LESSON 6-4

Rectangles

Then
- You used properties of parallelograms and determined whether quadrilaterals were parallelograms.

Now
1. Recognize and apply properties of rectangles.
2. Determine whether parallelograms are rectangles.

Why?
- Leonardo is in charge of set design for a school play. He needs to use paint to create the appearance of a doorway on a lightweight solid wall. The doorway is to be a rectangle 36 inches wide and 80 inches tall. How can Leonardo be sure that he paints a rectangle?

New Vocabulary
rectangle

Common Core State Standards

Content Standards
G.CO.11 Prove theorems about parallelograms.
G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.

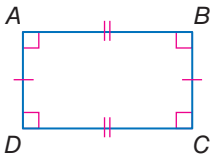
Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.
5 Use appropriate tools strategically.

1 Properties of Rectangles

A **rectangle** is a parallelogram with four right angles. By definition, a rectangle has the following properties.

- All four angles are right angles.
- Opposite sides are parallel and congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- Diagonals bisect each other.

In addition, the diagonals of a rectangle are congruent.



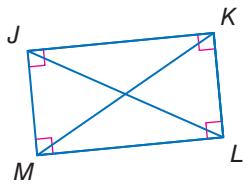
Rectangle *ABCD*

Theorem 6.13 Diagonals of a Rectangle

If a parallelogram is a rectangle, then its diagonals are congruent.

Abbreviation *If a ▱ is a rectangle, diag. are ≅.*

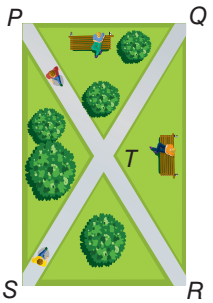
Example If ▱*JKLM* is a rectangle, then $\overline{JL} \cong \overline{MK}$.



You will prove Theorem 6.13 in Exercise 33.

Real-World Example 1 Use Properties of Rectangles

EXERCISE **A rectangular park has two walking paths as shown. If $PS = 180$ meters and $PR = 200$ meters, find QT.**



$\overline{QS} \cong \overline{PR}$ If a ▱ is a rectangle, diag. are ≅.
$QS = PR$ Definition of congruence
$QS = 200$ Substitution

Since $PQRS$ is a rectangle, it is a parallelogram. The diagonals of a parallelogram bisect each other, so $QT = ST$.

$QT + ST = QS$ Segment Addition
$QT + QT = QS$ Substitution
$2QT = QS$ Simplify.
$QT = \frac{1}{2}QS$ Divide each side by 2.
$QT = \frac{1}{2}(200)$ or 100 Substitution

GuidedPractice **Refer to the figure in Example 1.**

1A. If $TS = 120$ meters, find PR.

1B. If $m\angle PRS = 64$, find $m\angle SQR$.

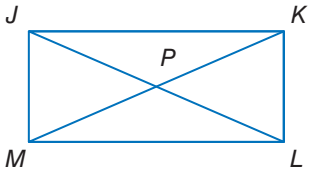
You can use the properties of rectangles along with algebra to find missing values.

StudyTip

Right Angles Recall from Theorem 6-6 that if a parallelogram has one right angle, then it has four right angles.

Example 2 Use Properties of Rectangles and Algebra

ALGEBRA **Quadrilateral *JKLM* is a rectangle. If $m\angle KJL = 2x + 4$ and $m\angle JLK = 7x + 5$, find x.**



Since *JKLM* is a rectangle, it has four right angles. So, $m\angle MLK = 90$. Since a rectangle is a parallelogram, opposite sides are parallel. Alternate interior angles of parallel lines are congruent, so $\angle JLM \cong \angle KJL$ and $m\angle JLM = m\angle KJL$.

$m\angle JLM + m\angle JLK = 90$ Angle Addition

$m\angle KJL + m\angle JLK = 90$ Substitution

$2x + 4 + 7x + 5 = 90$ Substitution

$9x + 9 = 90$ Add like terms.

$9x = 81$ Subtract 9 from each side.

$x = 9$ Divide each side by 9.

GuidedPractice

2. Refer to the figure in Example 2. If $JP = 3y - 5$ and $MK = 5y + 1$, find y.

2 Prove that Parallelograms are Rectangles

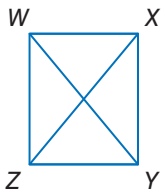
The converse of Theorem 6.13 is also true.

Theorem 6.14 Diagonals of a Rectangle

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Abbreviation *If diag. of a ▱ are ≅, then ▱ is a rectangle.*

Example If $\overline{WY} \cong \overline{XZ}$ in ▱*WXYZ*, then ▱*WXYZ* is a rectangle.



You will prove Theorem 6.14 in Exercise 34.

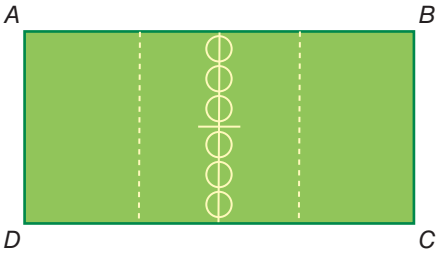
Real-WorldLink

The game of dodgeball is played on a rectangular playing field ideally 60 feet long and 30 feet wide. The field is divided into two equal sections by a center-line and attack-lines that are 3 meters (9.8 feet) from, and parallel to, the centerline.

Source: National Amateur Dodgeball Assoc.

Real-World Example 3 Providing Rectangle Relationships

DODGEBALL **A community recreation center has created an outdoor dodgeball playing field. To be sure that it meets the ideal playing field requirements, they measure the sides of the field and its diagonals. If $AB = 60$ feet, $BC = 30$ feet, $CD = 60$ feet, $AD = 30$ feet, $AC = 67$ feet, and $BD = 67$ feet, explain how the recreation center can be sure that the playing field is rectangular.**



Since $AB = CD$, $BC = AD$, and $AC = BD$, $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$, and $\overline{AC} \cong \overline{BD}$. Because $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$, *ABCD* is a parallelogram. Since $\overline{AC}$ and $\overline{BD}$ are congruent diagonals in ▱*ABCD*, ▱*ABCD* is a rectangle.

Micah Walter/Getty Images

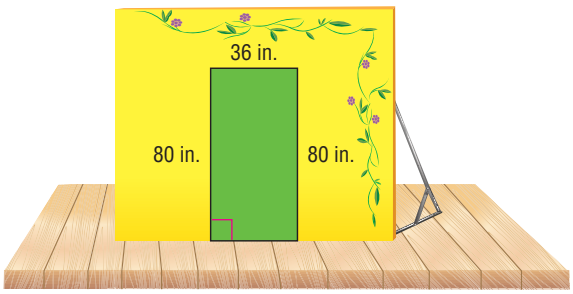
Real-World Link

The Mosaic Youth Theater in Detroit, Michigan, is a professional performing arts training program for young people ages 12 to 18. Students are involved in all aspects of performances, including set and lighting design, set construction, stage management, sound, and costumes.

Jim West/age fotostock

Guided Practice

3. SET DESIGN Refer to the beginning of the lesson. Leonardo measures the sides of his figure and confirms that they have the desired measures as shown. Using a carpenter's square, he also confirms that the measure of the bottom left corner of the figure is a right angle. Can he conclude that the figure is a rectangle? Explain.

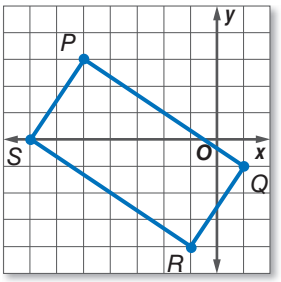


You can also use the properties of rectangles to prove that a quadrilateral positioned on a coordinate plane is a rectangle given the coordinates of the vertices.

Example 4 Rectangles and Coordinate Geometry

COORDINATE GEOMETRY **Quadrilateral $PQRS$ has vertices $P(-5, 3)$, $Q(1, -1)$, $R(-1, -4)$, and $S(-7, 0)$. Determine whether $PQRS$ is a rectangle by using the Distance Formula.**

Step 1 Use the Distance Formula to determine whether $PQRS$ is a parallelogram by determining if opposite sides are congruent.



$$PQ = \sqrt{(-5 - 1)^2 + [3 - (-1)]^2} \text{ or } \sqrt{52}$$

$$RS = \sqrt{[-1 - (-7)]^2 + (-4 - 0)^2} \text{ or } \sqrt{52}$$

$$PS = \sqrt{[-5 - (-7)]^2 + (3 - 0)^2} \text{ or } \sqrt{13}$$

$$QR = \sqrt{[1 - (-1)^2 + [-1 - (-4)]^2} \text{ or } \sqrt{13}$$

Study Tip

Rectangles and Parallelograms A rectangle is a parallelogram, but a parallelogram is not necessarily a rectangle.

Since opposite sides of the quadrilateral have the same measure, they are congruent. So, quadrilateral $PQRS$ is a parallelogram.

Step 2 Determine whether the diagonals of $\square PQRS$ are congruent.

$$PR = \sqrt{[-5 - (-1)]^2 + [3 - (-4)]^2} \text{ or } \sqrt{65}$$

$$QS = \sqrt{[1 - (-7)]^2 + (-1 - 0)^2} \text{ or } \sqrt{65}$$

Since the diagonals have the same measure, they are congruent. So, $\square PQRS$ is a rectangle.

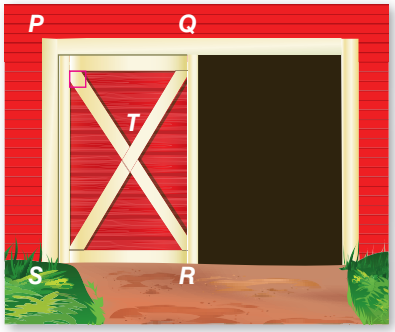
Guided Practice

4. Quadrilateral $JKLM$ has vertices $J(-10, 2)$, $K(-8, -6)$, $L(5, -3)$, and $M(2, 5)$. Determine whether $JKLM$ is a rectangle using the Slope Formula.

Check Your Understanding

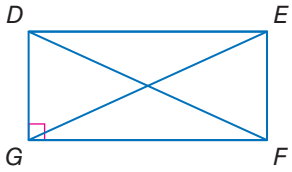
= Step-by-Step Solutions begin on page R14.

Example 1 **FARMING** An X-brace on a rectangular barn door is both decorative and functional. It helps to prevent the door from warping over time. If $ST = 3\frac{13}{16}$ feet, $PS = 7$ feet, and $m\angle PTQ = 67$, find each measure.



1. QR
2. SQ
3. $m\angle TQR$
4. $m\angle TSR$

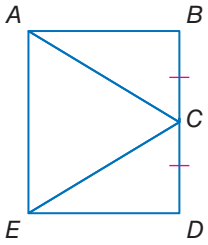
Example 2 **ALGEBRA** Quadrilateral $DEFG$ is a rectangle.



5. If $FD = 3x - 7$ and $EG = x + 5$, find EG.
6. If $m\angle EFD = 2x - 3$ and $m\angle DFG = x + 12$, find $m\angle EFD$.

Example 3

7. **PROOF** If $ABDE$ is a rectangle and $\overline{BC} \cong \overline{DC}$, prove that $\overline{AC} \cong \overline{EC}$.



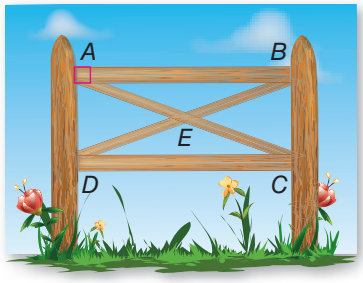
Example 4 **COORDINATE GEOMETRY** Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

8. $W(-4, 3)$, $X(1, 5)$, $Y(3, 1)$, $Z(-2, -2)$; Slope Formula
9. $A(4, 3)$, $B(4, -2)$, $C(-4, -2)$, $D(-4, 3)$; Distance Formula

Practice and Problem Solving

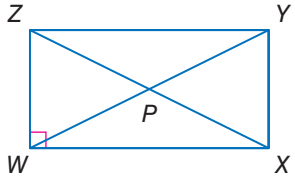
Extra Practice is on page R6.

Example 1 **FENCING** X-braces are also used to provide support in rectangular fencing. If $AB = 6$ feet, $AD = 2$ feet, and $m\angle DAE = 65$, find each measure.



10. BC
11. DB
12. $m\angle CEB$
13. $m\angle EDC$

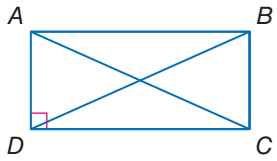
Example 2 CCSS **REGULARITY** Quadrilateral $WXYZ$ is a rectangle.



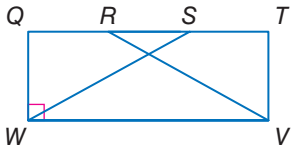
14. If $ZY = 2x + 3$ and $WX = x + 4$, find WX.
15. If $PY = 3x - 5$ and $WP = 2x + 11$, find ZP.
16. If $m\angle ZYW = 2x - 7$ and $m\angle WYX = 2x + 5$, find $m\angle ZYW$.
17. If $ZP = 4x - 9$ and $PY = 2x + 5$, find ZX.
18. If $m\angle XZY = 3x + 6$ and $m\angle XZW = 5x - 12$, find $m\angle YXZ$.
19. If $m\angle ZXW = x - 11$ and $m\angle WZX = x - 9$, find $m\angle ZXY$.

Example 3 **PROOF** **Write a two-column proof.**

20. **Given:** $ABCD$ is a rectangle.
Prove: $\triangle ADC \cong \triangle BCD$



21. **Given:** $QTVW$ is a rectangle.
$\overline{QR} \cong \overline{ST}$
Prove: $\triangle SWQ \cong \triangle RVT$



Example 4 **COORDINATE GEOMETRY** **Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.**

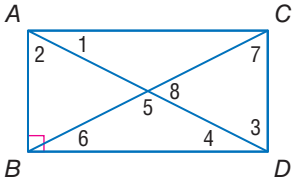
22. $W(-2, 4)$, $X(5, 5)$, $Y(6, -2)$, $Z(-1, -3)$; Slope Formula

23. $J(3, 3)$, $K(-5, 2)$, $L(-4, -4)$, $M(4, -3)$; Distance Formula

24. $Q(-2, 2)$, $R(0, -2)$, $S(6, 1)$, $T(4, 5)$; Distance Formula

25. $G(1, 8)$, $H(-7, 7)$, $J(-6, 1)$, $K(2, 2)$; Slope Formula

Quadrilateral $ABCD$ is a rectangle. Find each measure if $m\angle 2 = 40$.



26. $m\angle 1$

27. $m\angle 7$

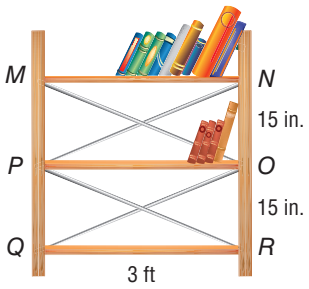
28. $m\angle 3$

29 $m\angle 5$

30. $m\angle 6$

31. $m\angle 8$

32. CCSS **MODELING** Jody is building a new bookshelf using wood and metal supports like the one shown. To what length should she cut the metal supports in order for the bookshelf to be *square*, which means that the angles formed by the shelves and the vertical supports are all right angles? Explain your reasoning.



PROOF **Write a two-column proof.**

33. Theorem 6.13

34. Theorem 6.14

PROOF **Write a paragraph proof of each statement.**

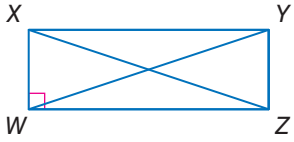
35. If a parallelogram has one right angle, then it is a rectangle.

36. If a quadrilateral has four right angles, then it is a rectangle.

37. **CONSTRUCTION** Construct a rectangle using the construction for congruent segments and the construction for a line perpendicular to another line through a point on the line. Justify each step of the construction.

38. **SPORTS** The end zone of a football field is 160 feet wide and 30 feet long. Kyle is responsible for painting the field. He has finished the end zone. Explain how Kyle can confirm that the end zone is the regulation size and be sure that it is also a rectangle using only a tape measure.

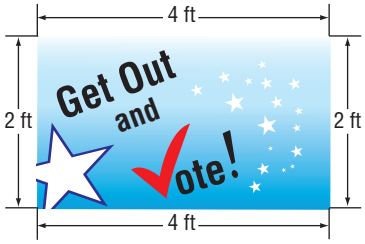
ALGEBRA **Quadrilateral $WXYZ$ is a rectangle.**



39. If $XW = 3$, $WZ = 4$, and $XZ = b$, find YW.

40. If $XZ = 2c$ and $ZY = 6$, and $XY = 8$, find WY.

41. SIGNS The sign below is in the foyer of Nyoko's school. Based on the dimensions given, can Nyoko be sure that the sign is a rectangle? Explain your reasoning.



PROOF Write a coordinate proof of each statement.

42. The diagonals of a rectangle are congruent.

43 If the diagonals of a parallelogram are congruent, then it is a rectangle.

44. MULTIPLE REPRESENTATIONS In the problem, you will explore properties of other special parallelograms.

a. Geometric Draw three parallelograms, each with all four sides congruent. Label one parallelogram *ABCD*, one *MNOP*, and one *WXYZ*. Draw the two diagonals of each parallelogram and label the intersections *R*.

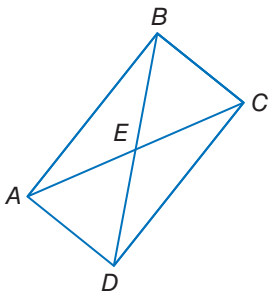
b. Tabular Use a protractor to measure the appropriate angles and complete the table below.

Parallelogram	*ABCD*		*MNOP*		*WXYZ*	
Angle	∠*ARB*	∠*BRC*	∠*MRN*	∠*NRO*	∠*WRX*	∠*XRY*
Angle Measure						

c. Verbal Make a conjecture about the diagonals of a parallelogram with four congruent sides.

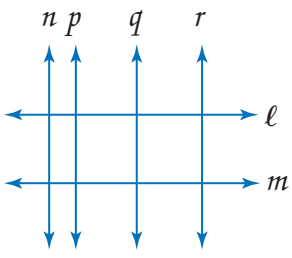
H.O.T. Problems Use Higher-Order Thinking Skills

45. CHALLENGE In rectangle *ABCD*, $m\angle EAB = 4x + 6$, $m\angle DEC = 10 - 11y$, and $m\angle EBC = 60$. Find the values of x and y.



46. CCSS CRITIQUE Parker says that any two congruent acute triangles can be arranged to make a rectangle. Tamika says that only two congruent right triangles can be arranged to make a rectangle. Is either of them correct? Explain your reasoning.

47. REASONING In the diagram at the right, lines n, p, q, and r are parallel and lines ℓ and m are parallel. How many rectangles are formed by the intersecting lines?

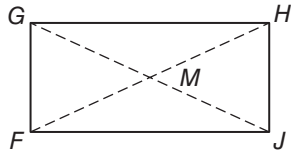


48. OPEN ENDED Write the equations of four lines having intersections that form the vertices of a rectangle. Verify your answer using coordinate geometry.

49. WRITING IN MATH Why are all rectangles parallelograms, but all parallelograms are not rectangles? Explain.

Standardized Test Practice

50. If $FJ = -3x + 5y$, $FM = 3x + y$, $GH = 11$, and $GM = 13$, what values of x and y make parallelogram $FGHJ$ a rectangle?

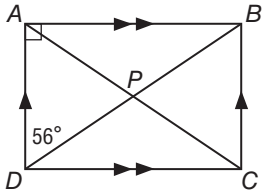


A $x = 3, y = 4$
B $x = 4, y = 3$
C $x = 7, y = 8$
D $x = 8, y = 7$

51. ALGEBRA A rectangular playground is surrounded by an 80-foot fence. One side of the playground is 10 feet longer than the other. Which of the following equations could be used to find r, the shorter side of the playground?

F $10r + r = 80$
G $4r + 10 = 80$
H $r(r + 10) = 80$
J $2(r + 10) + 2r = 80$

52. SHORT RESPONSE What is the measure of $\angle APB$?



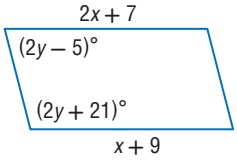
53. SAT/ACT If p is odd, which of the following must also be odd?

A $2p$
B $2p + 2$
C $\frac{p}{2}$
D $2p - 2$
E $p + 2$

Spiral Review

ALGEBRA Find x and y so that the quadrilateral is a parallelogram. (Lesson 6-3)

54.



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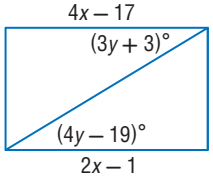
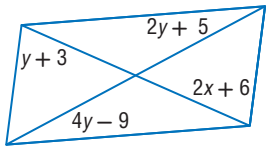


56.



57. COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of $\square ABCD$ with vertices $A(1, 3)$, $B(6, 2)$, $C(4, -2)$, and $D(-1, -1)$. (Lesson 6-2)

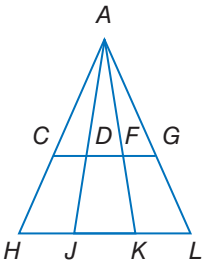
Refer to the figure at the right. (Lesson 4-6)

58. If $\overline{AC} \cong \overline{AF}$, name two congruent angles.

59. If $\angle AHJ \cong \angle AJH$, name two congruent segments.

60. If $\angle AJL \cong \angle ALJ$, name two congruent segments.

61. If $\overline{JA} \cong \overline{KA}$, name two congruent angles.



Skills Review

Find the distance between each pair of points.

62. (4, 2), (2, −5)
63. (0, 6), (−1, −4)
64. (−4, 3), (3, −4)