LESSON 5-5

The Triangle Inequality

Then

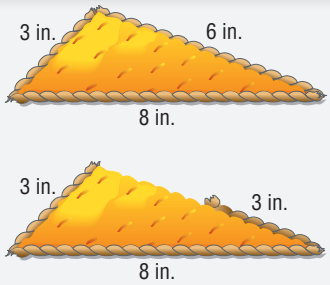
- You recognized and applied properties of inequalities to the relationships between the angles and sides of a triangle.

Now

1. Use the Triangle Inequality Theorem to identify possible triangles.
2. Prove triangle relationships using the Triangle Inequality Theorem.

Why?

- On a home improvement show, a designer wants to use scrap pieces of cording from another sewing project to decorate the triangular throw pillows that she and the homeowner have made. To minimize waste, she wants to use the scraps without cutting them. She selects three scraps at random and tries to form a triangle. Two such attempts are shown.



Common Core State Standards

Content Standards

G.CO.10 Prove theorems about triangles.

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

Mathematical Practices

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

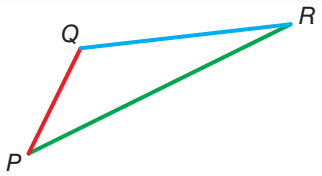
1 The Triangle Inequality

While a triangle is formed by three segments, a special relationship must exist among the lengths of the segments in order for them to form a triangle.

Theorem 5.11 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

Examples $PQ + QR > PR$
$QR + PR > PQ$
$PR + PQ > QR$



You will prove Theorem 5.11 in Exercise 23.

To show that it is not possible to form a triangle with three side lengths, you need only show that one of the three triangle inequalities is not true.

Example 1 Identify Possible Triangles Given Side Lengths

Is it possible to form a triangle with the given side lengths? If not, explain why not.

a. 8 in., 15 in., 17 in.

Check each inequality.

$8 + 15 \stackrel{?}{>} 17$
$23 > 17$ ✓

$8 + 17 \stackrel{?}{>} 15$
$25 > 15$ ✓

$15 + 17 \stackrel{?}{>} 8$
$32 > 8$ ✓

Since the sum of each pair of side lengths is greater than the third side length, sides with lengths 8, 15, and 17 inches will form a triangle.

b. 6 m, 8 m, 14 m

$6 + 8 \stackrel{?}{>} 14$
$14 \not> 14$ ✗

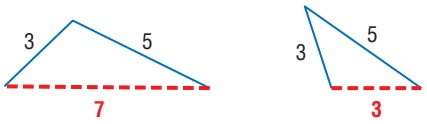
Since the sum of one pair of side lengths is not greater than the third side length, sides with lengths 6, 8, and 14 meters will not form a triangle.

GuidedPractice

1A. 15 yd, 16 yd, 30 yd

1B. 2 ft, 8 ft, 11 ft

When the lengths of two sides of a triangle are known, the third side can be any length in a range of values. You can use the Triangle Inequality Theorem to determine the range of possible lengths for the third side.



Standardized Test Example 2 Find Possible Side Lengths

Test-TakingTip

Testing Choices If you are short on time, you can test each choice to find the correct answer and eliminate any remaining choices.

If the measures of two sides of a triangle are 3 feet and 7 feet, which is the *least* possible whole number measure for the third side?

A 3 ft **B** 4 ft **C** 5 ft **D** 10 ft

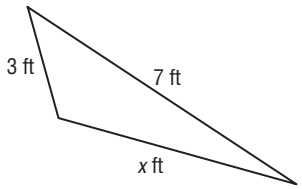
Read the Test Item

You need to determine which value is the least possible measure for the third side of a triangle with sides that measure 3 feet and 7 feet.

Solve the Test Item

To determine the least possible measure from the choices given, first determine the range of possible measures for the third side.

Draw a diagram and let x represent the length of the third side.



Next, set up and solve each of the three triangle inequalities.

$3 + 7 > x$ $\quad$ $3 + x > 7$ $\quad$ $x + 7 > 3$

$10 > x$ or $x < 10$ $\quad$ $x > 4$ $\quad$ $x > -4$

ReadingMath

Multiple Inequality Symbols The compound inequality $4 < x < 10$ is read *x is between 4 and 10.*

Notice that $x > -4$ is always true for any whole number measure for x. Combining the two remaining inequalities, the range of values that fit both inequalities is $x > 4$ and $x < 10$, which can be written as $4 < x < 10$.

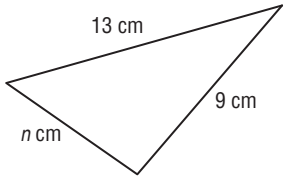
The least whole number value between 4 and 10 is 5. So the correct answer is choice C.

GuidedPractice

2. Which of the following could *not* be the value of n?

F 7 **H** 13

G 10 **J** 22



2 Proofs Using the Triangle Inequality Theorem

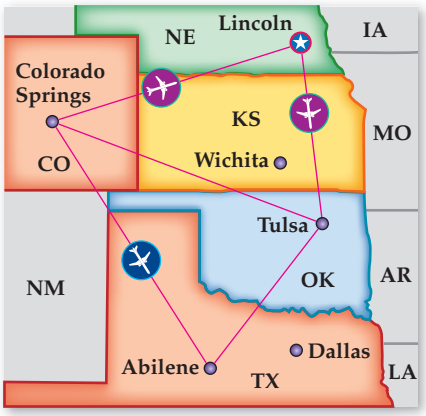
You can use the Triangle Inequality Theorem as a reason in proofs.

Real-WorldLink

A direct flight is not the same as a nonstop flight. For a direct flight, passengers do not change planes, but the plane may make one or more stops before continuing to its final destination.

Real-World Example 3 Proof Using Triangle Inequality Theorem

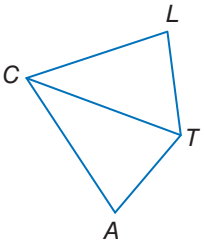
TRAVEL **The distance from Colorado Springs, Springs, Colorado, to Abilene, Texas, is the same as the distance from Colorado Springs to Tulsa, Oklahoma. Prove that a direct flight from Colorado Springs to Tulsa through Lincoln, Nebraska, is a greater distance than a nonstopflight from Colorado Springs to Abilene.**



Draw a simpler diagram of the situation and label the diagram. Draw in side $\overline{LT}$ to form $\triangle CTL$.

Given: $CA = CT$

Prove: $CL + LT > CA$



Proof:

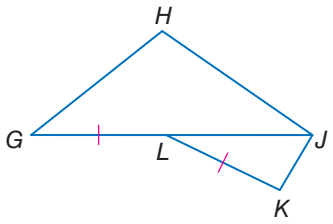
Statements	Reasons
1. $CA = CT$	**1.** Given
2. $CL + LT > CT$	**2.** Triangle Inequality Theorem
3. $CL + LT > CA$	**3.** Substitution

GuidedPractice

3. Write a two-column proof.

Given: $GL = LK$

Prove: $JH + GH > JK$



Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **Is it possible to form a triangle with the given side lengths? If not, explain why not.**

1. 5 cm, 7 cm, 10 cm **2.** 3 in., 4 in., 8 in. **3.** 6 m, 14 m, 10 m

Example 2

4. MULTIPLE CHOICE If the measures of two sides of a triangle are 5 yards and 9 yards, what is the least possible measure of the third side if the measure is an integer?

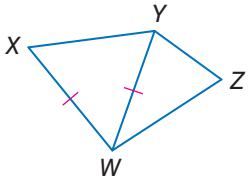
A 4 yd **B** 5 yd **C** 6 yd **D** 14 yd

Example 3

5. PROOF Write a two-column proof.

Given: $\overline{XW} \cong \overline{YW}$

Prove: $YZ + ZW > XW$



Ed Boettcher/CORBIS

Practice and Problem Solving

Extra Practice is on page R5.

Example 1 **Is it possible to form a triangle with the given side lengths? If not, explain why not.**

6. 4 ft, 9 ft, 15 ft

7. 11 mm, 21 mm, 16 mm

8. 9.9 cm, 1.1 cm, 8.2 cm

9. 2.1 in., 4.2 in., 7.9 in.

10. $2\frac{1}{2}$ m, $1\frac{3}{4}$ m, $5\frac{1}{8}$ m

11. $1\frac{1}{5}$ km, $4\frac{1}{2}$ km, $3\frac{3}{4}$ km

Example 2 **Find the range for the measure of the third side of a triangle given the measures of two sides.**

12. 4 ft, 8 ft

13. 5 m, 11 m

14. 2.7 cm, 4.2 cm

15. 3.8 in., 9.2 in.

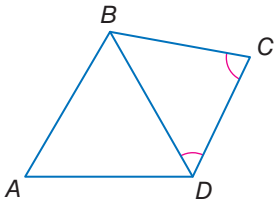
16. $\frac{1}{2}$ km, $3\frac{1}{4}$ km

17. $2\frac{1}{3}$ yd, $7\frac{2}{3}$ yd

Example 3 **PROOF Write a two-column proof.**

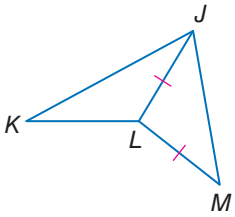
18. **Given:** $\angle BCD \cong \angle CDB$

Prove: $AB + AD > BC$



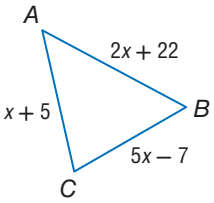
19. **Given:** $\overline{JL} \cong \overline{LM}$

Prove: $KJ + KL > LM$

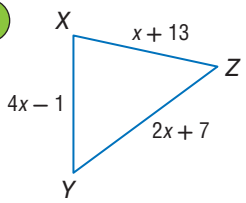


CCSS **SENSE-MAKING** **Determine the possible values of x.**

20.



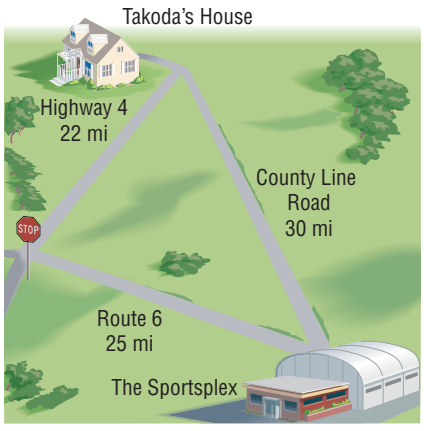
21



22. **DRIVING** Takoda wants to take the most efficient route from his house to a soccer tournament at The Sportsplex. He can take County Line Road or he can take Highway 4 and then Route 6 to the get to The Sportsplex.

a. Which of the two possible routes is the shortest? Explain your reasoning.

b. Suppose Takoda always drives below the speed limit. If the speed limit on County Line Road is 30 miles per hour and on both Highway 4 and Route 6 it is 55 miles per hour, which route will be faster? Explain.

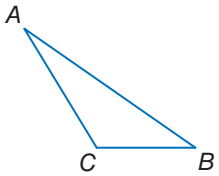


23. **PROOF Write a two-column proof.**

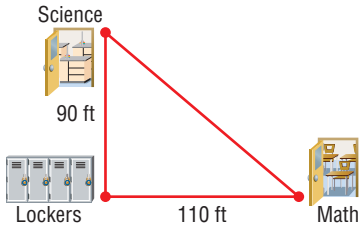
Given: $\triangle ABC$

Prove: $AC + BC > AB$ (Triangle Inequality Theorem)

(*Hint*: Draw auxiliary segment $\overline{CD}$, so that C is between B and D and $\overline{CD} \cong \overline{AC}$.)



24. SCHOOL When Toya goes from science class to math class, she usually stops at her locker. The distance from her science classroom to her locker is 90 feet, and the distance from her locker to her math classroom is 110 feet. What are the possible distances from science class to math class if she takes the hallway that goes directly between the two classrooms?



Find the range of possible measures of x if each set of expressions represents measures of the sides of a triangle.

25. $x, 4, 6$

26. $8, x, 12$

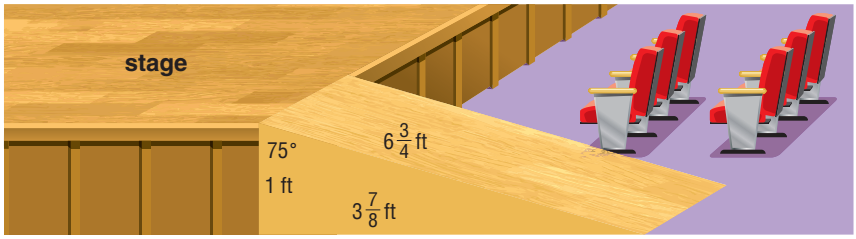
27. $x + 1, 5, 7$

28. $x - 2, 10, 12$

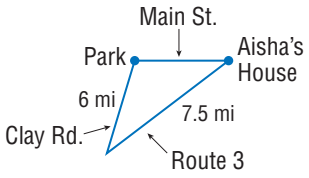
29. $x + 2, x + 4, x + 6$

30. $x, 2x + 1, x + 4$

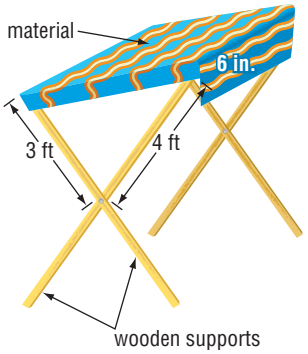
31. DRAMA CLUB Anthony and Catherine are working on a ramp up to the stage for the drama club's next production. Anthony's sketch of the ramp is shown below. Catherine is concerned about the measurements and thinks they should recheck the measures before they start cutting the wood. Is Catherine's concern valid? Explain your reasoning.



32. SENSE-MAKING Aisha is riding her bike to the park and can take one of two routes. The most direct route from her house is to take Main Street, but it is safer to take Route 3 and then turn right on Clay Road as shown. The additional distance she will travel if she takes Route 3 to Clay Road is between how many miles?



33. DESIGN Carlota designed an awning that she and her friends could take to the beach. Carlota decides to cover the top of the awning with material that will drape 6 inches over the front. What length of material should she buy to use with her design so that it covers the top of the awning, including the drape, when the supports are open as far as possible? Assume that the width of the material is sufficient to cover the awning.



ESTIMATION Without using a calculator, determine if it is possible to form a triangle with the given side lengths. Explain.

34. $\sqrt{8}$ ft, $\sqrt{2}$ ft, $\sqrt{35}$ ft

35. $\sqrt{99}$ yd, $\sqrt{48}$ yd, $\sqrt{65}$ yd

36. $\sqrt{3}$ m, $\sqrt{15}$ m, $\sqrt{24}$ m

37. $\sqrt{122}$ in., $\sqrt{5}$ in., $\sqrt{26}$ in.

REASONING **Determine whether the given coordinates are the vertices of a triangle. Explain.**

38. $X(1, -3)$, $Y(6, 1)$, $Z(2, 2)$

39 $F(-4, 3)$, $G(3, -3)$, $H(4, 6)$

40. $J(-7, -1)$, $K(9, -5)$, $L(21, -8)$

41. $Q(2, 6)$, $R(6, 5)$, $S(1, 2)$

42. **MULTIPLE REPRESENTATIONS** In this problem, you will use inequalities to make comparisons between the sides and angles of two triangles.

a. Geometric Draw three pairs of triangles that have two pairs of congruent sides and one pair of sides that is not congruent. Mark each pair of congruent sides. Label each triangle pair ABC and DEF, where $\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$.

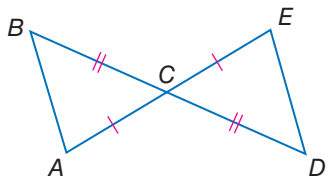
b. Tabular Copy the table below. Measure and record the values of BC, $m\angle A$, EF, and $m\angle D$ for each triangle pair.

Triangle Pair	BC	$m\angle A$	EF	$m\angle D$
1				
2				
3				

c. Verbal Make a conjecture about the relationship between the angles opposite the noncongruent sides of a pair of triangles that have two pairs of congruent legs.

H.O.T. Problems Use Higher-Order Thinking Skills

43. **CHALLENGE** What is the range of possible perimeters for figure $ABCDE$ if $AC = 7$ and $DC = 9$? Explain your reasoning.



44. **REASONING** What is the range of lengths of each leg of an isosceles triangle if the measure of the base is 6 inches? Explain.

45. **WRITING IN MATH** What can you tell about a triangle when given three side lengths? Include at least two items.

46. **CHALLENGE** The sides of an isosceles triangle are whole numbers, and its perimeter is 30 units. What is the probability that the triangle is equilateral?

47. **OPEN ENDED** The length of one side of a triangle is 2 inches. Draw a triangle in which the 2-inch side is the shortest side and one in which the 2-inch side is the longest side. Include side and angle measures on your drawing.

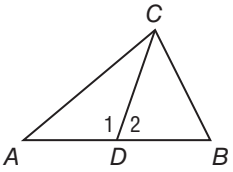
48. **WRITING IN MATH** Suppose your house is $\frac{3}{4}$ mile from a park and the park is 1.5 miles from a shopping center.

a. If your house, the park, and the shopping center are noncollinear, what do you know about the distance from your house to the shopping center? Explain your reasoning.

b. If the three locations are collinear, what do you know about the distance from your house to the shopping center? Explain your reasoning.

Standardized Test Practice

49. If $\overline{DC}$ is a median of $\triangle ABC$ and $m\angle 1 > m\angle 2$, which of the following statements is not true?



A $AD = BD$ **C** $AC > BC$

B $m\angle ADC = m\angle BDC$ **D** $m\angle 1 > m\angle B$

50. SHORT RESPONSE A high school soccer team has a goal of winning at least 75% of their 15 games this season. In the first three weeks, the team has won 5 games. How many more games must the team win to meet their goal?

51. Which of the following is a logical conclusion based on the statement and its converse below?

Statement: If a polygon is a rectangle, then it has four sides.

Converse: If a polygon has four sides, then it is a rectangle.

F The statement and its converse are both true.

G The statement is false; the converse is false.

H The statement is true; the converse is false.

J The statement is false; the converse is true.

52. SAT/ACT When 7 is subtracted from $14w$, the result is z. Which of the following equations represents this statement?

A $7 - 14w = z$ **D** $z = 14w - 7$

B $z = 14w + 7$ **E** $7 + 14w = 7z$

C $7 - z = 14w$

Spiral Review

State the assumption you would make to start an indirect proof of each statement. (Lesson 5-4)

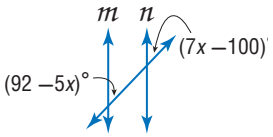
53. If $4y + 17 = 41$, then $y = 6$.

54. If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the two lines are parallel.

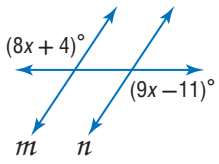
55. GEOGRAPHY The distance between San Jose, California, and Las Vegas, Nevada, is about 375 miles. The distance from Las Vegas to Carlsbad, California, is about 243 miles. Use the Triangle Inequality Theorem to find the possible distance between San Jose and Carlsbad. (Lesson 5-3)

Find x so that $m \parallel n$. Identify the postulate or theorem you used. (Lesson 3-5)

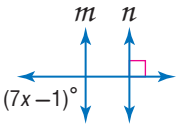
56.



57.



58.



ALGEBRA **Find x and JK if J is between K and L.** (Lesson 1-2)

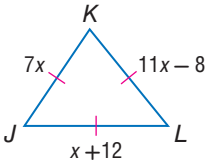
59. $KJ = 3x$, $JL = 6x$, and $KL = 12$

60. $KJ = 3x - 6$, $JL = x + 6$, and $KL = 24$

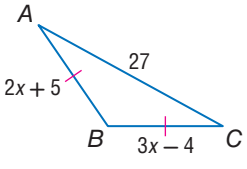
Skills Review

Find x and the measures of the unknown sides of each triangle.

61.



62.



63.