# LESSON 6-3 Tests for Parallelograms

**Then**

- You recognized and applied properties of parallelograms.

**Now**

1. Recognize the conditions that ensure a quadrilateral is a parallelogram.
2. Prove that a set of points forms a parallelogram in the coordinate plane.

**Why?**

- Lexi and Rosalinda cut strips of bulletin board paper at an angle to form the hallway display shown. Their friends asked them how they cut the strips so that their sides were parallel without using a protractor.

Rosalinda explained that since the left and right sides of the paper were parallel, she only needed to make sure that the sides were cut to the same length to guarantee that a strip would form a parallelogram.


**Common Core State Standards**

**Content Standards**

**G.CO.11** Prove theorems about parallelograms.

**G.GPE.4** Use coordinates to prove simple geometric theorems algebraically.

**Mathematical Practices**

3 Construct viable arguments and critique the reasoning of others.

2 Reason abstractly and quantitatively.

## 1 Conditions for Parallelograms

If a quadrilateral has each pair of opposite sides parallel, it is a parallelogram by definition.

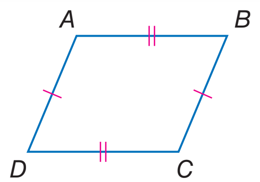
This is not the only test, however, that can be used to determine if a quadrilateral is a parallelogram.

### Theorems Conditions for Parallelograms

**6.9** If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

**Abbreviation** *If both pairs of opp. sides are ≅, then quad. is a ▱.*

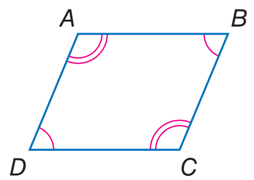
**Example** If $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$, then $ABCD$ is a parallelogram.




**6.10** If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

**Abbreviation** *If both pairs of opp. ∠s are ≅, then quad. is a ▱.*

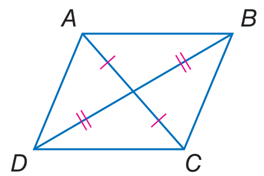
**Example** If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.




**6.11** If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

**Abbreviation** *If diag. bisect each other, then quad. is a ▱.*

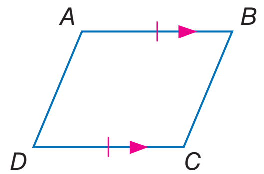
**Example** If $\overline{AC}$ and $\overline{DB}$ bisect each other, then $ABCD$ is a parallelogram.




**6.12** If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

**Abbreviation** *If one pair of opp. sides is ≅ and ||, then the quad. is a ▱.*

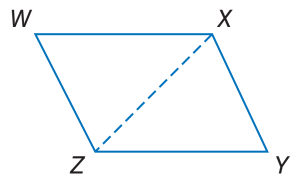
**Example** If $\overline{AB} \parallel \overline{DC}$ and $\overline{AB} \cong \overline{DC}$, then $ABCD$ is a parallelogram.




You will prove Theorems 6.10, 6.11, and 6.12 in Exercises 30, 32, and 33, respectively.

## Proof Theorem 6.9

Write a paragraph proof of Theorem 6.9.




**Given:** $\overline{WX} \cong \overline{ZY}$, $\overline{WZ} \cong \overline{XY}$

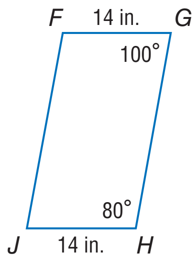
**Prove:** *WXYZ* is a parallelogram.

**Paragraph Proof:**

Two points determine a line, so we can draw auxiliary line $\overline{ZX}$ to form $\triangle ZWX$ and $\triangle XYZ$. We are given that $\overline{WX} \cong \overline{ZY}$ and $\overline{WZ} \cong \overline{XY}$. Also, $\overline{ZX} \cong \overline{XZ}$ by the Reflexive Property of Congruence. So $\triangle ZWX \cong \triangle XYZ$ by SSS. By CPCTC, $\angle WXZ \cong \angle YZX$ and $\angle WZX \cong \angle YXZ$. This means that $\overline{WX} \parallel \overline{ZY}$ and $\overline{WZ} \parallel \overline{XY}$ by the Alternate Interior Angles Converse. Opposite sides of *WXYZ* are parallel, so by definition *WXYZ* is a parallelogram.

## Example 1 Identify Parallelograms

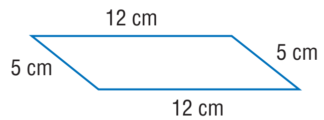
**Determine whether the quadrilateral is a parallelogram. Justify your answer.**



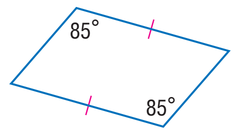

Opposite sides $\overline{FG}$ and $\overline{JH}$ are congruent because they have the same measure. Also, since $\angle FGH$ and $\angle GHJ$ are supplementary consecutive interior angles, $\overline{FG} \parallel \overline{JH}$. Therefore, by Theorem 6.12, *FGHJ* is a parallelogram.

### GuidedPractice

**1A.**




**1B.**



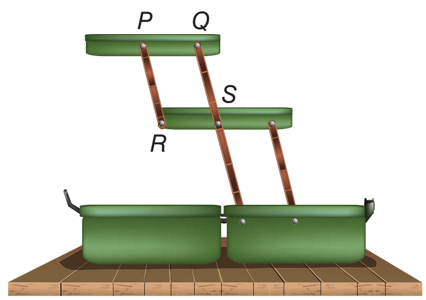

You can use the conditions of parallelograms to prove relationships in real-world situations.

**Real-WorldLink**

A 2- or 3-cantilever tackle box is often used to organize lures and other fishing supplies. The trays lift up and away so that all items in the box are easily accessible.

## Real-World Example 2 Use Parallelograms to Prove Relationships

**FISHING The diagram shows a side view of the tackle box at the left. In the diagram, $PQ = RS$ and $PR = QS$. Explain why the upper and middle trays remain parallel no matter to what height the trays are raised or lowered.**




Since both pairs of opposite sides of quadrilateral *PQSR* are congruent, *PQRS* is a parallelogram by Theorem 6.9. By the definition of a parallelogram, opposite sides are parallel, so $\overline{PQ} \parallel \overline{RS}$. Therefore, no matter the vertical position of the trays, they will always remain parallel.

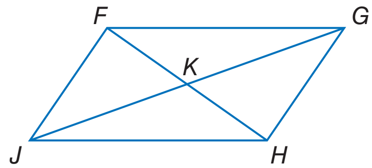
### GuidedPractice

2. **BANNERS** In the example at the beginning of the lesson, explain why the cuts made by Lexi and Rosalinda are parallel.

You can also use the conditions of parallelograms along with algebra to find missing values that make a quadrilateral a parallelogram.

## Example 3 Use Parallelograms and Algebra to Find Values

**If $FK = 3x - 1$, $KG = 4y + 3$, $JK = 6y - 2$, and $KH = 2x + 3$, find $x$ and $y$ so that the quadrilateral is a parallelogram.**




By Theorem 6.11, if the diagonals of a quadrilateral bisect each other, then it is a parallelogram. So find $x$ such that $\overline{FK} \cong \overline{KH}$ and $y$ such that $\overline{JK} \cong \overline{KG}$.

| | |
|---|---|
| $FK = KH$ | Definition of ≅ |
| $3x - 1 = 2x + 3$ | Substitution |
| $x - 1 = 3$ | Subtract $2x$ from each side. |
| $x = 4$ | Add 1 to each side. |
| $JK = KG$ | Definition of ≅ |
| $6y - 2 = 4y + 3$ | Substitution |
| $2y - 2 = 3$ | Subtract $4y$ from each side. |
| $2y = 5$ | Add 2 to each side. |
| $y = 2.5$ | Divide each side by 2. |

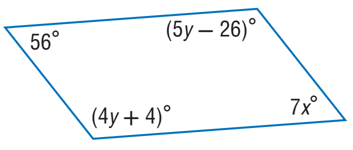
So, when $x$ is 4 and $y$ is 2.5, quadrilateral $FGHJ$ is a parallelogram.

> **WatchOut!**
>
> **Parallelograms** In Example 3, if $x$ is 4, then $y$ must be 2.5 in order for $FGHJ$ to be a parallelogram. In other words, if $x$ is 4 and $y$ is 1, then $FGHJ$ is not a parallelogram.

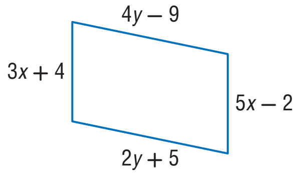
### GuidedPractice

**Find $x$ and $y$ so that each quadrilateral is a parallelogram.**

**3A.**




**3B.**




You have learned the conditions of parallelograms. The following list summarizes how to use the conditions to prove a quadrilateral is a parallelogram.

## Concept Summary

**Prove that a Quadrilateral Is a Parallelogram**

- Show that both pairs of opposite sides are parallel. (Definition)
- Show that both pairs of opposite sides are congruent. (Theorem 6.9)
- Show that both pairs of opposite angles are congruent. (Theorem 6.10)
- Show that the diagonals bisect each other. (Theorem 6.11)
- Show that a pair of opposite sides is both parallel and congruent. (Theorem 6.12)

**Study**Tip

**Midpoint Formula**
To show that a quadrilateral is a parallelogram, you can also use the Midpoint Formula. If the midpoint of each diagonal is the same point, then the diagonals bisect each other.

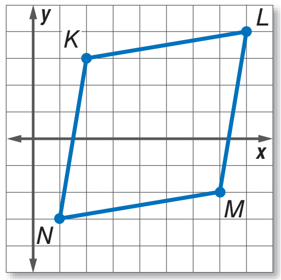
## 2 Parallelograms on the Coordinate Plane

We can use the Distance, Slope, and Midpoint Formulas to determine whether a quadrilateral in the coordinate plane is a parallelogram.

### Example 4 Parallelograms and Coordinate Geometry

**COORDINATE GEOMETRY** **Graph quadrilateral *KLMN* with vertices *K*(2, 3), *L*(8, 4), *M*(7, −2), and *N*(1, −3). Determine whether the quadrilateral is a parallelogram. Justify your answer using the Slope Formula.**

If the opposite sides of a quadrilateral are parallel, then it is a parallelogram.




slope of $\overline{KL} = \frac{4-3}{8-2}$ or $\frac{1}{6}$

slope of $\overline{NM} = \frac{-2-(-3)}{7-1}$ or $\frac{1}{6}$

slope of $\overline{KN} = \frac{-3-3}{1-2} = \frac{-6}{-1}$ or 6

slope of $\overline{LM} = \frac{-2-4}{7-8} = \frac{-6}{-1}$ or 6

Since opposite sides have the same slope, $\overline{KL} \parallel \overline{NM}$ and $\overline{KN} \parallel \overline{LM}$. Therefore, *KLMN* is a parallelogram by definition.

**Guided**Practice

**Determine whether the quadrilateral is a parallelogram. Justify your answer using the given formula.**

**4A.** *A*(3, 3), *B*(8, 2), *C*(6, −1), *D*(1, 0); Distance Formula

**4B.** *F*(−2, 4), *G*(4, 2), *H*(4, −2), *J*(−2, −1); Midpoint Formula

In Chapter 4, you learned that variable coordinates can be assigned to the vertices of triangles. Then the Distance, Slope, and Midpoint Formulas were used to write coordinate proofs of theorems. The same can be done with quadrilaterals.

**Review**Vocabulary

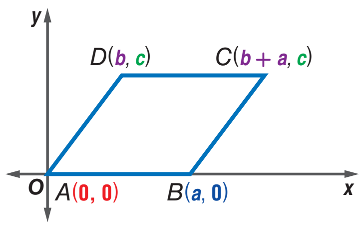
**coordinate proof** a proof that uses figures in the coordinate plane and algebra to prove geometric concepts

### Example 5 Parallelograms and Coordinate Proofs

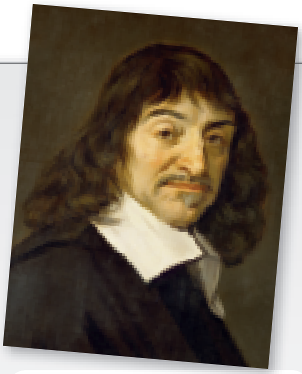
**Write a coordinate proof for the following statement.**

*If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.*

**Step 1** Position quadrilateral *ABCD* on the coordinate plane such that $\overline{AB} \parallel \overline{DC}$ and $\overline{AB} \cong \overline{DC}$.




- Begin by placing the vertex *A* at the **origin**.
- Let $\overline{AB}$ have a length of a units. Then *B* has coordinates **(*a*, 0)**.
- Since horizontal segments are parallel, position the endpoints of $\overline{DC}$ so that they have the same *y*-coordinate, ***c***.
- So that the distance from *D* to *C* is also *a* units, let the *x*-coordinate of *D* be ***b*** and of *C* be ***b* + *a***.



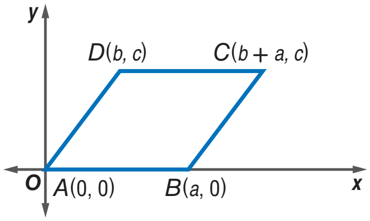
## Math HistoryLink

**René Descartes**
**(1596–1650)**
René Descartes was a French mathematician who was the first to use a coordinate grid. It has been said that he first thought of locating a point on a plane with a pair of numbers when he was watching a fly on the ceiling, but this is a myth.

**Step 2** Use your figure to write a proof.

**Given:** quadrilateral $ABCD$, $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$

**Prove:** $ABCD$ is a parallelogram.




**Coordinate Proof:**

By definition, a quadrilateral is a parallelogram if opposite sides are parallel. We are given that $\overline{AB} \parallel \overline{DC}$, so we need only show that $\overline{AD} \parallel \overline{BC}$.

Use the Slope Formula.

slope of $\overline{AD} = \frac{c-0}{b-0} = \frac{c}{b}$ slope of $\overline{BC} = \frac{c-0}{b+a-a} = \frac{c}{b}$

Since $\overline{AD}$ and $\overline{BC}$ have the same slope, $\overline{AD} \parallel \overline{BC}$. So quadrilateral $ABCD$ is a parallelogram because opposite sides are parallel.

### GuidedPractice

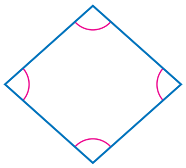
**5.** Write a coordinate proof of this statement: *If a quadrilateral is a parallelogram, then opposite sides are congruent.*

## Check Your Understanding

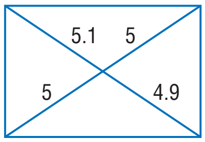
= Step-by-Step Solutions begin on page R14.

**Example 1** **Determine whether each quadrilateral is a parallelogram. Justify your answer.**

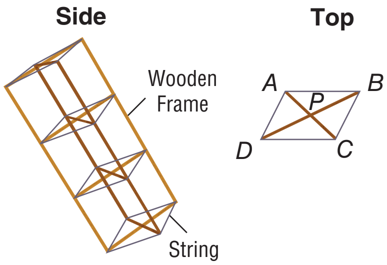
**1.**



**2.**

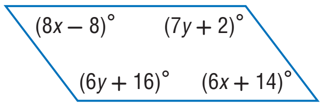



**Example 2** **3. KITES** Charmaine is building the kite shown below. She wants to be sure that the string around her frame forms a parallelogram before she secures the material to it. How can she use the measures of the wooden portion of the frame to prove that the string forms a parallelogram? Explain your reasoning.

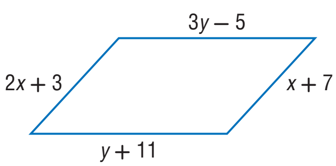



**Example 3** **ALGEBRA Find $x$ and $y$ so that the quadrilateral is a parallelogram.**

**4.**




**5**



**Example 4** **COORDINATE GEOMETRY** **Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.**

**6.** $A(-2, 4)$, $B(5, 4)$, $C(8, -1)$, $D(-1, -1)$; Slope Formula

**7.** $W(-5, 4)$, $X(3, 4)$, $Y(1, -3)$, $Z(-7, -3)$; Midpoint Formula

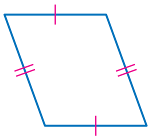
**Example 5** **8.** Write a coordinate proof for the statement: *If a quadrilateral is a parallelogram, then its diagonals bisect each other.*

## Practice and Problem Solving

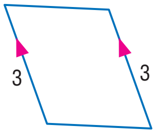
Extra Practice is on page R6.

**Example 1** **CCSS ARGUMENTS** **Determine whether each quadrilateral is a parallelogram. Justify your answer.**

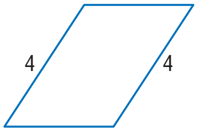
**9.**



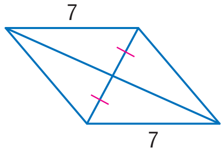
**10.**



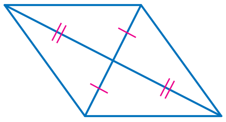

**11.**



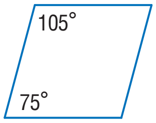

**12.**




**13.**

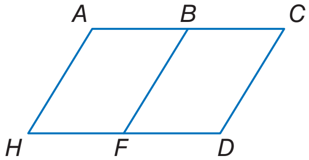


**14.**

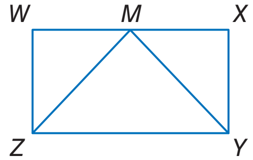



**Example 2**

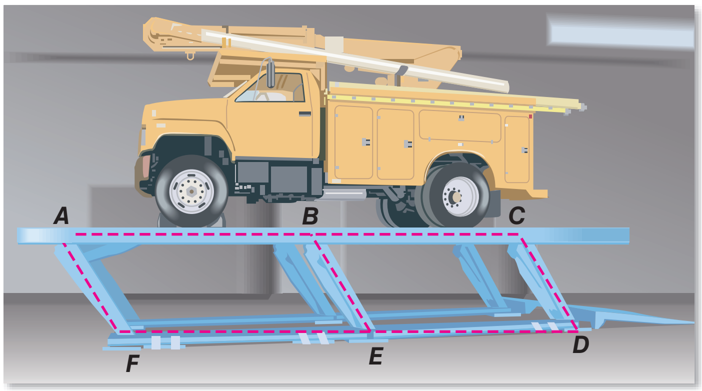
**15. PROOF** If *ACDH* is a parallelogram, *B* is the midpoint of $\overline{AC}$, and *F* is the midpoint of $\overline{HD}$, write a flow proof to prove that *ABFH* is a parallelogram.




**16. PROOF** If WXYZ is a parallelogram, $\angle W \cong \angle X$, and *M* is the midpoint of $\overline{WX}$, write a paragraph proof to prove that *ZMY* is an isosceles triangle.

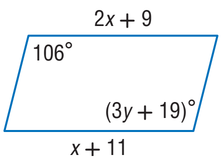



**17. REPAIR** Parallelogram lifts are used to elevate large vehicles for maintenance. In the diagram, *ABEF* and *BCDE* are parallelograms. Write a two-column proof to show that *ACDF* is also a parallelogram.

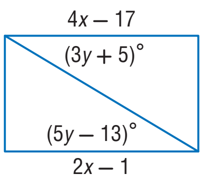



**Example 3** **ALGEBRA** **Find *x* and *y* so that the quadrilateral is a parallelogram.**

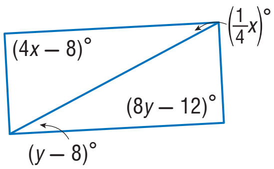
**18.**




**19.**

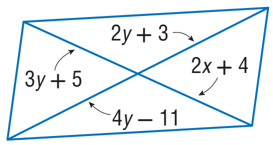



**20.**

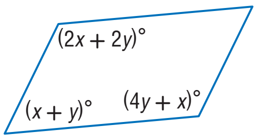


**ALGEBRA** **Find $x$ and $y$ so that the quadrilateral is a parallelogram.**

**21.**




**22.**

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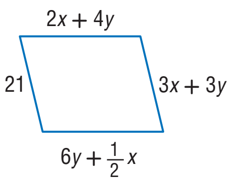



**Example 4** **COORDINATE GEOMETRY** **Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.**

**24.** $A(-3, 4)$, $B(4, 5)$, $C(5, -1)$, $D(-2, -2)$; Slope Formula

**25.** $J(-4, -4)$, $K(-3, 1)$, $L(4, 3)$, $M(3, -3)$; Distance Formula

**26.** $V(3, 5)$, $W(1, -2)$, $X(-6, 2)$, $Y(-4, 7)$; Slope Formula

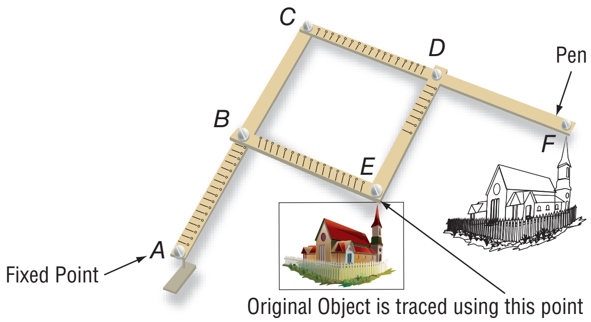
**27.** $Q(2, -4)$, $R(4, 3)$, $S(-3, 6)$, $T(-5, -1)$; Distance and Slope Formulas

**Example 5** **28.** Write a coordinate proof for the statement: *If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.*

**29.** Write a coordinate proof for the statement: *If a parallelogram has one right angle, it has four right angles.*

**30.** **PROOF** Write a paragraph proof of Theorem 6.10.

**31** **PANTOGRAPH** A pantograph is a device that can be used to copy an object and either enlarge or reduce it based on the dimensions of the pantograph.




**a.** If $\overline{AC} \cong \overline{CF}$, $\overline{AB} \cong \overline{CD} \cong \overline{BE}$, and $\overline{DF} \cong \overline{DE}$, write a paragraph proof to show that $\overline{BE} \parallel \overline{CD}$.

**b.** The scale of the copied object is the ratio of $CF$ to $BE$. If $AB$ is 12 inches, $DF$ is 8 inches, and the width of the original object is 5.5 inches, what is the width of the copy?

**PROOF** **Write a two-column proof.**

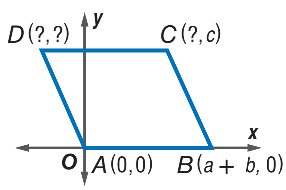
**32.** Theorem 6.11

**33.** Theorem 6.12

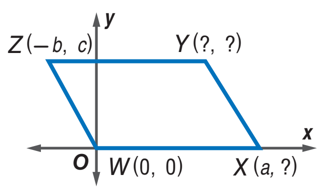
**34.** **CONSTRUCTION** Explain how you can use Theorem 6.11 to construct a parallelogram. Then construct a parallelogram using your method.

**CCSS** **REASONING** **Name the missing coordinates for each parallelogram.**

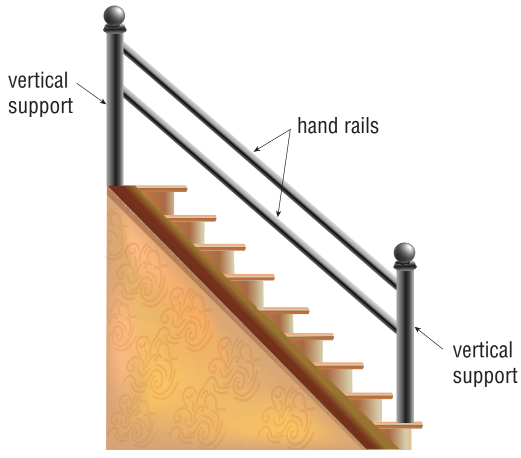
**35**



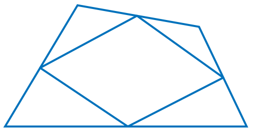

**36.**



**37** **SERVICE** While replacing a hand rail, a contractor uses a carpenter's square to confirm that the vertical supports are perpendicular to the top step and the ground, respectively. How can the contractor prove that the two hand rails are parallel using the fewest measurements? Assume that the top step and the ground are both level.




**38.** **PROOF** Write a coordinate proof to prove that the segments joining the midpoints of the sides of any quadrilateral form a parallelogram.



**39.** **MULTIPLE REPRESENTATIONS** In this problem, you will explore the properties of rectangles. A rectangle is a quadrilateral with four right angles.

**a. Geometric** Draw three rectangles with varying lengths and widths. Label one rectangle *ABCD*, one *MNOP*, and one *WXYZ*. Draw the two diagonals for each rectangle.

**b. Tabular** Measure the diagonals of each rectangle, and complete the table at the right.

**c. Verbal** Write a conjecture about the diagonals of a rectangle.

| Rectangle | Side | Length |
|---|---|---|
| *ABCD* | $\overline{AC}$ | |
| | $\overline{BD}$ | |
| *MNOP* | $\overline{MO}$ | |
| | $\overline{NP}$ | |
| *WXYZ* | $\overline{WY}$ | |
| | $\overline{XZ}$ | |

## H.O.T. Problems Use Higher-Order Thinking Skills

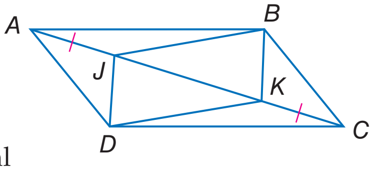
**40.** **CHALLENGE** The diagonals of a parallelogram meet at the point (0, 1). One vertex of the parallelogram is located at (2, 4), and a second vertex is located at (3, 1). Find the locations of the remaining vertices.

**41.** **WRITING IN MATH** Compare and contrast Theorem 6.9 and Theorem 6.3.

**42.** CCSS **ARGUMENTS** If two parallelograms have four congruent corresponding angles, are the parallelograms *sometimes*, *always*, or *never* congruent?

**43.** **OPEN ENDED** Position and label a parallelogram on the coordinate plane differently than shown in either Example 5, Exercise 35, or Exercise 36.

**44.** **CHALLENGE** If *ABCD* is a parallelogram and $\overline{AJ} \cong \overline{KC}$, show that quadrilateral *JBKD* is a parallelogram.




**45.** **WRITING IN MATH** How can you prove that a quadrilateral is a parallelogram?

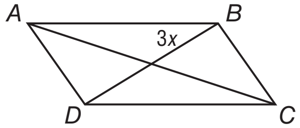
## Standardized Test Practice

**46.** If sides $\overline{AB}$ and $\overline{DC}$ of quadrilateral $ABCD$ are parallel, which additional information would be sufficient to prove that quadrilateral $ABCD$ is a parallelogram?

**A** $\overline{AB} \cong \overline{AC}$  **C** $\overline{AC} \cong \overline{BD}$

**B** $\overline{AB} \cong \overline{DC}$  **D** $\overline{AD} \cong \overline{BC}$

**47. SHORT RESPONSE** Quadrilateral $ABCD$ is shown. $AC$ is 40 and $BD$ is $\frac{3}{5}AC$. $\overline{BD}$ bisects $\overline{AC}$. For what value of $x$ is $ABCD$ a parallelogram?




**48. ALGEBRA** Jarod's average driving speed for a 5-hour trip was 58 miles per hour. During the first 3 hours, he drove 50 miles per hour. What was his average speed in miles per hour for the last 2 hours of his trip?

**F** 70  **H** 60

**G** 66  **J** 54

**49. SAT/ACT** A parallelogram has vertices at (0, 0), (3, 5), and (0, 5). What are the coordinates of the fourth vertex?

**A** (0, 3)  **D** (0, −3)

**B** (5, 3)  **E** (3, 0)

**C** (5, 0)

## Spiral Review

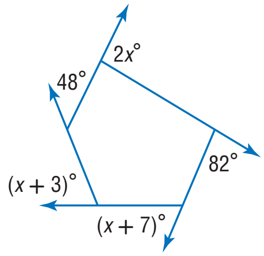
**COORDINATE GEOMETRY** **Find the coordinates of the intersection of the diagonals of ▱*ABCD* with the given vertices.** (Lesson 6-2)

**50.** $A(-3, 5)$, $B(6, 5)$, $C(5, -4)$, $D(-4, -4)$

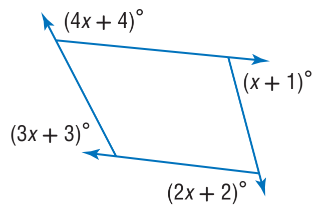
**51.** $A(2, 5)$, $B(10, 7)$, $C(7, -2)$, $D(-1, -4)$

**Find the value of *x*.** (Lesson 6-1)

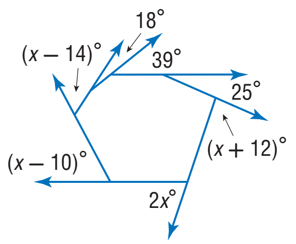
**52.**




**53.**



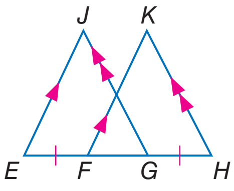

**54.**



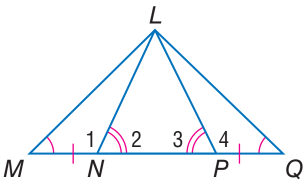

**55. FITNESS** Toshiro was at the gym for just over two hours. He swam laps in the pool and lifted weights. Prove that he did one of these activities for more than an hour. (Lesson 5-4)

**PROOF** **Write a flow proof.** (Lesson 4-5)

**56. Given:** $\overline{EJ} \parallel \overline{FK}$, $\overline{JG} \parallel \overline{KH}$, $\overline{EF} \cong \overline{GH}$
**Prove:** $\triangle EJG \cong \triangle FKH$




**57. Given:** $\overline{MN} \cong \overline{PQ}$, $\angle M \cong \angle Q$, $\angle 2 \cong \angle 3$
**Prove:** $\triangle MLP \cong \triangle QLN$




## Skills Review

**Use slope to determine whether *XY* and *YZ* are *perpendicular* or *not perpendicular*.**

**58.** $X(-2, 2)$, $Y(0, 1)$, $Z(4, 1)$

**59.** $X(4, 1)$, $Y(5, 3)$, $Z(6, 2)$