LESSON 6-2

Parallelograms

Then

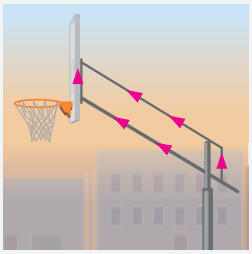
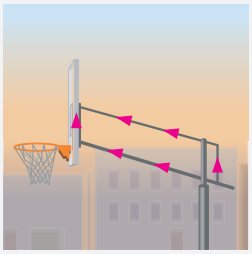
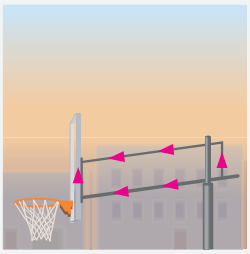
- You classified polygons with four sides as quadrilaterals.

Now

1. Recognize and apply properties of the sides and angles of parallelograms.
2. Recognize and apply properties of the diagonals of parallelograms.

Why?

- The arm of the basketball goal shown can be adjusted to a height of 10 feet or 5 feet. Notice that as the height is adjusted, each pair of opposite sides of the quadrilateral formed by the arms remains parallel.

New Vocabulary

parallelogram

Common Core State Standards

Content Standards

G.CO.11 Prove theorems about parallelograms.

G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.

Mathematical Practices

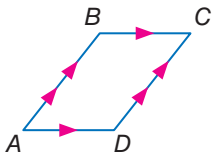
4 Model with mathematics.

3 Construct viable arguments and critique the reasoning of others.

1 Sides and Angles of Parallelograms

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. To name a parallelogram, use the symbol $\square$. In $\square ABCD$, $\overline{BC} \parallel \overline{AD}$ and $\overline{AB} \parallel \overline{DC}$ by definition.

Other properties of parallelograms are given in the theorems below.



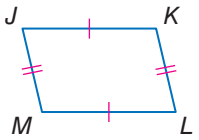
$\square ABCD$

Theorem Properties of Parallelograms

6.3 If a quadrilateral is a parallelogram, then its opposite sides are congruent.

Abbreviation *Opp. sides of a* $\square$ *are* $\cong$.

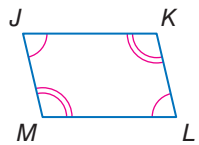
Example If *JKLM* is a parallelogram, then $\overline{JK} \cong \overline{ML}$ and $\overline{JM} \cong \overline{KL}$.



6.4 If a quadrilateral is a parallelogram, then its opposite angles are congruent.

Abbreviation *Opp.* $\measuredangle$ *of a* $\square$ *are* $\cong$.

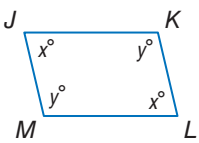
Example If *JKLM* is a parallelogram, then $\angle J \cong \angle L$ and $\angle K \cong \angle M$.



6.5 If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

Abbreviation *Cons.* $\measuredangle$ *in a* $\square$ *are supplementary.*

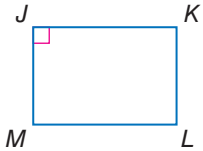
Example If *JKLM* is a parallelogram, then $x + y = 180$.



6.6 If a parallelogram has one right angle, then it has four right angles.

Abbreviation *If a* $\square$ *has 1 rt.* $\angle$, *it has 4 rt.* $\angle$*s.*

Example In $\square JKLM$, if $\angle J$ is a right angle, then $\angle K$, $\angle L$, and $\angle M$ are also right angles.



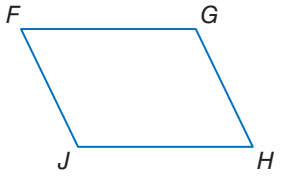
You will prove Theorems 6.3, 6.5, and 6.6 in Exercises 28, 26, and 7, respectively.

StudyTip

Including a Figure
Theorems are presented in general terms. In a proof, you must include a drawing so that you can refer to segments and angles specifically.

Proof Theorem 6.4

Write a two-column proof of Theorem 6.4.



Given: $\square FGHJ$

Prove: $\angle F \cong \angle H$, $\angle J \cong \angle G$

Proof:

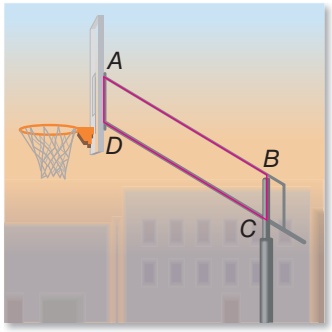
Statements	Reasons
1. $\square FGHJ$	1. Given
2. $\overline{FG} \parallel \overline{JH}$; $\overline{FJ} \parallel \overline{GH}$	2. Definition of parallelogram
3. $\angle F$ and $\angle J$ are supplementary. $\angle J$ and $\angle H$ are supplementary. $\angle H$ and $\angle G$ are supplementary.	3. If parallel lines are cut by a transversal, consecutive interior angles are supplementary.
4. $\angle F \cong \angle H$, $\angle J \cong \angle G$	4. Supplements of the same angles are congruent.

Real-WorldCareer

Coach Coaches organize amateur and professional atheletes, teaching them the fundamentals of a sport. They manage teams during both practice sessions and competitions. Additional tasks may include selecting and issuing sports equiment, materials, and supplies. Head coaches at public secondary schools usually have a bachelor's degree.

Real-World Example 1 Use Properties of Parallelograms

BASKETBALL In $\square ABCD$, suppose $m\angle A = 55$, $AB = 2.5$ feet, and $BC = 1$ foot. Find each measure.



a. DC

$DC = AB$	Opp. sides of a $\square$ are $\cong$.
$= 2.5$ ft	Substitution

b. $m\angle B$

$m\angle B + m\angle A = 180$	Cons. $\angle$s in a $\square$ are supplementary.
$m\angle B + 55 = 180$	Substitution
$m\angle B = 125$	Subtract 55 from each side.

c. $m\angle C$

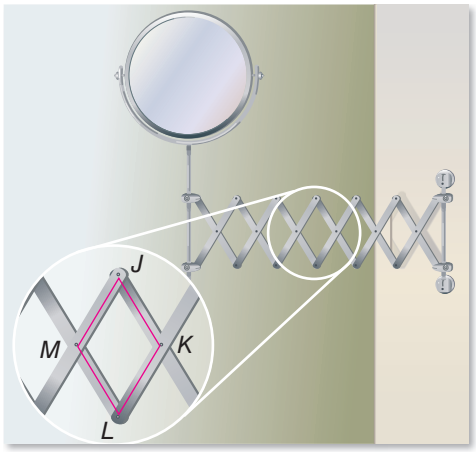
$m\angle C = m\angle A$	Opp. $\angle$s of a $\square$ are $\cong$.
$= 55$	Substitution

GuidedPractice

1. MIRRORS The wall-mounted mirror shown uses parallelograms that change shape as the arm is extended. In $\square JKLM$, suppose $m\angle J = 47$. Find each measure.

A. $m\angle L$ **B.** $m\angle M$

C. Suppose the arm was extended further so that $m\angle J = 90$. What would be the measure of each of the other angles? Justify your answer.



moodboard/SuperStock

2 Diagonals of Parallelograms

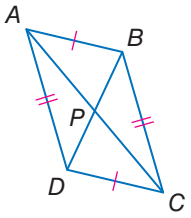
The diagonals of a parallelogram have special properties as well.

Theorem Diagonals of Parallelograms

6.7 If a quadrilateral is a parallelogram, then its diagonals bisect each other.

Abbreviation *Diag. of a ▱ bisect each other.*

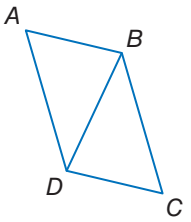
Example If *ABCD* is a parallelogram, then $\overline{AP} \cong \overline{PC}$ and $\overline{DP} \cong \overline{PB}$.



6.8 If a quadrilateral is a parallelogram, then each diagonal separates the parallelogram into two congruent triangles.

Abbreviation *Diag. separates a ▱ into 2 ≅ △.*

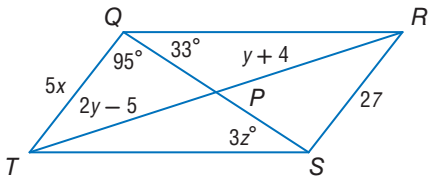
Example If *ABCD* is a parallelogram, then $\triangle ABD \cong \triangle CDB$.



You will prove Theorems 6.7 and 6.8 in Exercises 29 and 27, respectively.

Example 2 Use Properties of Parallelograms and Algebra

ALGEBRA If *QRST* is a parallelogram, find the value of the indicated variable.



a. x

$\overline{QT} \cong \overline{RS}$	Opp. sides of a ▱ are ≅.
$QT = RS$	Definition of congruence
$5x = 27$	Substitution
$x = 5.4$	Divide each side by 5.

b. y

$\overline{TP} \cong \overline{PR}$	Diag. of a ▱ bisect each other.
$TP = PR$	Definition of congruence
$2y - 5 = y + 4$	Substitution
$y = 9$	Subtract y and add 5 to each side.

c. z

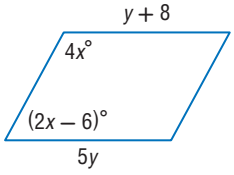
$\triangle TQS \cong \triangle RSQ$	Diag. separates a ▱ into 2 ≅ △.
$\angle QST \cong \angle SQR$	CPCTC
$m\angle QST = m\angle SQR$	Definition of congruence
$3z = 33$	Substitution
$z = 11$	Divide each side by 3.

StudyTip
Congruent Triangles
A parallelogram with two diagonals divides the figure into two pairs of congruent triangles.

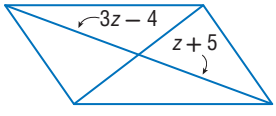
GuidedPractice

Find the value of each variable in the given parallelogram.

2A.



2B.



You can use Theorem 6.7 to determine the coordinates of the intersection of the diagonals of a parallelogram on a coordinate plane given the coordinates of the vertices.

Example 3 Parallelograms and Coordinate Geometry

COORDINATE GEOMETRY **Determine the coordinates of the intersection of the diagonals of $\square FGHJ$ with vertices $F(-2, 4)$, $G(3, 5)$, $H(2, -3)$, and $J(-3, -4)$.**

Since the diagonals of a parallelogram bisect each other, their intersection point is the midpoint of $\overline{FH}$ and $\overline{GJ}$. Find the midpoint of $\overline{FH}$ with endpoints $(-2, 4)$ and $(2, -3)$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 2}{2}, \frac{4 + (-3)}{2}\right) \quad \text{Midpoint Formula}$$

$$= (0, 0.5) \quad \text{Simplify.}$$

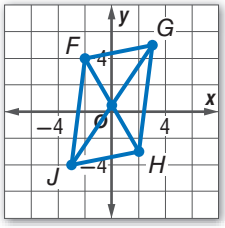
The coordinates of the intersection of the diagonals of $\square FGHJ$ are $(0, 0.5)$.

CHECK Find the midpoint of $\overline{GJ}$ with endpoints $(3, 5)$ and $(-3, -4)$.

$$\left(\frac{3 + (-3)}{2}, \frac{5 + (-4)}{2}\right) = (0, 0.5) \checkmark$$

StudyTip

CCSS Regularity Graph the parallelogram in Example 3 and the point of intersection of the diagonals you found. Draw the diagonals. The point of intersection appears to be correct.



GuidedPractice

3. COORDINATE GEOMETRY Determine the coordinates of the intersection of the diagonals of $RSTU$ with vertices $R(-8, -2)$, $S(-6, 7)$, $T(6, 7)$, and $U(4, -2)$.

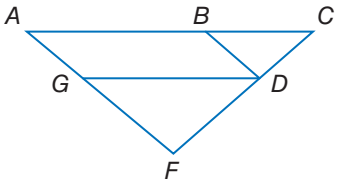
You can use the properties of parallelograms and their diagonals to write proofs.

Example 4 Proofs Using the Properties of Parallelograms

Write a paragraph proof.

Given: $\square ABDG$, $\overline{AF} \cong \overline{CF}$

Prove: $\angle BDG \cong \angle C$



Proof:

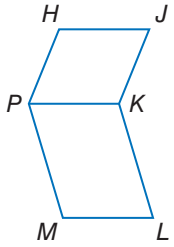
We are given $ABDG$ is a parallelogram. Since opposite angles in a parallelogram are congruent, $\angle BDG \cong \angle A$. We are also given that $\overline{AF} \cong \overline{CF}$. By the Isosceles Triangle Theorem, $\angle A \cong \angle C$. So, by the Transitive Property of Congruence, $\angle BDG \cong \angle C$.

GuidedPractice

4. Write a two-column proof.

Given: $\square HJKP$ and $\square PKLM$

Prove: $\overline{HJ} \cong \overline{ML}$

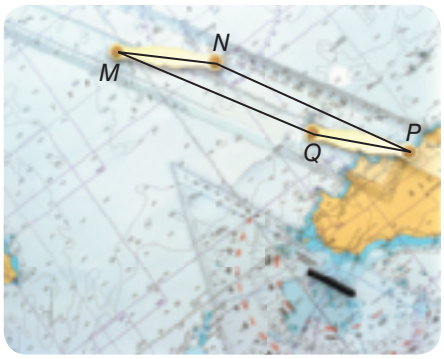


Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1

1. NAVIGATION To chart a course, sailors use a *parallel ruler.* One edge of the ruler is placed along the line representing the direction of the course to be taken. Then the other ruler is moved until its edge reaches the compass rose printed on the chart. Reading the compass determines which direction to travel. The rulers and the crossbars of the tool form $\square MNPQ$.



a. If $m\angle NMQ = 32$, find $m\angle MNP$.

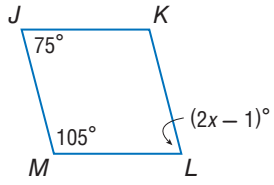
b. If $m\angle MQP = 125$, find $m\angle MNP$.

c. If $MQ = 4$, what is NP?

Example 2

ALGEBRA Find the value of each variable in each parallelogram.

2.



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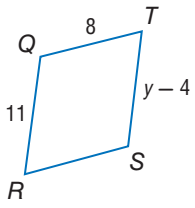

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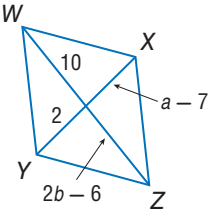

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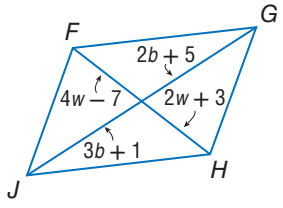


Example 3

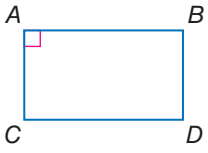
6. COORDINATE GEOMETRY Determine the coordinates of the intersection of the diagonals of $\square ABCD$ with vertices $A(-4, 6)$, $B(5, 6)$, $C(4, -2)$, and $D(-5, -2)$.

Example 4

CCSS **ARGUMENTS Write the indicated type of proof.**

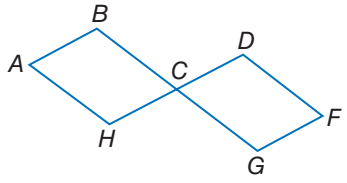
7. paragraph

Given: $\square ABCD$, $\angle A$ is a right angle.
Prove: $\angle B$, $\angle C$, and $\angle D$ are right angles. (Theorem 6.6)



8. two-column

Given: $ABCH$ and $DCGF$ are parallelograms.
Prove: $\angle A \cong \angle F$



Practice and Problem Solving

Extra Practice is on page R6.

Example 1

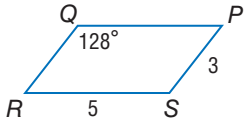
Use $\square PQRS$ to find each measure.

9. $m\angle R$

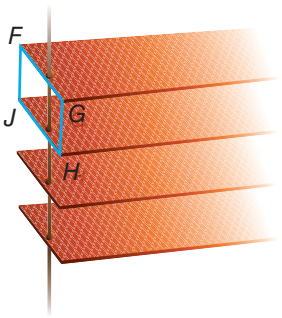
10. QR

11. QP

12. $m\angle S$



13 **HOME DECOR** The slats on Venetian blinds are designed to remain parallel in order to direct the path of light coming in a window. In $\square FGHJ$, $FJ = \frac{3}{4}$ inch, $FG = 1$ inch, and $m\angle JHG = 62$. Find each measure.



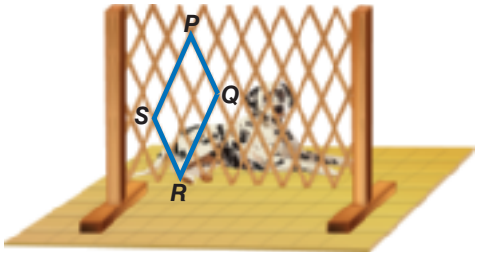
a. JH

b. GH

c. $m\angle JFG$

d. $m\angle FJH$

14. **CCSS MODELING** Wesley is a member of the kennel club in his area. His club uses accordion fencing like the section shown at the right to block out areas at dog shows.

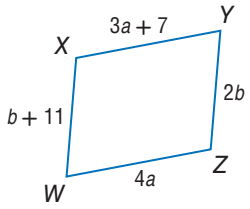


a. Identify two pairs of congruent segments.

b. Identify two pairs of supplementary angles.

Example 2 **ALGEBRA** **Find the value of each variable in each parallelogram.**

15.



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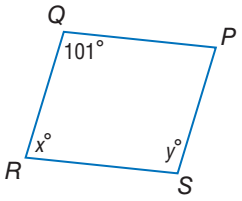

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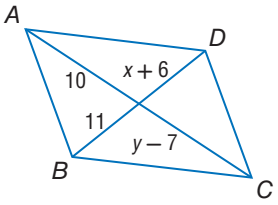

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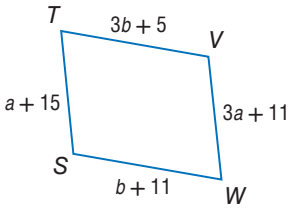

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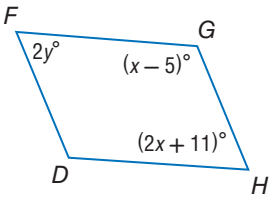

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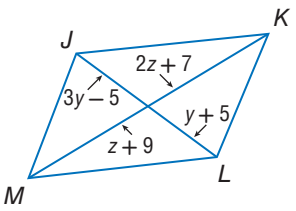


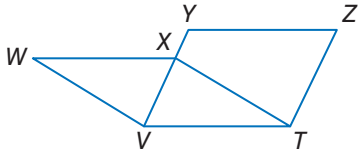
Example 3 **COORDINATE GEOMETRY** **Find the coordinates of the intersection of the diagonals of $\square WXYZ$ with the given vertices.**

21. $W(-1, 7)$, $X(8, 7)$, $Y(6, -2)$, $Z(-3, -2)$

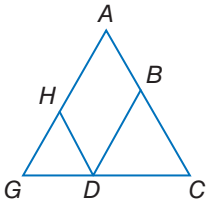
22. $W(-4, 5)$, $X(5, 7)$, $Y(4, -2)$, $Z(-5, -4)$

Example 4 **PROOF** **Write a two-column proof.**

23. **Given:** $WXTV$ and $ZYVT$ are parallelograms.
Prove: $\overline{WX} \cong \overline{ZY}$



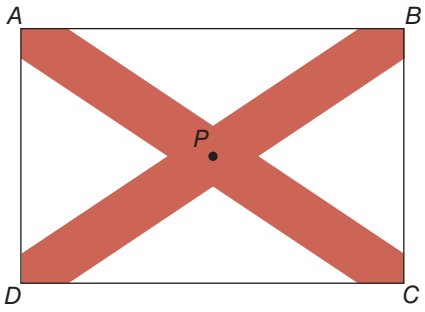
24. **Given:** $\square BDHA$, $\overline{CA} \cong \overline{CG}$
Prove: $\angle BDH \cong \angle G$



25. FLAGS Refer to the Alabama state flag at the right.

Given: $\triangle ACD \cong \triangle CAB$

Prove: $\overline{DP} \cong \overline{PB}$

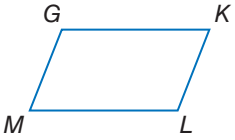


CCSS ARGUMENTS Write the indicated type of proof.

26. two-column

Given: $\square GKLM$

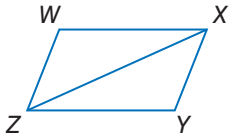
Prove: $\angle G$ and $\angle K$, $\angle K$ and $\angle L$, $\angle L$ and $\angle M$, and $\angle M$ and $\angle G$ are supplementary. (Theorem 6.5)



27. two-column

Given: $\square WXYZ$

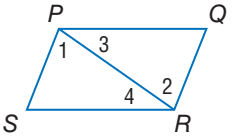
Prove: $\triangle WXZ \cong \triangle YZX$ (Theorem 6.8)



28. two-column

Given: $\square PQRS$

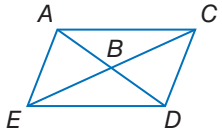
Prove: $\overline{PQ} \cong \overline{RS}$, $\overline{QR} \cong \overline{SP}$ (Theorem 6.3)



29. paragraph

Given: $\square ACDE$ is a parallelogram.

Prove: $\overline{EC}$ bisects $\overline{AD}$. (Theorem 6.7)

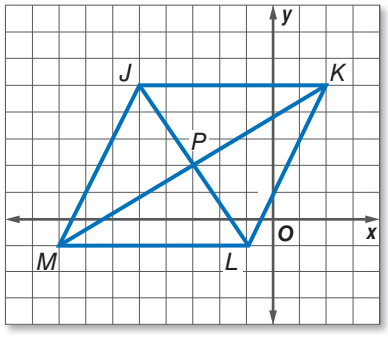


30. COORDINATE GEOMETRY Use the graph shown.

a. Use the Distance Formula to determine if the diagonals of $JKLM$ bisect each other. Explain.

b. Determine whether the diagonals are congruent. Explain.

c. Use slopes to determine if the consecutive sides are perpendicular. Explain.



ALGEBRA Use $\square ABCD$ to find each measure or value.

31. x

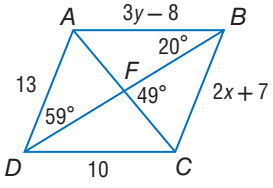
32. y

33 $m\angle AFB$

34. $m\angle DAC$

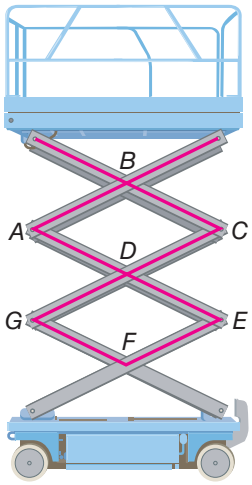
35. $m\angle ACD$

36. $m\angle DAB$



37. COORDINATE GEOMETRY $\square ABCD$ has vertices $A(-3, 5)$, $B(1, 2)$, and $C(3, -4)$. Determine the coordinates of vertex D if it is located in Quadrant III.

38. MECHANICS Scissor lifts are variable elevation work platforms. One is shown at the right. In the diagram, $ABCD$ and $DEFG$ are congruent parallelograms.



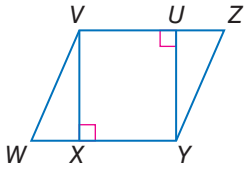
a. List the angle(s) congruent to $\angle A$. Explain your reasoning.

b. List the segment(s) congruent to $\overline{BC}$. Explain your reasoning.

c. List the angle(s) supplementary to $\angle C$. Explain your reasoning.

PROOF **Write a two-column proof.**

39 **Given:** $\square YWVZ$, $\overline{VX} \perp \overline{WY}$, $\overline{YU} \perp \overline{VZ}$
Prove: $\triangle YUZ \cong \triangle VXW$



40. MULTIPLE REPRESENTATIONS In this problem, you will explore tests for parallelograms.

a. Geometric Draw three pairs of segments that are both congruent and parallel and connect the endpoints to form quadrilaterals. Label one quadrilateral $ABCD$, one $MNOP$, and one $WXYZ$. Measure and label the sides and angles of the quadrilaterals.

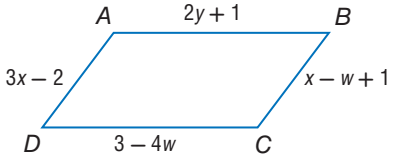
b. Tabular Copy and complete the table below.

Quadrilateral	Opposite Sides Congruent?	Opposite Angles Congruent?	Parallelogram
$ABCD$			
$MNOP$			
$WXYZ$			

c. Verbal Make a conjecture about quadrilaterals with one pair of segments that are both congruent and parallel.

H.O.T. Problems Use Higher-Order Thinking Skills

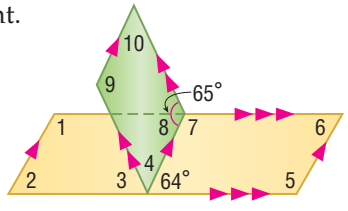
41. CHALLENGE $ABCD$ is a parallelogram with side lengths as indicated in the figure at the right. The perimeter of $ABCD$ is 22. Find AB.



42. WRITING IN MATH Explain why parallelograms are *always* quadrilaterals, but quadrilaterals are *sometimes* parallelograms.

43. OPEN ENDED Provide a counterexample to show that parallelograms are not always congruent if their corresponding sides are congruent.

44. CCSS REASONING Find $m\angle 1$ and $m\angle 10$ in the figure at the right. Explain.



45. WRITING IN MATH Summarize the properties of the sides, angles, and diagonals of a parallelogram.

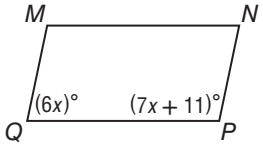
Standardized Test Practice

46. Two consecutive angles of a parallelogram measure $3x + 42$ and $9x - 18$. What are the measures of the angles?

A 13, 167 **C** 39, 141

B 58.5, 31.5 **D** 81, 99

47. GRIDDED RESPONSE Parallelogram $MNPQ$ is shown. What is the value of x?



48. ALGEBRA In a history class with 32 students, the ratio of girls to boys is 5 to 3. How many more girls are there than boys?

F 2 **G** 8 **H** 12 **J** 15

49. SAT/ACT The table shows the heights of the tallest buildings in Kansas City, Missouri. To the nearest tenth, what is the positive difference between the median and the mean of the data?

Name	Height (m)
One Kansas City Place	193
Town Pavillion	180
Hyatt Regency	154
Power and Light Building	147
City Hall	135
1201 Walnut	130

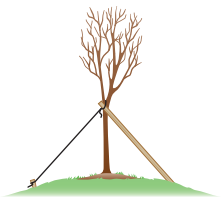
A 5
B 6
C 7
D 8
E 10

Spiral Review

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon. (Lesson 6-1)

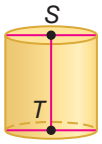
50. 108 **51.** 140 **52.** ≈ 147.3 **53.** 160 **54.** 135 **55.** 176.4

56. LANDSCAPING When landscapers plant new trees, they usually brace the tree using a stake tied to the trunk of the tree. Use the SAS or SSS Inequality to explain why this is an effective method for keeping a newly planted tree perpendicular to the ground. Assume that the tree does not lean forward or backward. (Lesson 5-6)

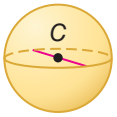


Determine whether the solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the bases, faces, edges, and vertices. (Lesson 1-7)

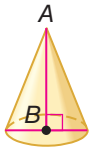
57.



58.



59.



Skills Review

The vertices of a quadrilateral are $W(3, -1)$, $X(4, 2)$, $Y(-2, 3)$ and $Z(-3, 0)$. Determine whether each segment is a side or diagonal of the quadrilateral, and find the slope of each segment.

60. $\overline{YZ}$ **61.** $\overline{YW}$ **62.** $\overline{ZW}$