LESSON 5-3

Inequalities in One Triangle

Then

- You found the relationship between the angle measures of a triangle.

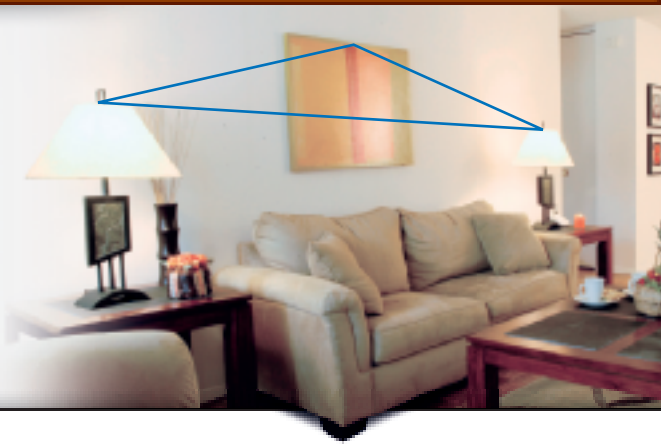
Now

1. Recognize and apply properties of inequalities to the measures of the angles of a triangle.
2. Recognize and apply properties of inequalities to the relationships between the angles and sides of a triangle.

Why?

- To create the appearance of depth in a room, interior designers use a technique called *triangulation*. A basic example of this technique is the placement of an end table on each side of a sofa with a painting over the sofa.

The measures of the base angles of the triangle should be less than the measure of the other angle.



Common Core State Standards

Content Standards

G.CO.10 Prove theorems about triangles.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

3 Construct viable arguments and critique the reasoning of others.

1 Angle Inequalities

In algebra, you learned about the inequality relationship between two real numbers. This relationship is often used in proofs.

KeyConcept Definition of Inequality

Words	For any real numbers a and b, $a > b$ if and only if there is a positive number c such that $a = b + c$.
Example	If $5 = 2 + 3$, then $5 > 2$ and $5 > 3$.

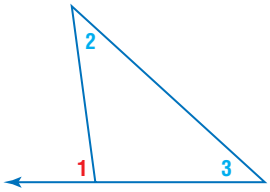
The table below lists some of the properties of inequalities you studied in algebra.

KeyConcept Properties of Inequality for Real Numbers

The following properties are true for any real numbers a, b, and c.

Comparison Property of Inequality	$a < b$, $a = b$, or $a > b$
Transitive Property of Inequality	**1.** If $a < b$ and $b < c$, then $a < c$. **2.** If $a > b$ and $b > c$, then $a > c$.
Addition Property of Inequality	**1.** If $a > b$, then $a + c > b + c$. **2.** If $a < b$, then $a + c < b + c$.
Subtraction Property of Inequality	**1.** If $a > b$, then $a - c > b - c$. **2.** If $a < b$, then $a - c < b - c$.

The definition of inequality and the properties of inequalities can be applied to the measures of angles and segments, since these are real numbers. Consider $\angle 1$, $\angle 2$, and $\angle 3$ in the figure shown.



By the Exterior Angle Theorem, you know that $m\angle 1 = m\angle 2 + m\angle 3$.

Since the angle measures are positive numbers, we can also say that

$$m\angle 1 > m\angle 2 \quad \text{and} \quad m\angle 1 > m\angle 3$$

by the definition of inequality. This result suggests the following theorem.

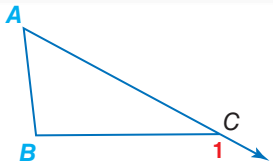
ReviewVocabulary

Remote Interior Angle Each exterior angle of a triangle has two *remote interior angles* that are not adjacent to the exterior angle.

Theorem 5.8 Exterior Angle Inequality

The measure of an exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles.

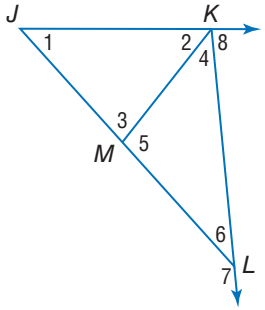
Example: $m\angle 1 > m\angle A$
$m\angle 1 > m\angle B$



The proof of Theorem 5.8 is in Lesson 5-4.

Example 1 Use the Exterior Angle Inequality Theorem

Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.



a. measures less than $m\angle 7$

$\angle 7$ is an exterior angle to $\triangle KML$, with $\angle 4$ and $\angle 5$ as corresponding remote interior angles. By the Exterior Angle Inequality Theorem, $m\angle 7 > m\angle 4$ and $m\angle 7 > m\angle 5$.

$\angle 7$ is also an exterior angle to $\triangle JKL$, with $\angle 1$ and $\angle JKL$ as corresponding remote interior angles. So, $m\angle 7 > m\angle 1$ and $m\angle 7 > m\angle JKL$. Since $m\angle JKL = m\angle 2 + m\angle 4$, by substitution $m\angle 7 > m\angle 2 + m\angle 4$. Therefore, $m\angle 7 > m\angle 2$.

So, the angles with measures less than $m\angle 7$ are $\angle 1$, $\angle 2$, $\angle 4$, $\angle 5$.

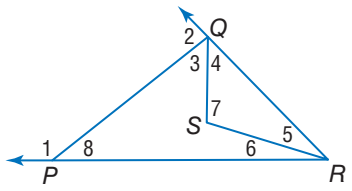
b. measures greater than $m\angle 6$

$\angle 3$ is an exterior angle to $\triangle KLM$. So by the Exterior Angle Inequality Theorem, $m\angle 3 > m\angle 6$. Because $\angle 8$ is an exterior angle to $\triangle JKL$, $m\angle 8 > m\angle 6$. Thus, the measures of $\angle 3$ and $\angle 8$ are greater than $m\angle 6$.

GuidedPractice

1A. measures less than $m\angle 1$

1B. measures greater than $m\angle 8$

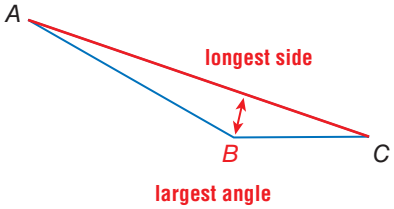


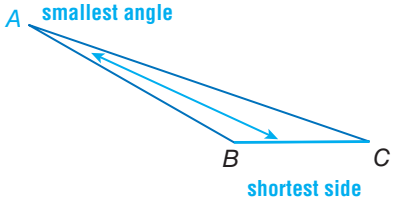
WatchOut!

Identifying Side Opposite Be careful to correctly identify the side opposite an angle. The sides that form the angle cannot be the sides opposite the angle.

2 Angle-Side Inequalities

In Lesson 4-6, you learned that if two sides of a triangle are congruent, or the triangle is isosceles, then the angles opposite those sides are congruent. What relationship exists if the sides are not congruent? Examine the longest and shortest sides and smallest and largest angles of a scalene obtuse triangle.





Notice that the longest side and largest angle of $\triangle ABC$ are opposite each other. Likewise, the shortest side and smallest angle are opposite each other.

WatchOut!

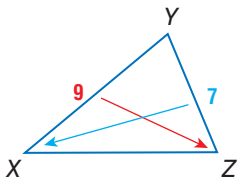
Symbols for Angles and Inequalities The symbol for angle ($\angle$) looks similar to the symbol for less than ($<$), especially when handwritten. Be careful to write the symbols correctly in situations where both are used.

The side-angle relationships in an obtuse scalene triangle are true for all triangles, and are stated using inequalities in the theorems below.

Theorems Angle-Side Relationships in Triangles

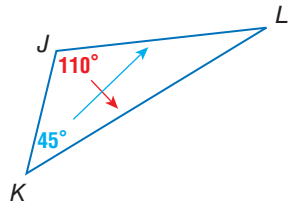
5.9 If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.

Example: $XY > YZ$, so $m\angle Z > m\angle X$.



5.10 If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.

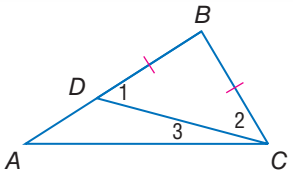
Example: $m\angle J > m\angle K$, so $KL > JL$.



Proof Theorem 5.9

Given: $\triangle ABC$, $AB > BC$

Prove: $m\angle BCA > m\angle A$



Proof:

Since $AB > BC$ in the given $\triangle ABC$, there exists a point D on $\overline{AB}$ such that $BD = BC$. Draw $\overline{CD}$ to form isosceles $\triangle BCD$. By the Isosceles Triangle Theorem, $\angle 1 \cong \angle 2$, so $m\angle 1 = m\angle 2$ by the definition of congruent angles.

By the Angle Addition Postulate, $m\angle BCA = m\angle 2 + m\angle 3$, so $m\angle BCA > m\angle 2$ by the definition of inequality. By substitution, $m\angle BCA > m\angle 1$.

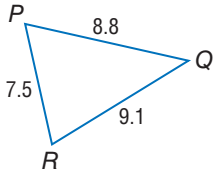
By the Exterior Angle Inequality Theorem, $m\angle 1 > m\angle A$. Therefore, because $m\angle BCA > m\angle 1$ and $m\angle 1 > m\angle A$, by the Transitive Property of Inequality, $m\angle BCA > m\angle A$.

You will prove Theorem 5.10 in Lesson 5-4, Exercise 31.

PT

Example 2 Order Triangle Angle Measures

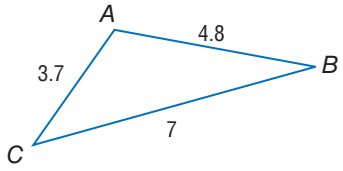
List the angles of $\triangle PQR$ in order from smallest to largest.



The sides from shortest to longest are $\overline{PR}$, $\overline{PQ}$, $\overline{QR}$. The angles opposite these sides are $\angle Q$, $\angle R$, and $\angle P$, respectively. So the angles from smallest to largest are $\angle Q$, $\angle R$, and $\angle P$.

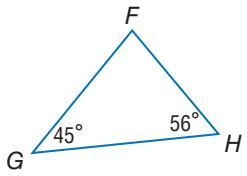
GuidedPractice

2. List the angles and sides of $\triangle ABC$ in order from smallest to largest.



Example 3 Order Triangle Side Lengths

List the sides of $\triangle FGH$ in order from shortest to longest.



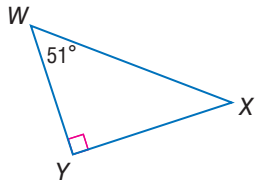
First find the missing angle measure using the Triangle Angle Sum Theorem.

$m\angle F = 180 - (45 + 56)$ or 79

So, the angles from smallest to largest are $\angle G$, $\angle H$, and $\angle F$. The sides opposite these angles are $\overline{FH}$, $\overline{FG}$, and $\overline{GH}$, respectively. So, the sides from shortest to longest are $\overline{FH}$, $\overline{FG}$, $\overline{GH}$.

GuidedPractice

3. List the angles and sides of $\triangle WXY$ in order from smallest to largest.



You can use angle-side relationships in triangles to solve real-world problems.

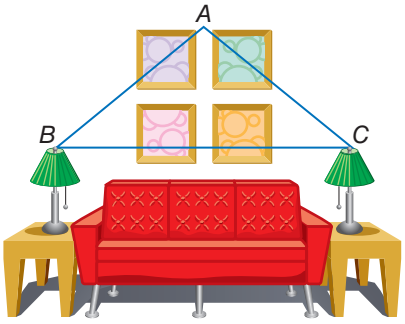
Stuart O'Sullivan/Getty Images

Real-WorldCareer

Interior Designer An interior designer decorates a space so that it is visually pleasing and comfortable for people to live or work in. Designers must know color and paint theory, lighting design, and space planning. A bachelor's degree is recommended for entry-level positions. Graduates usually enter a 1- to 3-year apprenticeship before taking a licensing exam.

Real-World Example 4 Angle-Side Relationships

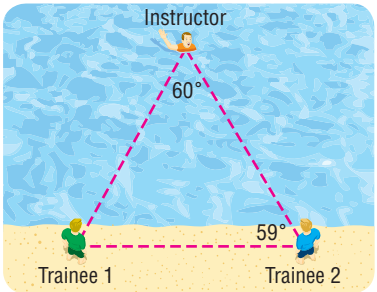
INTERIOR DESIGN **An interior designer uses triangulation to create depth in a client's living room. If $m\angle B$ is to be less than $m\angle A$, which distance should be longer—the distance between the two lamps or the distance from the lamp at B to the midpoint of the top of the artwork? Explain.**



According to Theorem 5.10, in order for $m\angle B < m\angle A$, the length of the side opposite $\angle B$ must be less than the length of the side opposite $\angle A$. Since $\overline{AC}$ is opposite $\angle B$, and $\overline{BC}$ is opposite $\angle A$, then $AC < BC$ and $BC > AC$. So BC, the distance between the lamps, must be greater than the distance from the lamp at B to the midpoint of the top of the artwork.

GuidedPractice

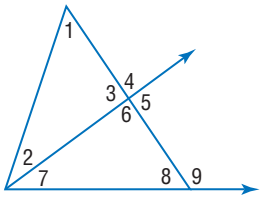
4. LIFEGUARDING During lifeguard training, an instructor simulates a person in distress so that trainees can practice their rescue skills. If the instructor, Trainee 1, and Trainee 2 are located in the positions shown on the diagram, which of the two trainees is closest to the instructor?



Check Your Understanding

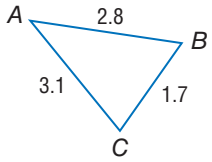
= Step-by-Step Solutions begin on page R14.

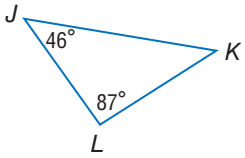
Example 1 **Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.**



1. measures less than $m\angle 4$
2. measures greater than $m\angle 7$
3. measures greater than $m\angle 2$
4. measures less than $m\angle 9$

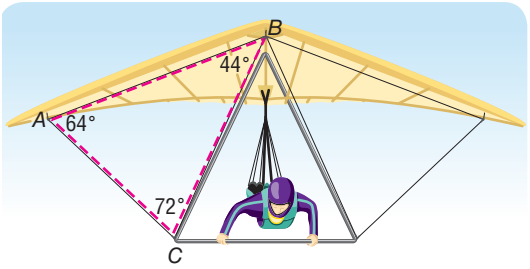
Examples 2–3 **List the angles and sides of each triangle in order from smallest to largest.**

5. 

6. 

Example 4

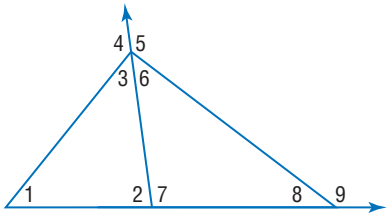
7. **HANG GLIDING** The supports on a hang glider form triangles like the one shown. Which is longer—the support represented by $\overline{AC}$ or the support represented by $\overline{BC}$? Explain your reasoning.



Practice and Problem Solving

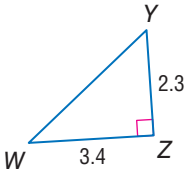
Extra Practice is on page R5.

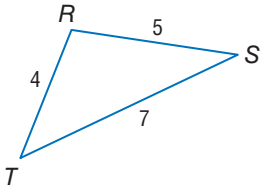
Example 1 CCSS **SENSE-MAKING** **Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.**

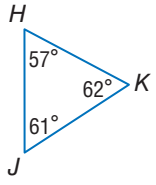


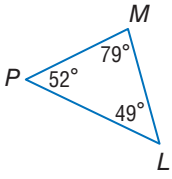
8. measures greater than $m\angle 2$
9. measures less than $m\angle 4$
10. measures less than $m\angle 5$
11. measures less than $m\angle 9$
12. measures greater than $m\angle 8$
13. measures greater than $m\angle 7$

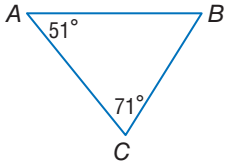
Examples 2–3 **List the angles and sides of each triangle in order from smallest to largest.**

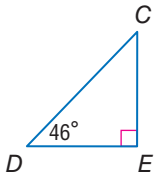
14. 

15. 

16. 

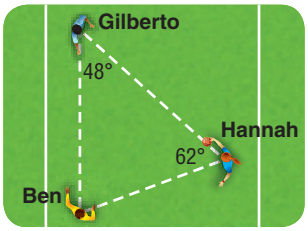
17. 

18. 

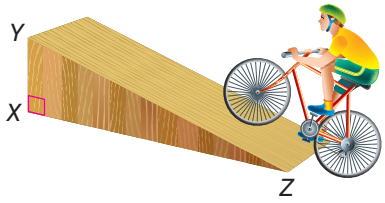
19. 

Example 4

20. SPORTS Ben, Gilberto, and Hannah are playing Ultimate. Hannah is trying to decide if she should pass to Ben or Gilberto. Which player should she choose in order to have the shorter passing distance? Explain your reasoning.

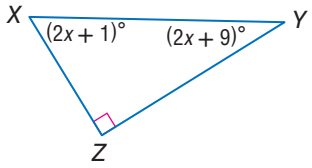


21. RAMPS The wedge below represents a bike ramp. Which is longer, the length of the ramp $\overline{XZ}$ or the length of the top surface of the ramp $\overline{YZ}$? Explain your reasoning using Theorem 5.9.

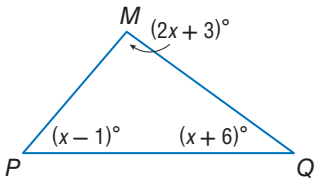


List the angles and sides of each triangle in order from smallest to largest.

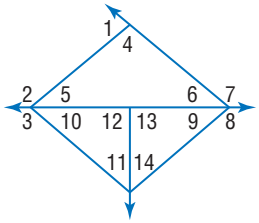
22.



23.



Use the figure at the right to determine which angle has the greatest measure.



24. $\angle 1, \angle 5, \angle 6$

25. $\angle 2, \angle 4, \angle 6$

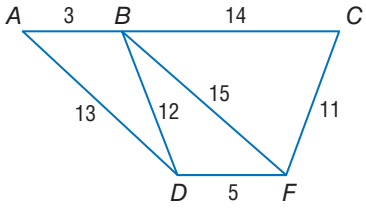
26. $\angle 7, \angle 4, \angle 5$

27. $\angle 3, \angle 11, \angle 12$

28. $\angle 3, \angle 9, \angle 14$

29. $\angle 8, \angle 10, \angle 11$

CCSS SENSE-MAKING **Use the figure at the right to determine the relationship between the measures of the given angles.**



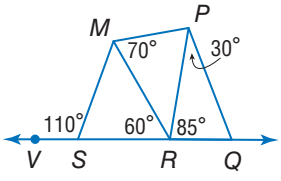
30. $\angle ABD, \angle BDA$

31. $\angle BCF, \angle CFB$

32. $\angle BFD, \angle BDF$

33. $\angle DBF, \angle BFD$

Use the figure at the right to determine the relationship between the given lengths.



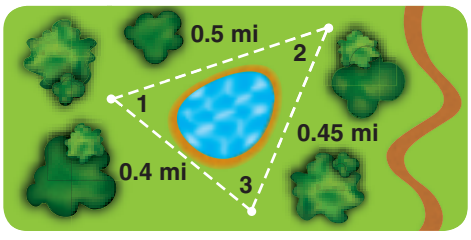
34. SM, MR

35. RP, MP

36. RQ, PQ

37. RM, RQ

38. HIKING Justin and his family are hiking around a lake as shown in the diagram at the right. Order the angles of the triangle formed by their path from largest to smallest.

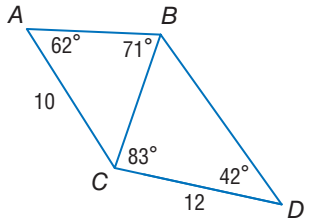


COORDINATE GEOMETRY **List the angles of each triangle with the given vertices in order from smallest to largest. Justify your answer.**

39 $A(-4, 6), B(-2, 1), C(5, 6)$

40. $X(-3, -2), Y(3, 2), Z(-3, -6)$

41. List the side lengths of the triangles in the figure from shortest to longest. Explain your reasoning.



42. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship between the sides of a triangle.

a. Geometric Draw three triangles, including one acute, one obtuse, and one right angle. Label the vertices of each triangle A, B, and C.

b. Tabular Measure the length of each side of the three triangles. Then copy and complete the table.

Triangle	AB	BC	$AB + BC$	CA
Acute				
Obtuse				
Right				

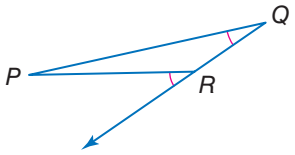
c. Tabular Create two additional tables like the one above, finding the sum of BC and CA in one table and the sum of AB and CA in the other.

d. Algebraic Write an inequality for each of the tables you created relating the measure of the sum of two of the sides to the measure of the third side of a triangle.

e. Verbal Make a conjecture about the relationship between the measure of the sum of two sides of a triangle and the measure of the third side.

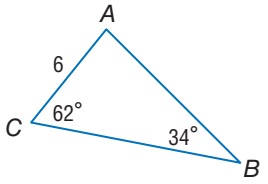
H.O.T. Problems Use Higher-Order Thinking Skills

43. **WRITING IN MATH** Analyze the information given in the diagram and explain why the markings must be incorrect.



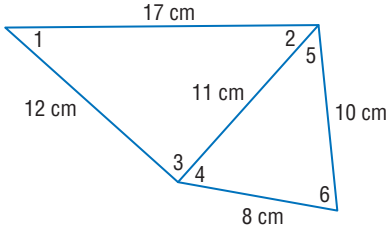
44. **CHALLENGE** Using only a ruler, draw $\triangle ABC$ such that $m\angle A > m\angle B > m\angle C$. Justify your drawing.

45. **OPEN ENDED** Give a possible measure for $\overline{AB}$ in $\triangle ABC$ shown. Explain your reasoning.



46. **CCSS ARGUMENTS** Is the base of an isosceles triangle *always*, *sometimes*, or *never* the longest side of the triangle? Explain.

47. **CHALLENGE** Use the side lengths in the figure to list the numbered angles in order from smallest to largest given that $m\angle 2 = m\angle 5$. Explain your reasoning.



48. **WRITING IN MATH** Why is the hypotenuse always the longest side of a triangle?

Standardized Test Practice

49. STATISTICS The chart shows the number and types of DVDs sold at three stores.

DVD Type	Store 1	Store 2	Store 3
Comedy	75	80	92
Action	54	37	65
Horror	30	48	62
Science Fiction	21	81	36
Total	180	246	255

According to the information in the chart, which of these statements is true?

A The mean number of DVDs sold per store was 56.

B Store 1 sold twice as many action and horror films as store 3 sold of science fiction.

C Store 2 sold fewer comedy and science fiction than store 3 sold.

D The mean number of science fiction DVDs sold per store was 46.

50. Two angles of a triangle have measures 45° and 92°. What type of triangle is it?

F obtuse scalene **H** acute scalene

G obtuse isosceles **J** acute isosceles

51. EXTENDED RESPONSE At a five-star restaurant, a waiter earns a total of t dollars for working h hours in which he receives $198 in tips and makes $2.50 per hour.

a. Write an equation to represent the total amount of money the waiter earns.

b. If the waiter earned a total of $213, how many hours did he work?

c. If the waiter earned $150 in tips and worked for 12 hours, what is the total amount of money he earned?

52. SAT/ACT Which expression has the *least* value?

A $|-99|$ **D** $|-28|$

B $|45|$ **E** $|15|$

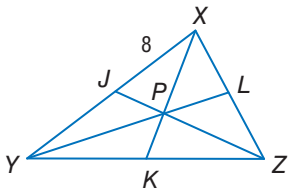
C $|-39|$

Spiral Review

In $\triangle XYZ$, P is the centroid, $KP = 3$, and $XJ = 8$. Find each length. (Lesson 5-2)

53. XK

54. YJ

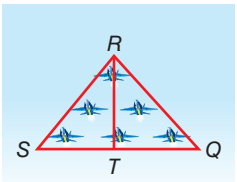


COORDINATE GEOMETRY **Write an equation in slope-intercept form for the perpendicular bisector of the segment with the given endpoints. Justify your answer.** (Lesson 5-1)

55. $D(-2, 4)$ and $E(3, 5)$

56. $D(-2,-4)$ and $E(2, 1)$

57. JETS The United States Navy Flight Demonstration Squadron, the Blue Angels, flies in a formation that can be viewed as two triangles with a common side. Write a two-column proof to prove that $\triangle SRT \cong \triangle QRT$ if T is the midpoint of $\overline{SQ}$ and $\overline{SR} \cong \overline{QR}$. (Lesson 4-4)



58. POOLS A rectangular pool is 20 feet by 30 feet. The depth of the pool is 60 inches, but the depth of the water is $\frac{3}{4}$ of the depth of the pool. Find each measure to the nearest tenth. (Lesson 1-7)

a. the surface area of the pool

b. the volume of water in the pool

Skills Review

Determine whether each statement is true or false if $x = 8$, $y = 2$, and $z = 3$.

59. $z(x - y) = 13$

60. $2x = 3yz$

61. $x + y > z + y$