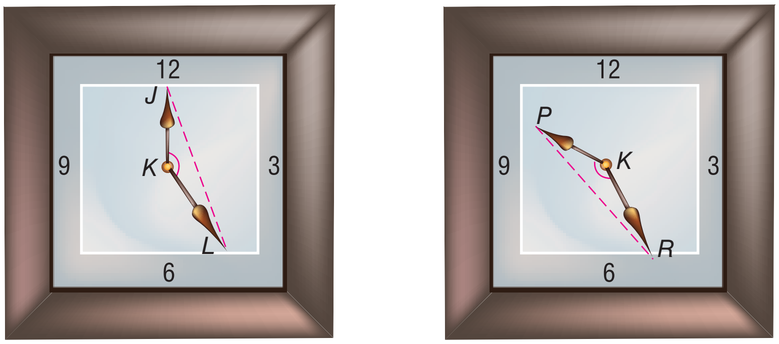
Step 2 Construct one arc with radius AB centered at point X and another arc with radius BC centered at point Z.



Step 3 Label the point of intersection of the two arcs Y. Draw $\overline{XY}$ and $\overline{ZY}$ to form $\triangle XYZ$.

2 SAS Postulate

The angle formed by two adjacent sides of a polygon is called an **included angle**. Consider included angle JKL formed by the hands on the first clock shown below. Any time the hands form an angle with the same measure, the distance between the ends of the hands $\overline{JL}$ and $\overline{PR}$ will be the same.



$\triangle PKR \cong \triangle JKL$

Any two triangles formed using the same side lengths and included angle measure will be congruent. This illustrates the following postulate.

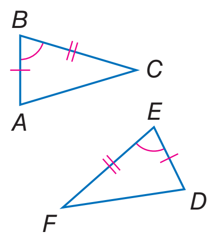
StudyTip

Side-Side-Angle The measures of two sides and a nonincluded angle are not sufficient to prove two triangles congruent.

Postulate 4.2 Side-Angle-Side (SAS) Congruence

Words If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.

Example If **S**ide $\overline{AB} \cong \overline{DE}$,
Angle $\angle B \cong \angle E$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.

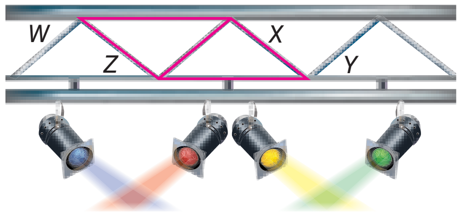


Real-World Career

Lighting Technicians In the motion picture industry, gaffers, or lighting technicians, place the lighting required for a film. Gaffers make sure the angles the lights form are in the correct positions. They may have college or technical school degrees, or they may have completed a formal training program.

Real-World Example 3 Use SAS to Prove Triangles are Congruent

LIGHTING The scaffolding for stage lighting shown appears to be made up of congruent triangles. If $\overline{WX} \cong \overline{YZ}$ and $\overline{WX} \parallel \overline{ZY}$, write a two-column proof to prove that $\triangle WXZ \cong \triangle YZX$.

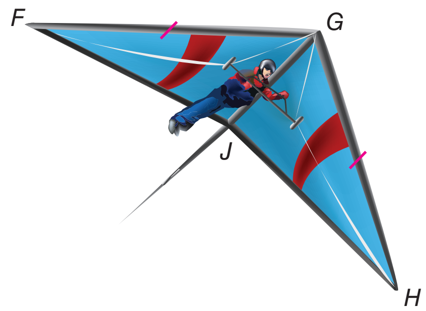


Proof:

Statements	Reasons
1. $\overline{WX} \cong \overline{YZ}$	**1.** Given
2. $\overline{WX} \parallel \overline{ZY}$	**2.** Given
3. $\angle WXZ \cong \angle XZY$	**3.** Alternate Interior Angle Theorem
4. $\overline{XZ} \cong \overline{ZX}$	**4.** Reflexive Property of Congruence
5. $\triangle WXZ \cong \triangle YZX$	**5.** SAS

Guided Practice

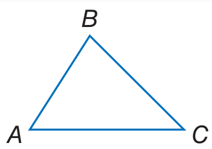
3. **EXTREME SPORTS** The wings of the hang glider shown appear to be congruent triangles. If $\overline{FG} \cong \overline{GH}$ and $\overline{JG}$ bisects $\angle FGH$, prove that $\triangle FGJ \cong \triangle HGJ$.

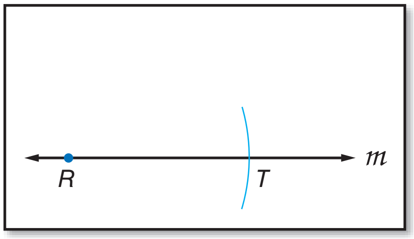


You can also construct congruent triangles given two sides and the included angle.

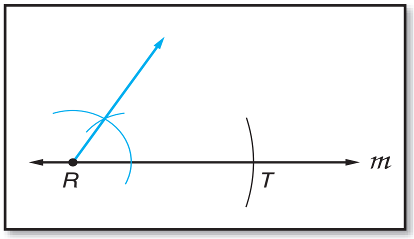
Construction Congruent Triangles Using Two Sides and the Included Angle

Draw a triangle and label it $\triangle ABC$. Then use the SAS Postulate to construct $\triangle RST \cong \triangle ABC$.

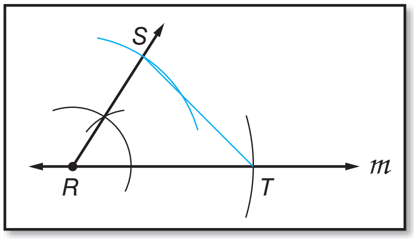




Step 1 Draw point R on a line m. Then construct $\overline{RT} \cong \overline{AC}$ on line m.



Step 2 Construct $\angle R \cong \angle A$ using $\overline{RT}$ as a side of the angle and point R.

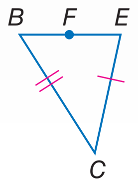


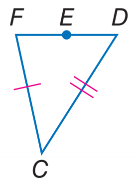
Step 3 Construct $\overline{RS} \cong \overline{AB}$. Then draw $\overline{ST}$ to form $\triangle RST$.

Example 4 Use SAS or SSS in Proofs

StudyTip

Overlapping Figures When triangles overlap, it can be helpful to draw each triangle separately and label the congruent parts. In Example 4, the figure could have been separated as shown.

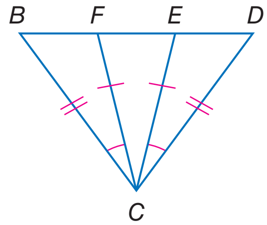




Write a paragraph proof.

Given: $\overline{BC} \cong \overline{DC}$, $\angle BCF \cong \angle DCE$, $\overline{FC} \cong \overline{EC}$

Prove: $\angle CFD \cong \angle CEB$



Proof:

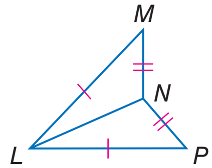
Since $\overline{BC} \cong \overline{DC}$, $\angle BCF \cong \angle DCE$, and $\overline{FC} \cong \overline{EC}$, then $\triangle BCF \cong \triangle DCE$ by SAS. By CPCTC, $\angle CFB \cong \angle CED$. $\angle CFD$ forms a linear pair with $\angle CFB$, and $\angle CEB$ forms a linear pair with $\angle CED$. By the Congruent Supplements Theorem, $\angle CFD$ is supplementary to $\angle CFB$ and $\angle CEB$ is supplementary to $\angle CED$. Since angles supplementary to the same angle or congruent angles are congruent, $\angle CFD \cong \angle CEB$.

GuidedPractice

4. Write a two-column proof.

Given: $\overline{MN} \cong \overline{PN}$, $\overline{LM} \cong \overline{LP}$

Prove: $\angle LNM \cong \angle LNP$

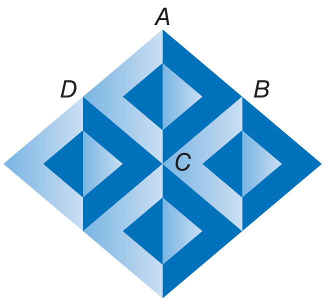


Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1

1. OPTICAL ILLUSION The figure shown is a pattern formed using four large congruent squares and four small congruent squares.



a. How many different-sized triangles are used to create the illusion?

b. Use the Side-Side-Side Congruence Postulate to prove that $\triangle ABC \cong \triangle CDA$.

c. What is the relationship between $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$? Explain your reasoning.

Example 2

2. EXTENDED RESPONSE Triangle ABC has vertices $A(-3, -5)$, $B(-1, -1)$, and $C(-1, -5)$. Triangle XYZ has vertices $X(5, -5)$, $Y(3, -1)$, and $Z(3, -5)$.

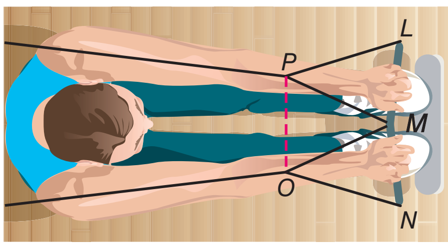
a. Graph both triangles on the same coordinate plane.

b. Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning.

c. Write a logical argument using coordinate geometry to support your conjecture.

Example 3

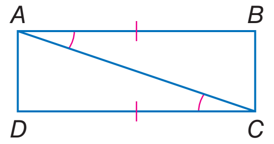
3. EXERCISE In the exercise diagram, if $\overline{LP} \cong \overline{NO}$, $\angle LPM \cong \angle NOM$, and $\triangle MOP$ is equilateral, write a paragraph proof to show that $\triangle LMP \cong \triangle NMO$.



Example 4 **4.** **Write a two-column proof.**

Given: $\overline{BA} \cong \overline{DC}$, $\angle BAC \cong \angle DCA$

Prove: $\overline{BC} \cong \overline{DA}$



Practice and Problem Solving

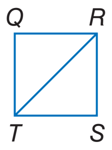
Extra Practice is on page R4.

Example 1 **PROOF** **Write the specified type of proof.**

5. paragraph proof

Given: $\overline{QR} \cong \overline{SR}$,
$\overline{ST} \cong \overline{QT}$

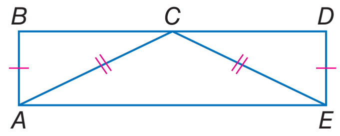
Prove: $\triangle QRT \cong \triangle SRT$



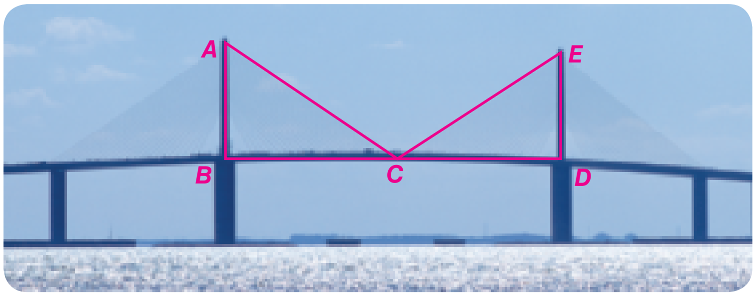
6. two-column proof

Given: $\overline{AB} \cong \overline{ED}$, $\overline{CA} \cong \overline{CE}$;
$\overline{AC}$ bisects $\overline{BD}$.

Prove: $\triangle ABC \cong \triangle EDC$



7. **BRIDGES** The Sunshine Skyway Bridge in Florida is the world's longest cable-stayed bridge, spanning 4.1 miles of Tampa Bay. It is supported using steel cables suspended from two concrete supports. If the supports are the same height above the roadway and perpendicular to the roadway, and the topmost cables meet at a point midway between the supports, prove that the two triangles shown in the photo are congruent.



Example 2 **SENSE-MAKING** **Determine whether $\triangle MNO \cong \triangle QRS$. Explain.**

8. $M(2, 5), N(5, 2), O(1, 1), Q(-4, 4), R(-7, 1), S(-3, 0)$

9. $M(0, -1), N(-1, -4), O(-4, -3), Q(3, -3), R(4, -4), S(3, 3)$

10. $M(0, -3), N(1, 4), O(3, 1), Q(4, -1), R(6, 1), S(9, -1)$

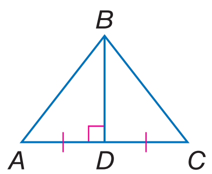
11. $M(4, 7), N(5, 4), O(2, 3), Q(2, 5), R(3, 2), S(0, 1)$

Example 3 **PROOF** **Write the specified type of proof.**

12. two-column proof

Given: $\overline{BD} \perp \overline{AC}$,
$\overline{BD}$ bisects $\overline{AC}$.

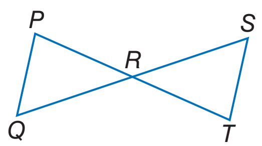
Prove: $\triangle ABD \cong \triangle CBD$



13. paragraph proof

Given: R is the midpoint of
$\overline{QS}$ and $\overline{PT}$.

Prove: $\triangle PRQ \cong \triangle TRS$

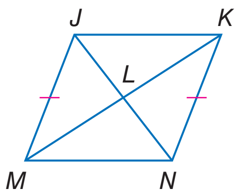


Example 4 **PROOF** **Write the specified type of proof.**

14. flow proof

Given: $\overline{JM} \cong \overline{NK}$; L is the midpoint of $\overline{JN}$ and $\overline{KM}$.

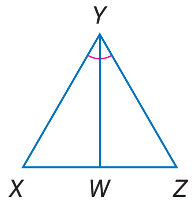
Prove: $\angle MJL \cong \angle KNL$



15. paragraph proof

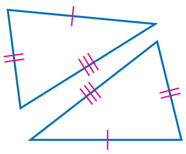
Given: $\triangle XYZ$ is equilateral. $\overline{WY}$ bisects $\angle XYZ$.

Prove: $\overline{XW} \cong \overline{ZW}$



CCSS ARGUMENTS **Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write *not possible*.**

16.



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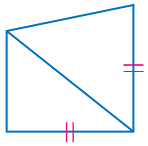

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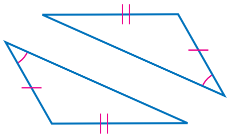

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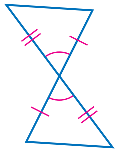
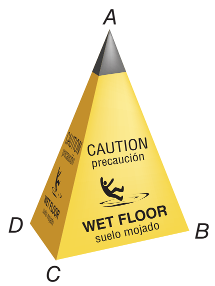


20. **SIGNS** Refer to the diagram at the right.

a. Identify the three-dimensional figure represented by the wet floor sign.

b. If $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{DC}$, prove that $\triangle ACB \cong \triangle ACD$.

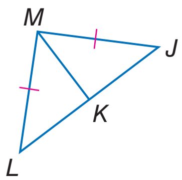
c. Why do the triangles not look congruent in the diagram?



PROOF **Write a flow proof.**

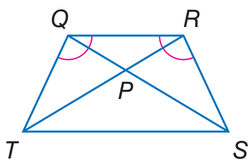
21. **Given:** $\overline{MJ} \cong \overline{ML}$; K is the midpoint of $\overline{JL}$.

Prove: $\triangle MJK \cong \triangle MLK$



22. **Given:** $\triangle TPQ \cong \triangle SPR$, $\angle TQR \cong \angle SRQ$

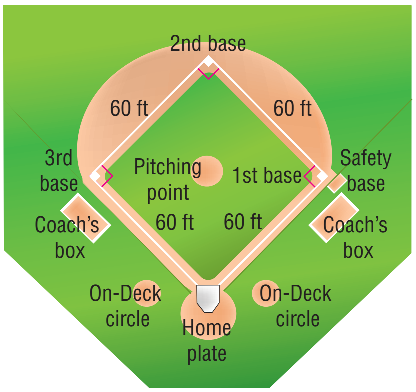
Prove: $\triangle TQR \cong \triangle SRQ$



23. **SOFTBALL** Use the diagram of a fast-pitch softball diamond shown. Let F = first base, S = second base, T = third base, P = pitching point, and R = home plate.

a. Write a two-column proof to prove that the distance from first base to third base is the same as the distance from home plate to second base.

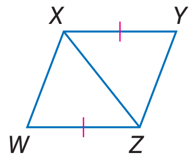
b. Write a two-column proof to prove that the angle formed between second base, home plate, and third base is the same as the angle formed between second base, home plate, and first base.



PROOF **Write a two-column proof.**

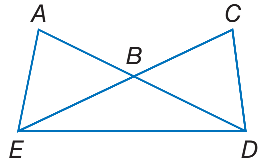
24. **Given:** $\overline{YX} \cong \overline{WZ}$, $\overline{YX} \parallel \overline{ZW}$

Prove: $\triangle YXZ \cong \triangle WZX$



25. **Given:** $\triangle EAB \cong \triangle DCB$

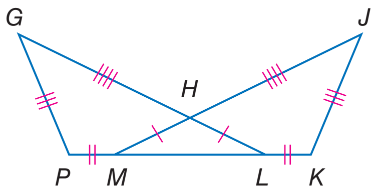
Prove: $\triangle EAD \cong \triangle DCE$



26. CCSS **ARGUMENTS** Write a paragraph proof.

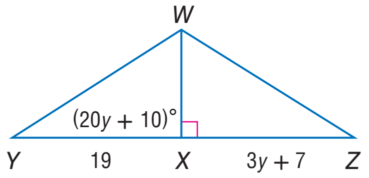
Given: $\overline{HL} \cong \overline{HM}$, $\overline{PM} \cong \overline{KL}$, $\overline{PG} \cong \overline{KJ}$, $\overline{GH} \cong \overline{JH}$

Prove: $\angle G \cong \angle J$

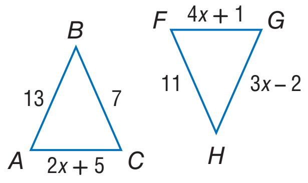


ALGEBRA **Find the value of the variable that yields congruent triangles. Explain.**

27 $\triangle WXY \cong \triangle WXZ$

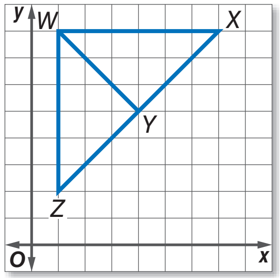


28. $\triangle ABC \cong \triangle FGH$



H.O.T. Problems Use Higher-Order Thinking Skills

29. **CHALLENGE** **Refer to the graph shown.**



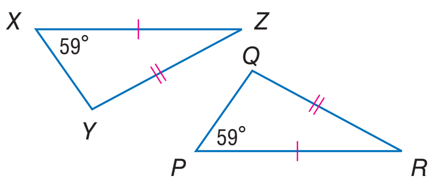
a. Describe two methods you could use to prove that $\triangle WYZ$ is congruent to $\triangle WYX$. You may not use a ruler or a protractor. Which method do you think is more efficient? Explain.

b. Are $\triangle WYZ$ and $\triangle WYX$ congruent? Explain your reasoning.

30. **REASONING** Determine whether the following statement is *true* or *false*. If true, explain your reasoning. If *false*, provide a counterexample.

If the congruent sides in one isosceles triangle have the same measure as the congruent sides in another isosceles triangle, then the triangles are congruent.

31. **ERROR ANALYSIS** Bonnie says that $\triangle PQR \cong \triangle XYZ$ by SAS. Shada disagrees. She says that there is not enough information to prove that the two triangles are congruent. Is either of them correct? Explain.



32. **OPEN ENDED** Use a straightedge to draw obtuse triangle ABC. Then construct $\triangle XYZ$ so that it is congruent to $\triangle ABC$ using either SSS or SAS. Justify your construction mathematically and verify it using measurement.

33. **WRITING IN MATH** Two pairs of corresponding sides of two right triangles are congruent. Are the triangles congruent? Explain your reasoning.

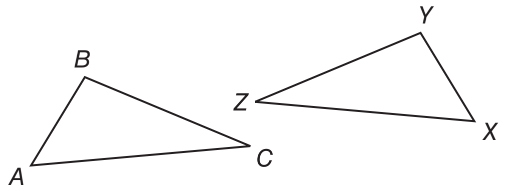
Standardized Test Practice

34. ALGEBRA The Ross Family drove 300 miles to visit their grandparents. Mrs. Ross drove 70 miles per hour for 65% of the trip and 35 miles per hour or less for 20% of the trip that was left. Assuming that Mrs. Ross never went over 70 miles per hour, how many miles did she travel at a speed between 35 and 70 miles per hour?

A 195 **C** 21

B 84 **D** 18

35. In the figure, $\angle C \cong \angle Z$ and $\overline{AC} \cong \overline{XZ}$.



What additional information could be used to prove that $\triangle ABC \cong \triangle XYZ$?

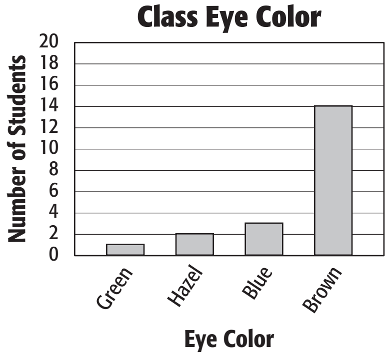
F $\overline{BC} \cong \overline{YZ}$

G $\overline{AB} \cong \overline{XY}$

H $\overline{BC} \cong \overline{XZ}$

J $\overline{XZ} \cong \overline{XY}$

36. EXTENDED RESPONSE The graph below shows the eye colors of all of the students in a class. What is the probability that a student chosen at random from this class will have blue eyes? Explain your reasoning.



37. SAT/ACT If $4a + 6b = 6$ and $-2a + b = -7$, what is the value of a?

A −2

B −1

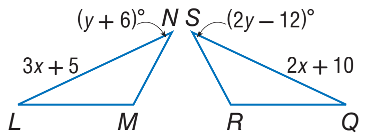
C 2

D 3

E 4

Spiral Review

In the diagram, $\triangle LMN \cong \triangle QRS$. (Lesson 4-3)



38. Find x.

39. Find y.

40. ASTRONOMY The Big Dipper is a part of the larger constellation Ursa Major. Three of the brighter stars in the constellation form $\triangle RSA$. If $m\angle R = 41$ and $m\angle S = 109$, find $m\angle A$. (Lesson 4-2)

Write an equation in slope-intercept form for each line. (Lesson 3-4)

41. (−5, −3) and (10, −6)

42. (4, −1) and (−2, −1)

43. (−4, −1) and (−8, −5)

Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample. (Lesson 2-3)

44. If $x^2 = 25$, then $x = 5$.

45. If you are 16, you are a junior in high school.

Skills Review

State the property that justifies each statement.

46. $AB = AB$

47. If $EF = GH$ and $GH = JK$, then $EF = JK$.

48. If $a^2 = b^2 - c^2$, then $b^2 - c^2 = a^2$.

49. If $XY + 20 = YW$ and $XY + 20 = DT$, then $YW = DT$.