LESSON 4-5

# Proving Triangles Congruent—ASA, AAS

**Then**

- You proved triangles congruent using SSS and SAS.

**Now**

1. Use the ASA Postulate to test for congruence.
2. Use the AAS Theorem to test for congruence.

**Why?**

- Competitive sweep rowing, also called *crew*, involves two or more people who sit facing the stern of the boat, with each rower pulling one oar. In high school competitions, a race, called a *regatta*, usually requires a body of water that is more than 1500 meters long. Congruent triangles can be used to measure distances that are not easily measured directly, like the length of a regatta course.

**New Vocabulary**

included side

**Common Core State Standards**

**Content Standards**

G.CO.10 Prove theorems about triangles.

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

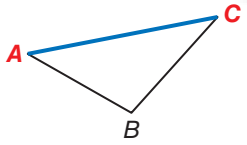
**Mathematical Practices**

3 Construct viable arguments and critique the reasoning of others.

5 Use appropriate tools strategically.

## 1 ASA Postulate

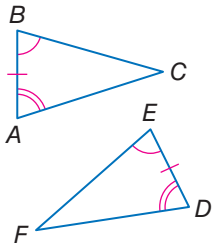
An **included side** is the side located between two consecutive angles of a polygon. In $\triangle ABC$ at the right, $\overline{AC}$ is the included side between $\angle A$ and $\angle C$.




### Postulate 4.3 Angle-Side-Angle (ASA) Congruence

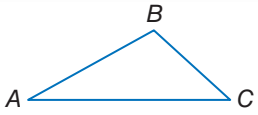
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

**Example** If **A**ngle $\angle A \cong \angle D$,
**S**ide $\overline{AB} \cong \overline{DE}$, and
**A**ngle $\angle B \cong \angle E$,
then $\triangle ABC \cong \triangle DEF$.




### Construction Congruent Triangles Using Two Angles and Included Side

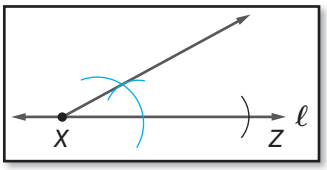
Draw a triangle and label it $\triangle ABC$. Then use the ASA Postulate to construct $\triangle XYZ \cong \triangle ABC$.




**Step 1**

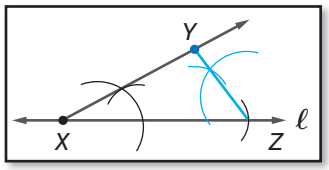

Draw a line $\ell$ and select a point $X$. Construct $\overline{XZ}$ such that $\overline{XZ} \cong \overline{AC}$.

**Step 2**




Construct an angle congruent to $\angle A$ at $X$ using $\overleftrightarrow{XZ}$ as a side of the angle.

**Step 3**




Construct an angle congruent to $\angle C$ at $Z$ using $\overleftrightarrow{XZ}$ as a side of the angle. Label the point where the new sides of the angles meet as $Y$.

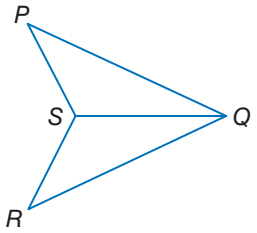
Ben Blankenburg/CORBIS

## Example 1 Use ASA to Prove Triangles Congruent

**Write a two-column proof.**

**Given:** $\overline{QS}$ bisects $\angle PQR$; $\angle PSQ \cong \angle RSQ$.

**Prove:** $\triangle PQS \cong \triangle RQS$




**Proof:**

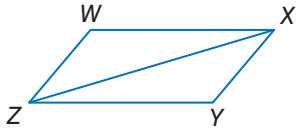
| Statements | Reasons |
|---|---|
| 1. $\overline{QS}$ bisects $\angle PQR$; $\angle PSQ \cong \angle RSQ$. | 1. Given |
| 2. $\angle PQS \cong \angle RQS$ | 2. Definition of Angle Bisector |
| 3. $\overline{QS} \cong \overline{QS}$ | 3. Reflexive Property of Congruence |
| 4. $\triangle PQS \cong \triangle RQS$ | 4. ASA |

### GuidedPractice

1. Write a flow proof.

**Given:** $\overline{ZX}$ bisects $\angle WZY$; $\overline{XZ}$ bisects $\angle YXW$.

**Prove:** $\triangle WXZ \cong \triangle XZY$

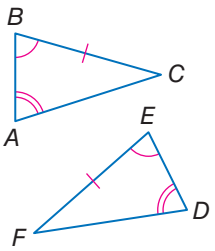



**2 AAS Theorem** The congruence of two angles and a nonincluded side are also sufficient to prove two triangles congruent. This congruence relationship is a theorem because it can be proved using the Third Angles Theorem.

## Theorem 4.5 Angle-Angle-Side (AAS) Congruence

If two angles and the nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

**Example** If Angle $\angle A \cong \angle D$,
Angle $\angle B \cong \angle E$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.

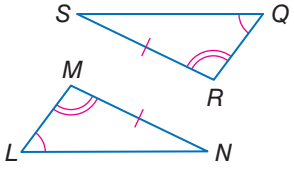


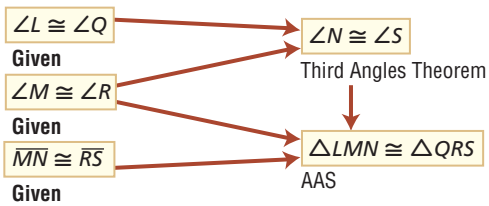

## Proof Angle-Angle-Side Theorem

**Given:** $\angle L \cong \angle Q$, $\angle M \cong \angle R$, $\overline{MN} \cong \overline{RS}$

**Prove:** $\triangle LMN \cong \triangle QRS$

**Proof:**



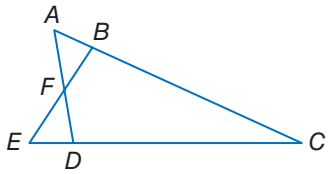



## Example 2 Use AAS to Prove Triangles Congruent

**Write a two-column proof.**

**Given:** $\angle DAC \cong \angle BEC$
$\overline{DC} \cong \overline{BC}$

**Prove:** $\triangle ACD \cong \triangle ECB$

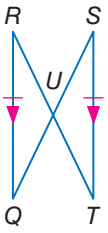



**Proof:** We are given that $\angle DAC \cong \angle BEC$ and $\overline{DC} \cong \overline{BC}$. $\angle C \cong \angle C$ by the Reflexive Property. By AAS, $\triangle ACD \cong \triangle ECB$.

### GuidedPractice

**2.** Write a flow proof.
**Given:** $\overline{RQ} \cong \overline{ST}$ and $\overline{RQ} \parallel \overline{ST}$

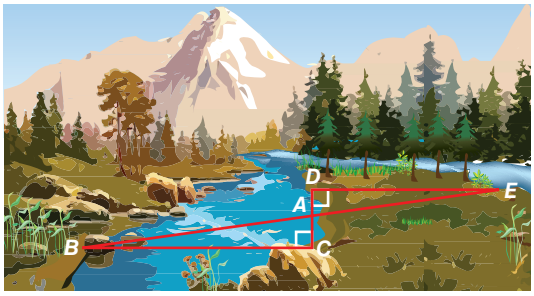
**Prove:** $\triangle RUQ \cong \triangle TUS$




You can use congruent triangles to measure distances that are difficult to measure directly.

## Real-World Example 3 Apply Triangle Congruence

**COMMUNITY SERVICE** **Jeremias is working with a community service group to build a bridge across a creek at a local park. The bridge will span the creek between points *C* and *B*. Jeremias located a fixed point *D* to use as a reference point so that the segments have the relationships shown. *A* is the midpoint of $\overline{CD}$ and *DE* is 15 feet. How long does the bridge need to be?**




In order to determine the length of $\overline{CB}$, we must first prove that the two triangles Jeremias has created are congruent.

- Since $\overline{CD}$ is perpendicular to both $\overline{CB}$ and $\overline{DE}$, the segments form right angles as shown on the diagram.
- All right angles are congruent, so $\angle BCA \cong \angle EDA$.
- Point *A* is the midpoint of $\overline{CD}$, so $\overline{CA} \cong \overline{AD}$.
- $\angle BAC$ and $\angle EAD$ are vertical angles, so they are congruent.

Therefore, by ASA, $\triangle BAC \cong \triangle EAD$.

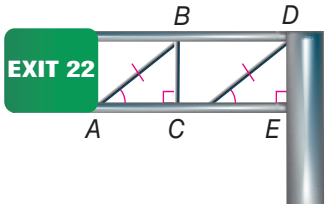
Since $\triangle BAC \cong \triangle EAD$, $\overline{DE} \cong \overline{CB}$ by CPCTC. Since the measure of $\overline{DE}$ is 15 feet, the measure of $\overline{CB}$ is also 15 feet. Therefore, the bridge needs to be 15 feet long.

**StudyTip**

**Angle-Angle-Angle** In Example 3, $\angle B$ and $\angle E$ are congruent by the Third Angles Theorem. Congruence of all three corresponding angles is not sufficient, however, to prove two triangles congruent.

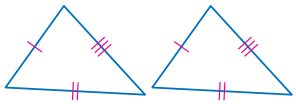
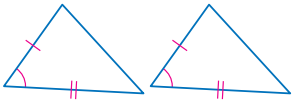
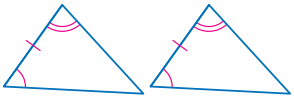
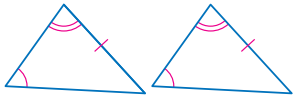
## GuidedPractice

**3.** In the sign scaffold shown at the right, $\overline{BC} \perp \overline{AC}$ and $\overline{DE} \perp \overline{CE}$. $\angle BAC \cong \angle DCE$, and $\overline{AB} \cong \overline{CD}$. Write a paragraph proof to show that $\overline{BC} \cong \overline{DE}$.




You have learned several methods for proving triangle congruence.

## ConceptSummary Proving Triangles Congruent

| SSS | SAS | ASA | AAS |
|---|---|---|---|
|  |  |  |  |
| Three pairs of corresponding sides are congruent. | Two pairs of corresponding sides and their included angles are congruent. | Two pairs of corresponding angles and their included sides are congruent. | Two pairs of corresponding angles and the corresponding nonincluded sides are congruent. |

## Check Your Understanding

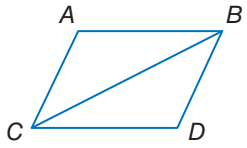
= Step-by-Step Solutions begin on page R14.

**Example 1** **PROOF** **Write the specified type of proof.**

**1.** two-column proof

**Given:** $\overline{CB}$ bisects $\angle ABD$ and $\angle ACD$.

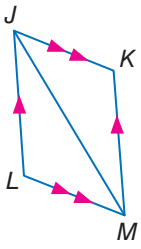
**Prove:** $\triangle ABC \cong \triangle DBC$




**2.** flow proof

**Given:** $\overline{JK} \parallel \overline{LM}$, $\overline{JL} \parallel \overline{KM}$

**Prove:** $\triangle JML \cong \triangle MJK$

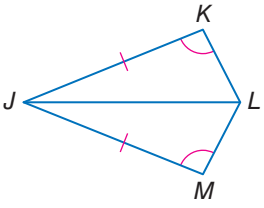



**Example 2**

**3.** paragraph proof

**Given:** $\angle K \cong \angle M$, $\overline{JK} \cong \overline{JM}$, $\overline{JL}$ bisects $\angle KLM$.

**Prove:** $\triangle JKL \cong \triangle JML$

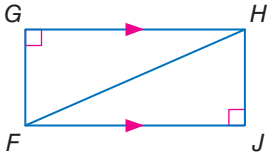



**4.** two-column proof

**Given:** $\overline{GH} \parallel \overline{FJ}$

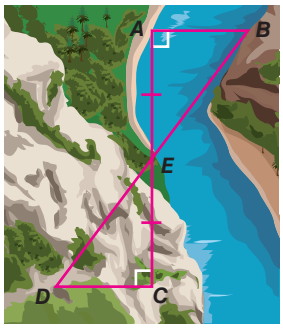
$m\angle G = m\angle J = 90$

**Prove:** $\triangle HJF \cong \triangle FGH$



**Example 3**

**5** **BRIDGE BUILDING** A surveyor needs to find the distance from point $A$ to point $B$ across a canyon. She places a stake at $A$, and a coworker places a stake at $B$ on the other side of the canyon. The surveyor then locates $C$ on the same side of the canyon as $A$ such that $\overline{CA} \perp \overline{AB}$. A fourth stake is placed at $E$, the midpoint of $\overline{CA}$. Finally, a stake is placed at $D$ such that $\overline{CD} \perp \overline{CA}$ and $D$, $E$, and $B$ are sited as lying along the same line.




**a.** Explain how the surveyor can use the triangles formed to find $AB$.

**b.** If $AC = 1300$ meters, $DC = 550$ meters, and $DE = 851.5$ meters, what is $AB$? Explain your reasoning.

## Practice and Problem Solving

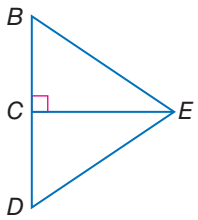
Extra Practice is on page R4.

**Example 1**

**PROOF** **Write a paragraph proof.**

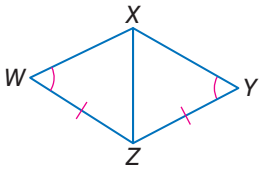
**6.** **Given:** $\overline{CE}$ bisects $\angle BED$; $\angle BCE$ and $\angle ECD$ are right angles.

**Prove:** $\triangle ECB \cong \triangle ECD$

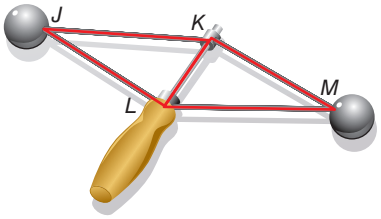



**7.** **Given:** $\angle W \cong \angle Y$, $\overline{WZ} \cong \overline{YZ}$, $\overline{XZ}$ bisects $\angle WZY$.

**Prove:** $\triangle XWZ \cong \triangle XYZ$




**8.** **TOYS** The object of the toy shown is to make the two spheres meet and strike each other repeatedly on one side of the wand and then again on the other side. If $\angle JKL \cong \angle MLK$ and $\angle JLK \cong \angle MKL$, prove that $\overline{JK} \cong \overline{ML}$.

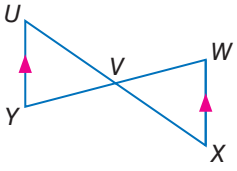



**Example 2**

**PROOF** **Write a two-column proof.**

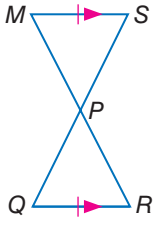
**9** **Given:** $V$ is the midpoint of $\overline{YW}$; $\overline{UY} \parallel \overline{XW}$.

**Prove:** $\triangle UVY \cong \triangle XVW$




**10.** **Given:** $\overline{MS} \cong \overline{RQ}$, $\overline{MS} \parallel \overline{RQ}$

**Prove:** $\triangle MSP \cong \triangle RQP$

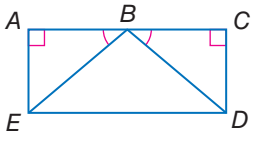



**11.** CCSS **ARGUMENTS** Write a flow proof.

**Given:** $\angle A$ and $\angle C$ are right angles.

$\angle ABE \cong \angle CBD$, $\overline{AE} \cong \overline{CD}$

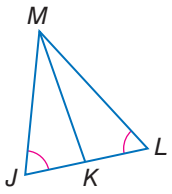
**Prove:** $\overline{BE} \cong \overline{BD}$



**12. PROOF** Write a flow proof.

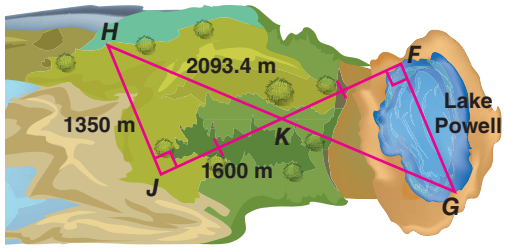
**Given:** $\overline{KM}$ bisects $\angle JML$; $\angle J \cong \angle L$.

**Prove:** $\overline{JM} \cong \overline{LM}$




Example 3

**13. CCSS MODELING** A high school wants to hold a 1500-meter regatta on Lake Powell but is unsure if the lake is long enough. To measure the distance across the lake, the crew members locate the vertices of the triangles below and find the measures of the lengths of $\triangle HJK$ as shown below.

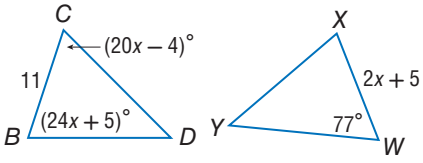



**a.** Explain how the crew team can use the triangles formed to estimate the distance $FG$ across the lake.

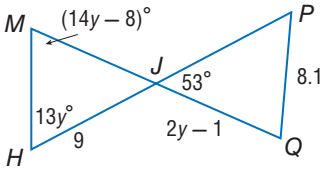
**b.** Using the measures given, is the lake long enough for the team to use as the location for their regatta? Explain your reasoning.

**ALGEBRA Find the value of the variable that yields congruent triangles.**

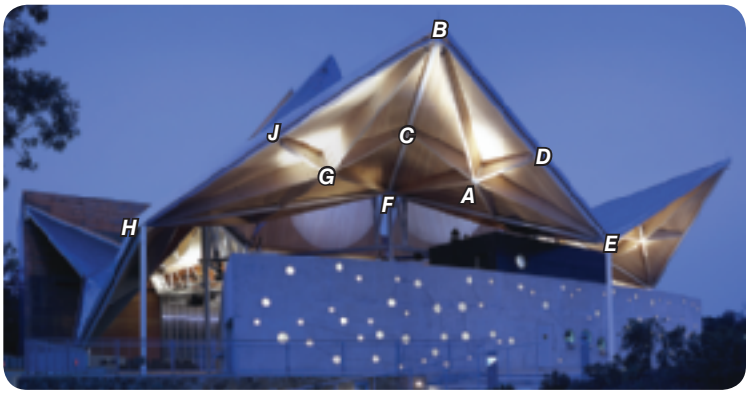
**14.** $\triangle BCD \cong \triangle WXY$




**15** $\triangle MHJ \cong \triangle PQJ$




**16. THEATER DESIGN** The trusses of the roof of the outdoor theater shown below appear to be several different pairs of congruent triangles. Assume that trusses that appear to lie on the same line actually lie on the same line.




**a.** If $\overline{AB}$ bisects $\angle CBD$ and $\angle CAD$, prove that $\triangle ABC \cong \triangle ABD$.

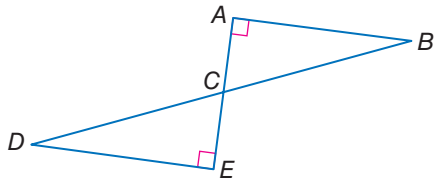
**b.** If $\triangle ABC \cong \triangle ABD$ and $\angle FCA \cong \angle EDA$, prove that $\triangle CAF \cong \triangle DAE$.

**c.** If $\overline{HB} \cong \overline{EB}$, $\angle BHG \cong \angle BEA$, $\angle HGJ \cong \angle EAD$, and $\angle JGB \cong \angle DAB$, prove that $\triangle BHG \cong \triangle BEA$.

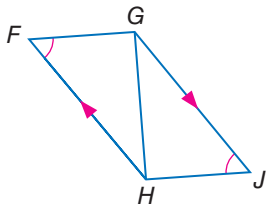
Greg Murphey

**PROOF** Write a paragraph proof.

**17.** **Given:** $\overline{AE} \perp \overline{DE}$, $\overline{EA} \perp \overline{AB}$, $C$ is the midpoint of $\overline{AE}$.
**Prove:** $\overline{CD} \cong \overline{CB}$

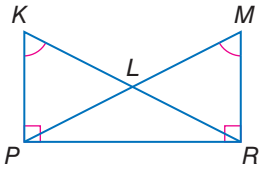



**18.** **Given:** $\angle F \cong \angle J$, $\overline{FH} \parallel \overline{GJ}$
**Prove:** $\overline{FH} \cong \overline{JG}$

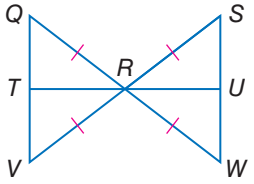



**PROOF** Write a two-column proof.

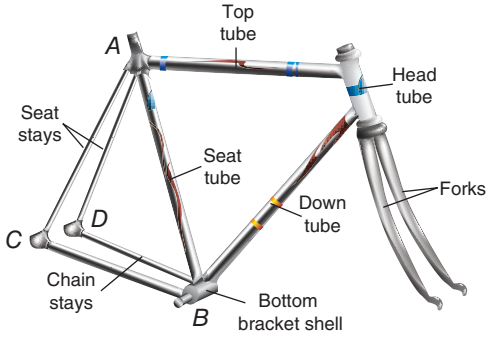
**19.** **Given:** $\angle K \cong \angle M$, $\overline{KP} \perp \overline{PR}$, $\overline{MR} \perp \overline{PR}$
**Prove:** $\angle KPL \cong \angle MRL$




**20.** **Given:** $\overline{QR} \cong \overline{SR} \cong \overline{WR} \cong \overline{VR}$
**Prove:** $\overline{QT} \cong \overline{WU}$



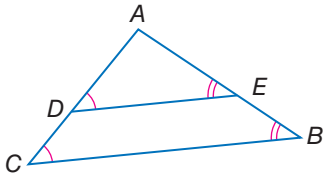

**21** **FITNESS** The seat tube of a bicycle forms a triangle with each seat and chain stay as shown. If each seat stay makes a 44° angle with its corresponding chain stay and each chain stay makes a 68° angle with the seat tube, show that the two seat stays are the same length.




## H.O.T. Problems Use Higher-Order Thinking Skills

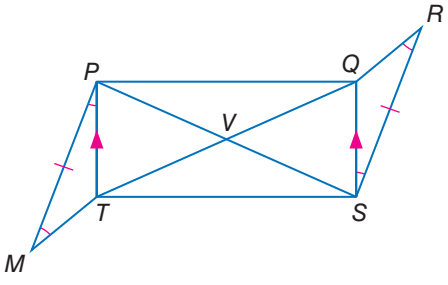
**22.** **OPEN ENDED** Draw and label two triangles that could be proved congruent by ASA.

**23.** **CCSS CRITIQUE** Tyrone says it is not possible to show that $\triangle ADE \cong \triangle ACB$. Lorenzo disagrees, explaining that since $\angle ADE \cong \angle ACB$, and $\angle A \cong \angle A$ by the Reflexive Property, $\triangle ADE \cong \triangle ACB$. Is either of them correct? Explain.




**24.** **REASONING** Find a counterexample to show why SSA (Side-Side-Angle) cannot be used to prove the congruence of two triangles.

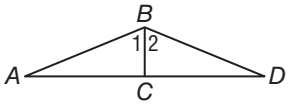
**25.** **CHALLENGE** Using the information given in the diagram, write a flow proof to show that $\triangle PVQ \cong \triangle SVT$.




**26.** **WRITING IN MATH** How do you know what method (SSS, SAS, etc.) to use when proving triangle congruence? Use a chart to explain your reasoning.

## Standardized Test Practice

**27.** Given: $\overline{BC}$ is perpendicular to $\overline{AD}$; $\angle 1 \cong \angle 2$.




Which theorem or postulate could be used to prove $\triangle ABC \cong \triangle DBC$?

**A** AAS **C** SAS

**B** ASA **D** SSS

**28. SHORT RESPONSE** Write an expression that can be used to find the values of $s(n)$ in the table.

| $n$ | −8 | −4 | −1 | 0 | 1 |
|---|---|---|---|---|---|
| $s(n)$ | 1.00 | 2.00 | 2.75 | 3.00 | 3.25 |

**29. ALGEBRA** If −7 is multiplied by a number greater than 1, which of the following describes the result?

**F** a number greater than 7

**G** a number between −7 and 7

**H** a number greater than −7

**J** a number less than −7

**30. SAT/ACT** $\sqrt{121 + 104} = ?$

**A** 15

**B** 21

**C** 25

**D** 125

**E** 225

## Spiral Review

**Determine whether $\triangle ABC \cong \triangle XYZ$. Explain.** (Lesson 4-4)

**31.** $A(6, 4)$, $B(1, -6)$, $C(-9, 5)$, $X(0, 7)$, $Y(5, -3)$, $Z(15, 8)$

**32.** $A(0, 5)$, $B(0, 0)$, $C(-2, 0)$, $X(4, 8)$, $Y(4, 3)$, $Z(6, 3)$

**33. ALGEBRA** If $\triangle RST \cong \triangle JKL$, $RS = 7$, $ST = 5$, $RT = 9 + x$, $JL = 2x - 10$, and $JK = 4y - 5$, draw and label a figure to represent the congruent triangles. Then find $x$ and $y$. (Lesson 4-3)

**34. FINANCIAL LITERACY** Maxine charges $5 to paint a mailbox and $4 per hour to mow a lawn. Write an equation to represent the amount of money Maxine can earn from a homeowner who has his or her mailbox painted and lawn mowed. (Lesson 3-4)

**Copy and complete each truth table.** (Lesson 2-2)

**35.**

| $p$ | $q$ | $\sim p$ | $\sim p \vee q$ |
|---|---|---|---|
| F | T | | |
| T | T | | |
| F | F | | |
| T | F | | |

**36.**

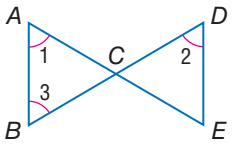
| $p$ | $q$ | $\sim q$ | $\sim q \wedge p$ |
|---|---|---|---|
| F | | F | |
| T | | T | |
| T | | F | |
| F | | T | |

## Skills Review

**PROOF Write a two-column proof for each of the following.**

**37. Given:** $\angle 2 \cong \angle 1$, $\angle 1 \cong \angle 3$

**Prove:** $\overline{AB} \parallel \overline{DE}$




**38. Given:** $\angle MJK \cong \angle KLM$, $\angle LMJ$ and $\angle KLM$ are supplementary.

**Prove:** $\overline{KJ} \parallel \overline{LM}$