# LESSON 6-1 Angles of Polygons

## Then

- You named and classified polygons.

## Now

1. Find and use the sum of the measures of the interior angles of a polygon.
2. Find and use the sum of the measures of the exterior angles of a polygon.

## Why?

- To create their honeycombs, young worker honeybees excrete flecks of wax that are carefully molded by other bees to form hexagonal cells. The cells are less than 0.1 millimeter thick, but they support almost 25 times their own weight. The cell walls all stand at exactly the same angle to one another. This angle is the measure of the interior angle of a regular hexagon.

### NewVocabulary

diagonal

### Common Core State Standards

**Content Standards**

**G.MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★

**Mathematical Practices**

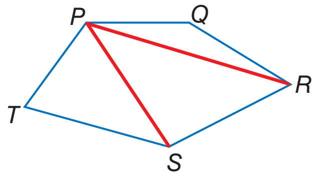
4 Model with mathematics.

3 Construct viable arguments and critique the reasoning of others.

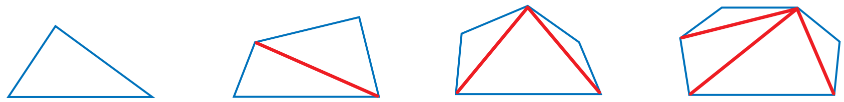
## 1 Polygon Interior Angles Sum

A **diagonal** of a polygon is a segment that connects any two nonconsecutive vertices.

The vertices of polygon *PQRST* that are not consecutive with vertex *P* are vertices *R* and *S*. Therefore, polygon *PQRST* has two diagonals from vertex *P*, $\overline{PR}$ and $\overline{PS}$. Notice that the diagonals from vertex *P* separate the polygon into three triangles.




The sum of the angle measures of a polygon is the sum of the angle measures of the triangles formed by drawing all the possible diagonals from one vertex.



**Triangle** **Quadrilateral** **Pentagon** **Hexagon**

Since the sum of the angle measures of a triangle is 180, we can make a table and look for a pattern to find the sum of the angle measures for any convex polygon.

| Polygon | Number of Sides | Number of Triangles | Sum of Interior Angle Measures |
|---|---|---|---|
| Triangle | 3 | 1 | (1)180 or 180 |
| Quadrilateral | 4 | 2 | (2)180 or 360 |
| Pentagon | 5 | 3 | (3)180 or 540 |
| Hexagon | 6 | 4 | (4)180 or 720 |
| *n*-gon | *n* | $n-2$ | $(n-2)180$ |

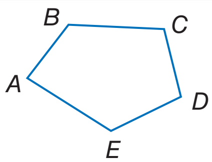
This leads to the following theorem.

**Theorem 6.1 Polygon Interior Angles Sum**

The sum of the interior angle measures of an *n*-sided convex polygon is $(n-2) \cdot 180$.

**Example** $m\angle A + m\angle B + m\angle C + m\angle D + m\angle E = (5-2) \cdot 180$

$= 540$




You will prove Theorem 6.1 for octagons in Exercise 42.

You can use the Polygon Interior Angles Sum Theorem to find the sum of the interior angles of a polygon and to find missing measures in polygons.

**StudyTip**

**Naming Polygons** Remember, a polygon with *n*-sides is an *n-gon*, but several polygons have special names.

| Number of Sides | Polygon |
|---|---|
| 3 | triangle |
| 4 | quadrilateral |
| 5 | pentagon |
| 6 | hexagon |
| 7 | heptagon |
| 8 | octagon |
| 9 | nonagon |
| 10 | decagon |
| 11 | hendecagon |
| 12 | dodecagon |
| *n* | *n*-gon |

## Example 1 Find the Interior Angles Sum of a Polygon

**a. Find the sum of the measures of the interior angles of a convex heptagon.**

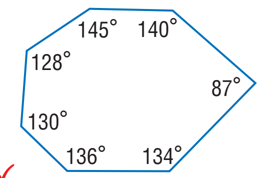
A heptagon has seven sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

$(n - 2) \cdot 180 = (7 - 2) \cdot 180$  $n = 7$

$= 5 \cdot 180$ or $900$  Simplify.

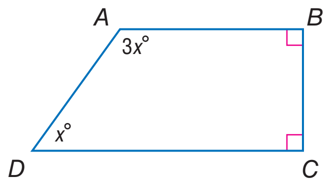
The sum of the measures is 900.

**CHECK** Draw a convex polygon with seven sides. Use a protractor to measure each angle to the nearest degree. Then find the sum of these measures.




$128 + 145 + 140 + 87 + 134 + 136 + 130 = 900$ ✓

**b. ALGEBRA Find the measure of each interior angle of quadrilateral *ABCD*.**




**Step 1** Find *x*.

Since there are 4 angles, the sum of the interior angle measures is $(4 - 2) \cdot 180$ or 360.

$360 = m\angle A + m\angle B + m\angle C + m\angle D$  Sum of interior angle measures

$360 = 3x + 90 + 90 + x$  Substitution

$360 = 4x + 180$  Combine like terms.

$180 = 4x$  Subtract 180 from each side.

$45 = x$  Divide each side by 4.

**Step 2** Use the value of *x* to find the measure of each angle.

$m\angle A = 3x$ $= 3(45)$ or $135$

$m\angle B = 90$

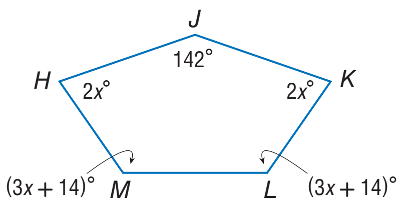
$m\angle C = 90$

$m\angle D = x$ $= 45$

**GuidedPractice**

**1A.** Find the sum of the measures of the interior angles of a convex octagon.

**1B.** Find the measure of each interior angle of pentagon *HJKLM* shown.



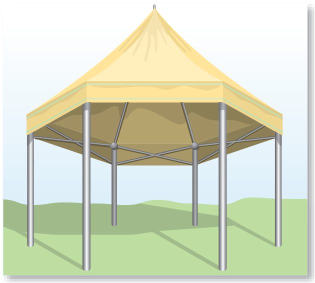

Recall from Lesson 1-6 that in a regular polygon, all of the interior angles are congruent. You can use this fact and the Polygon Interior Angle Sum Theorem to find the interior angle measure of any regular polygon.

**Review**Vocabulary

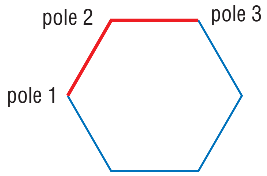
**regular polygon**
a convex polygon in which all of the sides are congruent and all of the angles are congruent

## Real-World Example 2 Interior Angle Measure of Regular Polygon

**TENTS The poles for a tent form the vertices of a regular hexagon. When the poles are properly positioned, what is the measure of the angle formed at a corner of the tent?**



**Understand** Draw a diagram of the situation.




The measure of the angle formed at a corner of the tent is an interior angle of a regular hexagon.

**Plan** Use the Polygon Interior Angles Sum Theorem to find the sum of the measures of the angles. Since the angles of a regular polygon are congruent, divide this sum by the number of angles to find the measure of each interior angle.

**Solve** **Step 1** Find the sum of the interior angle measures.

$$(n - 2) \cdot 180 = (6 - 2) \cdot 180 \qquad n = 6$$

$$= 4 \cdot 180 \text{ or } 720 \qquad \text{Simplify.}$$

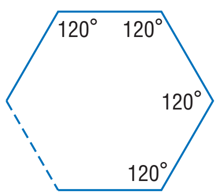
**Step 2** Find the measure of one interior angle.

$$\frac{\text{sum of interior angle measures}}{\text{number of congruent angles}} = \frac{720}{6} \qquad \text{Substitution}$$

$$= 120 \qquad \text{Divide.}$$

The angle at a corner of the tent measures 120.

**Check** To verify that this measure is correct, use a ruler and a protractor to draw a regular hexagon using 120 as the measure of each interior angle. The last side drawn should connect with the beginning point of the first segment drawn. ✓




PhotoSpin, Inc/Alamy

**Real-World**Link

Susan B. Anthony was a leader of the women's suffrage movement in the late 1800s, which eventually led to the Nineteenth Amendment giving women the right to vote. In 1979, the Susan B. Anthony one-dollar coin was first minted, making her the first woman to be depicted on U.S. currency.

**Source:** *Encyclopaedia Britannica*

### GuidedPractice

**2A.** **COINS** Find the measure of each interior angle of the regular hendecagon that appears on the face of a Susan B. Anthony one-dollar coin.

**2B.** **HOT TUBS** A certain company makes hot tubs in a variety of different shapes. Find the measure of each interior angle of the nonagon model.

Given the interior angle measure of a regular polygon, you can also use the Polygon Interior Angles Sum Theorem to find a polygon's number of sides.

## Example 3 Find Number of Sides Given Interior Angle Measure

**The measure of an interior angle of a regular polygon is 135. Find the number of sides in the polygon.**

Let $n$ = the number of sides in the polygon. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $135n$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n - 2) \cdot 180$.

| | |
|---|---|
| $135n = (n - 2) \cdot 180$ | Write an equation. |
| $135n = 180n - 360$ | Distributive Property |
| $-45n = -360$ | Subtract $180n$ from each side. |
| $n = 8$ | Divide each side by $-45$. |

The polygon has 8 sides.

### GuidedPractice

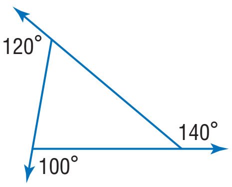
**3.** The measure of an interior angle of a regular polygon is 144. Find the number of sides in the polygon.

### ReviewVocabulary

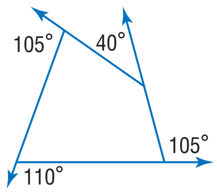
**exterior angle** an angle formed by one side of a polygon and the extension of another side

## 2 Polygon Exterior Angles Sum

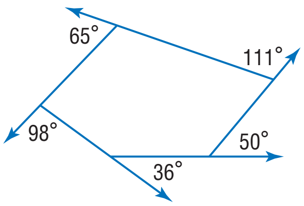
Does a relationship exist between the number of sides of a convex polygon and the sum of its exterior angle measures? Examine the polygons below in which an exterior angle has been measured at each vertex.




120 + 100 + 140 = **360**




105 + 110 + 105 + 40= **360**



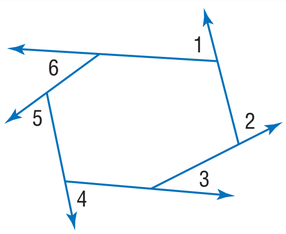

65 + 98 + 36 + 50 + 111= **360**

Notice that the sum of the exterior angle measures in each case is 360. This suggests the following theorem.

### Theorem 6.2 Polygon Exterior Angles Sum

The sum of the exterior angle measures of a convex polygon, one angle at each vertex, is 360.

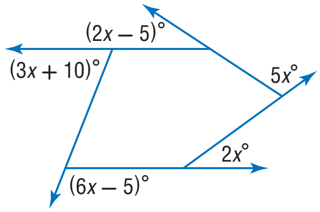
**Example**
$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 = 360$




You will prove Theorem 6.2 in Exercise 43.

## Example 4 Find Exterior Angle Measures of a Polygon

**a. ALGEBRA** Find the value of $x$ in the diagram.




Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for $x$.

$$(2x - 5) + 5x + 2x + (6x - 5) + (3x + 10) = 360$$
$$(2x + 5x + 2x + 6x + 3x) + [-5 + (-5) + 10] = 360$$
$$18x = 360$$
$$x = \frac{360}{18} \text{ or } 20$$

**b.** Find the measure of each exterior angle of a regular nonagon.

A regular nonagon has 9 congruent sides and 9 congruent interior angles. The exterior angles are also congruent, since angles supplementary to congruent angles are congruent. Let $n$ = the measure of each exterior angle and write and solve an equation.

$9n = 360$ Polygon Exterior Angles Sum Theorem

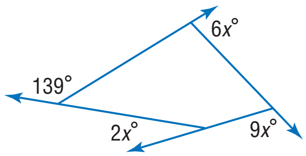
$n = 40$ Divide each side by 9.

The measure of each exterior angle of a regular nonagon is 40.

**StudyTip**

**CCSS Perseverance** To find the measure of each exterior angle of a regular polygon, you can find the measure of each interior angle and subtract this measure from 180, since an exterior angle and its corresponding interior angle are supplementary.

### GuidedPractice




**4A.** Find the value of $x$ in the diagram.

**4B.** Find the measure of each exterior angle of a regular dodecagon.

## Check Your Understanding

= Step-by-Step Solutions begin on page R14.

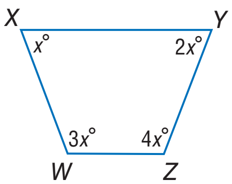
**Example 1** **Find the sum of the measures of the interior angles of each convex polygon.**

**1.** decagon

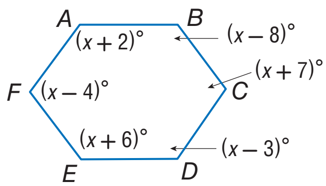
**2.** pentagon

**Find the measure of each interior angle.**

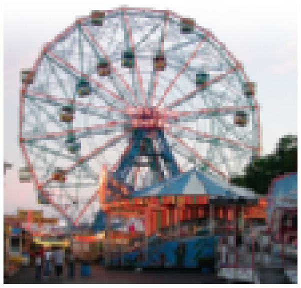
**3.**




**4.**




**Example 2** **5** **AMUSEMENT** The Wonder Wheel at Coney Island in Brooklyn, New York, is a regular polygon with 16 sides. What is the measure of each interior angle of the polygon?





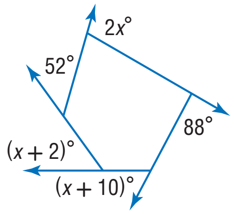
**Example 3** **The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.**

**6.** 150

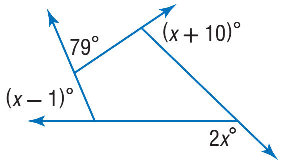
**7.** 170

**Example 4** **Find the value of $x$ in each diagram.**

**8.**




**9.**




**Find the measure of each exterior angle of each regular polygon.**

**10.** quadrilateral **11.** octagon

## Practice and Problem Solving

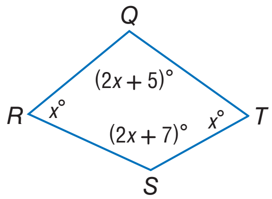
Extra Practice is on page R6.

**Example 1** **Find the sum of the measures of the interior angles of each convex polygon.**

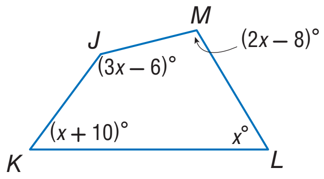
**12.** dodecagon **13.** 20-gon **14.** 29-gon **15.** 32-gon

**Find the measure of each interior angle.**

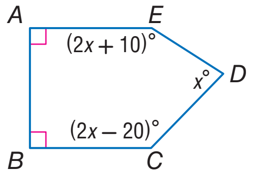
**16.**



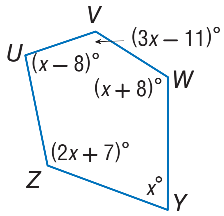

**17.**



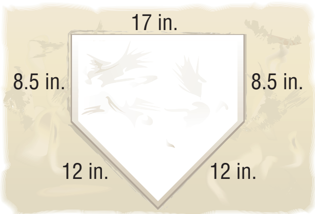

**18.**




**19.**




**20.** **BASEBALL** In baseball, home plate is a pentagon. The dimensions of home plate are shown. What is the sum of the measures of the interior angles of home plate?



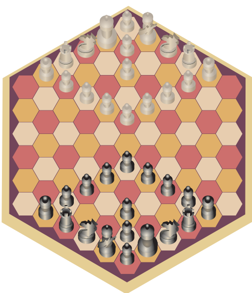

**Example 2** **Find the measure of each interior angle of each regular polygon.**

**21.** dodecagon **22.** pentagon **23.** decagon **24.** nonagon

**25.** CCSS **MODELING** Hexagonal chess is played on a regular hexagonal board comprised of 92 small hexagons in three colors. The chess pieces are arranged so that a player can move any piece at the start of a game.

**a.** What is the sum of the measures of the interior angles of the chess board?

**b.** Does each interior angle have the same measure? If so, give the measure. Explain your reasoning.

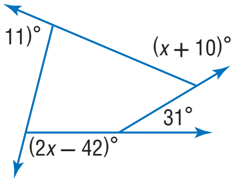


**Example 3** **The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.**

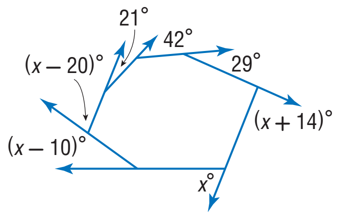
**26.** 60 **27.** 90 **28.** 120 **29.** 156

**Example 4** **Find the value of $x$ in each diagram.**

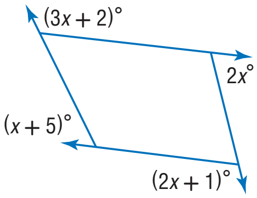
**30.**



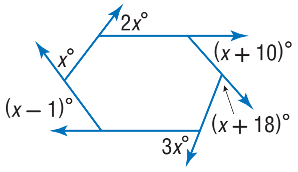

**31**




**32.**



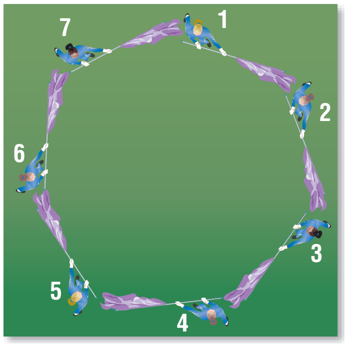

**33.**




**Find the measure of each exterior angle of each regular polygon.**

**34.** decagon **35.** pentagon **36.** hexagon **37.** 15-gon

**38.** **COLOR GUARD** During the halftime performance for a football game, the color guard is planning a new formation in which seven members stand around a central point and stretch their flag to the person immediately to their left as shown.




**a.** What is the measure of each exterior angle of the formation?

**b.** If the perimeter of the formation is 38.5 feet, how long is each flag?

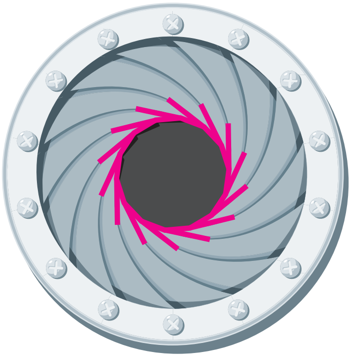
**Find the measures of an exterior angle and an interior angle given the number of sides of each regular polygon. Round to the nearest tenth, if necessary.**

**39.** 7 **40.** 13 **41.** 14

**42.** **PROOF** Write a paragraph proof to prove the Polygon Interior Angles Sum Theorem for octagons.

**43.** **PROOF** Use algebra to prove the Polygon Exterior Angles Sum Theorem.

**44.** **CCSS MODELING** The aperture on the camera lens shown is a regular 14-sided polygon.



**a.** What is the measure of each interior angle of the polygon?

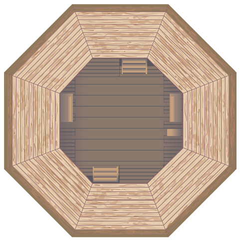
**b.** What is the measure of each exterior angle of the polygon?

**ALGEBRA** **Find the measure of each interior angle.**

**45.** decagon, in which the measures of the interior angles are $x + 5$, $x + 10$, $x + 20$, $x + 30$, $x + 35$, $x + 40$, $x + 60$, $x + 70$, $x + 80$, and $x + 90$

**46.** polygon *ABCDE*, in which the measures of the interior angles are $6x$, $4x + 13$, $x + 9$, $2x - 8$, $4x - 1$

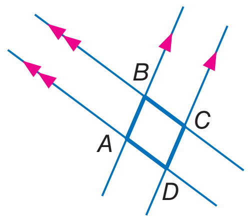
**47.** **THEATER** The drama club would like to build a theater in the round, so the audience can be seated on all sides of the stage, for its next production.



**a.** The stage is to be a regular octagon with a total perimeter of 60 feet. To what length should each board be cut to form the sides of the stage?

**b.** At what angle should each board be cut so that they will fit together as shown? Explain your reasoning.

**48.** **MULTIPLE REPRESENTATIONS** In this problem, you will explore angle and side relationships in special quadrilaterals.




**a. Geometric** Draw two pairs of parallel lines that intersect like the ones shown. Label the quadrilateral formed by *ABCD*. Repeat these steps to form two additional quadrilaterals, *FGHJ* and *QRST*.

**b. Tabular** Copy and complete the table below.

| Quadrilateral | Lengths and Measures | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *ABCD* | $m\angle A$ | | $m\angle B$ | | $m\angle C$ | | $m\angle D$ | |
| | *AB* | | *BC* | | *CD* | | *DA* | |
| *FGHJ* | $m\angle F$ | | $m\angle G$ | | $m\angle H$ | | $m\angle J$ | |
| | *FG* | | *GH* | | *HJ* | | *JF* | |
| *QRST* | $m\angle Q$ | | $m\angle R$ | | $m\angle S$ | | $m\angle T$ | |
| | *QR* | | *RS* | | *ST* | | *TQ* | |

**c. Verbal** Make a conjecture about the relationship between the angles opposite each other in a quadrilateral formed by two pairs of parallel lines.

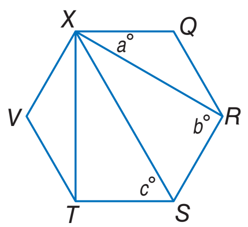
**d. Verbal** Make a conjecture about the relationship between two consecutive angles in a quadrilateral formed by two pairs of parallel lines.

**e. Verbal** Make a conjecture about the relationship between the sides opposite each other in a quadrilateral formed by two pairs of parallel lines.

## H.O.T. Problems Use Higher-Order Thinking Skills

**49.** **ERROR ANALYSIS** Marcus says that the sum of the exterior angles of a decagon is greater than that of a heptagon because a decagon has more sides. Liam says that the sum of the exterior angles for both polygons is the same. Is either of them correct? Explain your reasoning.

**50.** **CHALLENGE** Find the values of *a*, *b*, and *c* if *QRSTVX* is a regular hexagon. Justify your answer.



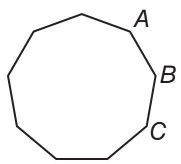

**51.** **CCSS ARGUMENTS** If two sides of a regular hexagon are extended to meet at a point in the exterior of the polygon, will the triangle formed *always*, *sometimes*, or *never* be equilateral? Justify your answer.

**52.** **OPEN ENDED** Sketch a polygon and find the sum of its interior angles. How many sides does a polygon with twice this interior angles sum have? Justify your answer.

**53.** **WRITING IN MATH** Explain how triangles are related to the Interior Angles Sum Theorem.

## Standardized Test Practice

**54.** If the polygon shown is regular, what is $m\angle ABC$?



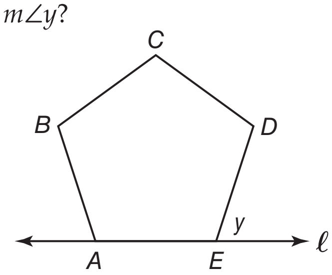

**A** 140

**B** 144

**C** 162

**D** 180

**55. SHORT RESPONSE** Figure *ABCDE* is a regular pentagon with line $\ell$ passing through side *AE*. What is $m\angle y$?




**56. ALGEBRA** $\dfrac{3^2 \cdot 4^5 \cdot 5^3}{5^3 \cdot 3^3 \cdot 4^6} =$

**F** $\frac{1}{60}$

**G** $\frac{1}{12}$

**H** $\frac{3}{4}$

**J** 12

**57. SAT/ACT** The sum of the measures of the interior angles of a polygon is twice the sum of the measures of its exterior angles. What type of polygon is it?

**A** square

**B** pentagon

**C** hexagon

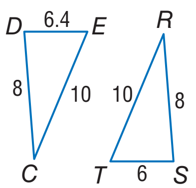
**D** octagon

**E** nonagon

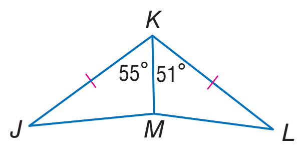
## Spiral Review

**Compare the given measures.** (Lesson 5-6)

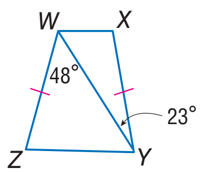
**58.** $m\angle DCE$ and $m\angle SRT$




**59.** *JM* and *ML*



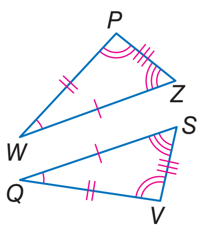

**60.** *WX* and *ZY*



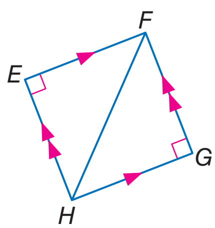

**61. HISTORY** The early Egyptians used to make triangles by using a rope with knots tied at equal intervals. Each vertex of the triangle had to occur at a knot. How many different triangles can be formed using the rope below? (Lesson 5-5)

**Show that the triangles are congruent by identifying all congruent corresponding parts. Then write a congruence statement.** (Lesson 4-3)

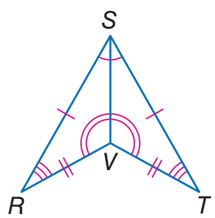
**62.**




**63.**




**64.**




## Skills Review

**In the figure, $\ell \parallel m$ and $\overline{AC} \parallel \overline{BD}$. Name all pairs of angles for each type indicated.**

**65.** alternate interior angles

**66.** consecutive interior angles