LESSON 6-5

Rhombi and Squares

Then

- You determined whether quadrilaterals were parallelograms and/or rectangles.

Now

- 1 Recognize and apply the properties of rhombi and squares.
- 2 Determine whether quadrilaterals are rectangles, rhombi, or squares.

Why?

- Some fruits, nuts, and vegetables are packaged using bags made out of rhombus-shaped tubular netting. Similar shaped nylon netting is used for goals in such sports as soccer, hockey, and football. A rhombus and a square are both types of equilateral parallelograms.

NewVocabulary

rhombus
square

Common Core State Standards

Content Standards

G.CO.11 Prove theorems about parallelograms.

G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.

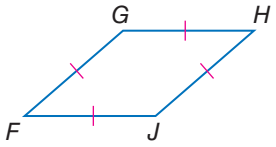
Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

2 Reason abstractly and quantitatively.

1 Properties of Rhombi and Squares

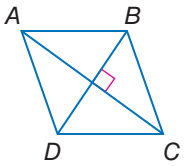
A **rhombus** is a parallelogram with all four sides congruent. A rhombus has all the properties of a parallelogram and the two additional characteristics described in the theorems below.



Theorems Diagonals of a Rhombus

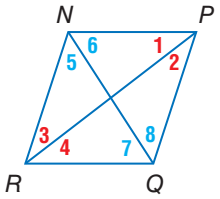
6.15 If a parallelogram is a rhombus, then its diagonals are perpendicular.

Example If $\square ABCD$ is a rhombus, then $\overline{AC} \perp \overline{BD}$.



6.16 If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.

Example If $\square NPQR$ is a rhombus, then $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$, $\angle 5 \cong \angle 6$, and $\angle 7 \cong \angle 8$.



You will prove Theorem 6.16 in Exercise 34.

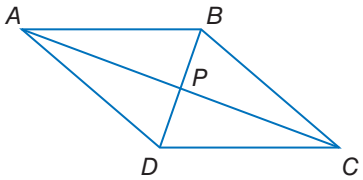
Proof Theorem 6.15

Given: $ABCD$ is a rhombus.

Prove: $\overline{AC} \perp \overline{BD}$

Paragraph Proof:

Since $ABCD$ is a rhombus, by definition $\overline{AB} \cong \overline{BC}$. A rhombus is a parallelogram and the diagonals of a parallelogram bisect each other, so $\overline{BD}$ bisects $\overline{AC}$ at P. Thus, $\overline{AP} \cong \overline{PC}$. $\overline{BP} \cong \overline{BP}$ by the Reflexive Property. So, $\triangle APB \cong \triangle CPB$ by SSS. $\angle APB \cong \angle CPB$ by CPCTC. $\angle APB$ and $\angle CPB$ also form a linear pair. Two congruent angles that form a linear pair are right angles. $\angle APB$ is a right angle, so $\overline{AC} \perp \overline{BD}$ by the definition of perpendicular lines.

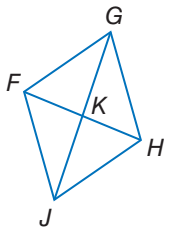


ReadingMath

Rhombi The plural form of rhombus is *rhombi*, pronounced ROM-bye.

Example 1 Use Properties of a Rhombus

The diagonals of rhombus *FGHJ* intersect at *K*. Use the given information to find each measure or value.



a. If $m\angle FJH = 82$, find $m\angle KHJ$.

Since *FGHJ* is a rhombus, diagonal $\overline{JG}$ bisects $\angle FJH$. Therefore, $m\angle KJH = \frac{1}{2}m\angle FJH$. So $m\angle KJH = \frac{1}{2}(82)$ or 41. Since the diagonals of a rhombus are perpendicular, $m\angle JKH = 90$ by the definition of perpendicular lines.

$m\angle KJH + m\angle JKH + m\angle KHJ = 180$	Triangle Sum Theorem
$41 + 90 + m\angle KHJ = 180$	Substitution
$131 + m\angle KHJ = 180$	Simplify.
$m\angle KHJ = 49$	Subtract 131 from each side.

b. ALGEBRA If $GH = x + 9$ and $JH = 5x - 2$, find x.

$\overline{GH} \cong \overline{JH}$	By definition, all sides of a rhombus are congruent.
$GH = JH$	Definition of congruence
$x + 9 = 5x - 2$	Substitution
$9 = 4x - 2$	Subtract x from each side.
$11 = 4x$	Add 2 to each side.
$2.75 = x$	Divide each side by 4.

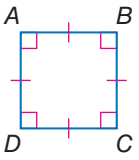
GuidedPractice

Refer to rhombus *FGHJ* above.

1A. If $FK = 5$ and $FG = 13$, find KJ.

1B. ALGEBRA If $m\angle JFK = 6y + 7$ and $m\angle KFG = 9y - 5$, find y.

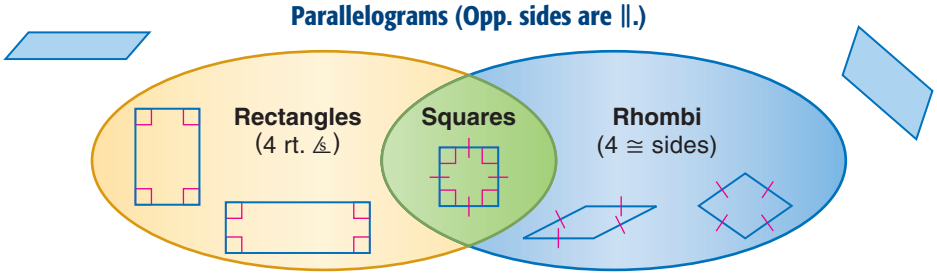
A **square** is a parallelogram with four congruent sides and four right angles. Recall that a parallelogram with four right angles is a rectangle, and a parallelogram with four congruent sides is a rhombus. Therefore, a parallelogram that is both a rectangle and a rhombus is also a square.



Square *ABCD*

The Venn diagram summarizes the relationships among parallelograms, rhombi, rectangles, and squares.

ConceptSummary Parallelograms



All of the properties of parallelograms, rectangles, and rhombi apply to squares. For example, the diagonals of a square bisect each other (parallelogram), are congruent (rectangle), and are perpendicular (rhombus).

2 Prove that Quadrilaterals are Rhombi or Squares

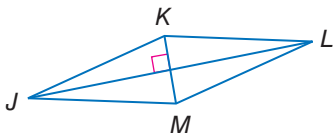
The theorems below provide conditions for rhombi and squares.

StudyTip
Common Misconception
Theorems 6.17, 6.18, and 6.19 apply only if you already know that a quadrilateral is a parallelogram.

Theorems Conditions for Rhombi and Squares

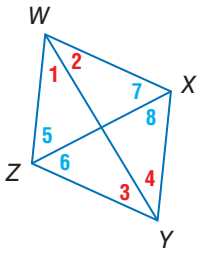
6.17 If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (Converse of Theorem. 6.15)

Example If $\overline{JL} \perp \overline{KM}$, then $\square JKLM$ is a rhombus.



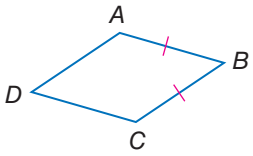
6.18 If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus. (Converse of Theorem. 6.16)

Example If $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, or $\angle 5 \cong \angle 6$ and $\angle 7 \cong \angle 8$, then $\square WXYZ$ is a rhombus.



6.19 If one pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.

Example If $\overline{AB} \cong \overline{BC}$, then $\square ABCD$ is a rhombus.



6.20 If a quadrilateral is both a rectangle and a rhombus, then it is a square.

You will prove Theorems 6.17–6.20 in Exercises 35–38, respectively.

You can use the properties of rhombi and squares to write proofs.

Example 2 Proofs Using Properties of Rhombi and Squares

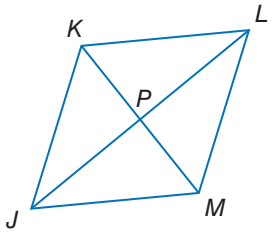
StudyTip
Congruent Triangles
Since a rhombus has four congruent sides, one diagonal separates the rhombus into two congruent isosceles triangles. Drawing two diagonals separates the rhombus into four congruent right triangles.

Write a paragraph proof.

Given: $JKLM$ is a parallelogram.

$\triangle JKL$ is isosceles.

Prove: $JKLM$ is a rhombus.



Paragraph Proof:

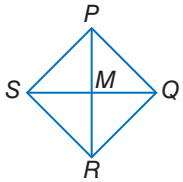
Since it is given that $\triangle JKL$ is isosceles, $\overline{KL} \cong \overline{JK}$ by definition. These are consecutive sides of the given parallelogram $JKLM$. So, by Theorem 6.19, $JKLM$ is a rhombus.

GuidedPractice

2. Write a paragraph proof.

Given: $\overline{SQ}$ is the perpendicular bisector of $\overline{PR}$.
$\overline{PR}$ is the perpendicular bisector of $\overline{SQ}$.
$\triangle RMS$ is isosceles.

Prove: $PQRS$ is a square.



Real-World Link

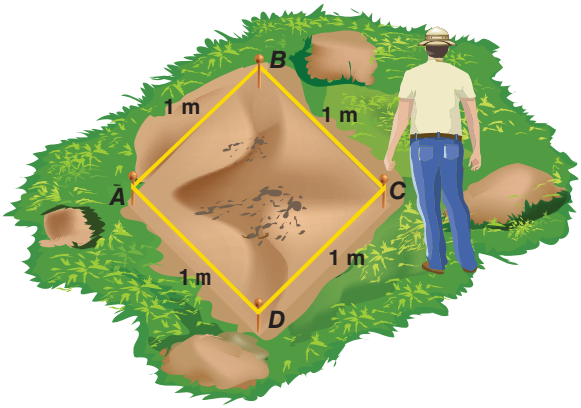
Archaeology is the study of artifacts that provide information about human life and activities in the past. Since humans only began writing about 5000 years ago, information from periods before that time must be gathered from the objects that archeologists locate.

Source: Encyclopeadia Britannica

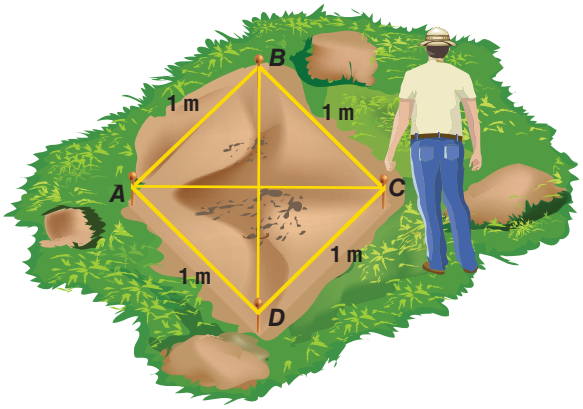
Kenneth Garrett/National Geographic/Getty Images

Real-World Example 3 Use Conditions for Rhombi and Squares

ARCHAEOLOGY **The key to the successful excavation of an archaeological site is accurate mapping. How can archaeologists be sure that the region they have marked off is a 1-meter by 1-meter square?**



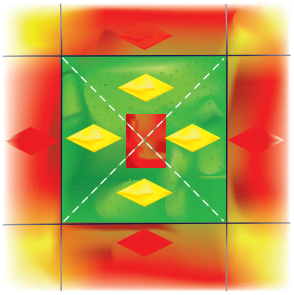
Each side of quadrilateral $ABCD$ measures 1 meter. Since opposite sides are congruent, $ABCD$ is a parallelogram. Since consecutive sides of $\square ABCD$ are congruent, it is a rhombus. If the archaeologists can show that $\square ABCD$ is also a rectangle, then by Theorem 6.20, $\square ABCD$ is a square.



If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle. So if the archeologists measure the length of string needed to form each diagonal and find that these lengths are equal, then $ABCD$ is a square.

GuidedPractice

3. QUILTING Kathy is designing a quilt with blocks like the one shown.



A. If she marks the diagonals of each yellow piece and determines that each pair of diagonals is perpendicular, can she conclude that each yellow piece is a rhombus? Explain.

B. If all four angles of the green piece have the same measure and the bottom and left sides have the same measure, can she conclude that the green piece is a square? Explain.

In Chapter 4, you used coordinate geometry to classify triangles. Coordinate geometry can also be used to classify quadrilaterals.

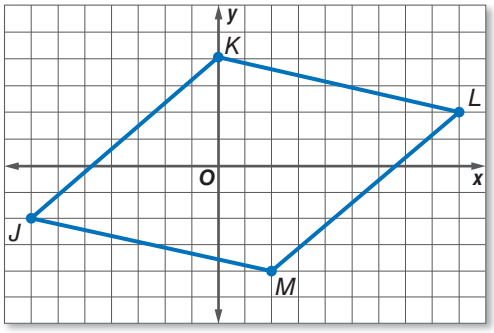
Example 4 Classify Quadrilaterals Using Coordinate Geometry

COORDINATE GEOMETRY **Determine whether $\square JKLM$ with vertices $J(-7, -2)$, $K(0, 4)$, $L(9, 2)$, and $M(2, -4)$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.**

Problem-Solving Tip
Make a Graph When analyzing a figure using coordinate geometry, graph the figure to help formulate a conjecture and also to help check the reasonableness of the answer you obtain algebraically.

Understand Plot and connect the vertices on a coordinate plane.

It appears from the graph that the parallelogram has four congruent sides, but no right angles. So, it appears that the figure is a rhombus, but not a square or a rectangle.



Plan If the diagonals of the parallelogram are congruent, then it is a rectangle. If they are perpendicular, then it is a rhombus. If they are both congruent and perpendicular, the parallelogram is a rectangle, a rhombus, and a square.

Solve **Step 1** Use the Distance Formula to compare the diagonal lengths.

$KM = \sqrt{(2 - 0)^2 + (-4 - 4)^2} = \sqrt{68}$ or $2\sqrt{17}$

$JL = \sqrt{[9 - (-7)]^2 + [2 - (-2)]^2} = \sqrt{272}$ or $4\sqrt{17}$

Since $2\sqrt{17} \neq 4\sqrt{17}$, the diagonals are not congruent. So, $\square JKLM$ is *not* a rectangle. Since the figure is not a rectangle, it also *cannot* be a square.

Study Tip
Square and Rhombus
A square is a rhombus, but a rhombus is not necessarily a square.

Step 2 Use the Slope Formula to determine whether the diagonals are perpendicular.

slope of $\overline{KM} = \frac{-4 - 4}{2 - 0} = \frac{-8}{2}$ or -4

slope of $\overline{JL} = \frac{2 - (-2)}{9 - (-7)} = \frac{4}{16}$ or $\frac{1}{4}$

Since the product of the slopes of the diagonals is -1, the diagonals are perpendicular, so $\square JKLM$ is a rhombus.

Check $JK = \sqrt{[4 - (-2)]^2 + [0 - (-7)]^2}$ or $\sqrt{85}$

$KL = \sqrt{(9 - 0)^2 + (2 - 4)^2}$ or $\sqrt{85}$

So, $\square JKLM$ is a rhombus by Theorem 6.20.

Since the slope of $\overline{JK} = \frac{4 - (-2)}{0 - (-7)}$ or $\frac{6}{7}$, the slope of $\overline{KL} = \frac{2 - 4}{9 - 0}$ or $-\frac{2}{9}$, and the product of these slopes is not -1, consecutive sides $\overline{JK}$ and $\overline{KL}$ are not perpendicular. Therefore, $\angle JKL$ is not a right angle. So $\square JKLM$ is not a rectangle or a square. ✔

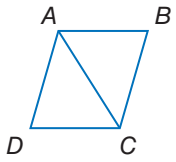
Guided Practice

4. Given $J(5, 0)$, $K(8, -11)$, $L(-3, -14)$, $M(-6, -3)$, determine whether parallelogram $JKLM$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

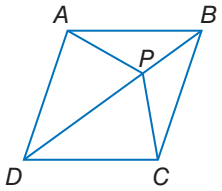
Example 1 **ALGEBRA** **Quadrilateral *ABCD* is a rhombus. Find each value or measure.**



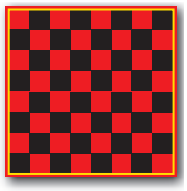
1. If $m\angle BCD = 64$, find $m\angle BAC$.

2. If $AB = 2x + 3$ and $BC = x + 7$, find CD.

Examples 2–3 **3.** **PROOF** Write a two-column proof to prove that if *ABCD* is a rhombus with diagonal $\overline{DB}$, then $\overline{AP} \cong \overline{CP}$.



4. **GAMES** The checkerboard below is made up of 64 congruent black and red squares. Use this information to prove that the board itself is a square.



Example 4 **COORDINATE GEOMETRY** **Given each set of vertices, determine whether $\square QRST$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.**

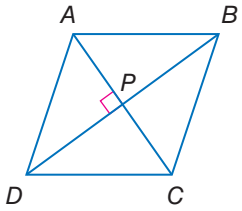
5. $Q(1, 2)$, $R(-2, -1)$, $S(1, -4)$, $T(4, -1)$

6. $Q(-2, -1)$, $R(-1, 2)$, $S(4, 1)$, $T(3, -2)$

Practice and Problem Solving

Extra Practice is on page R6.

Example 1 **ALGEBRA** **Quadrilateral *ABCD* is a rhombus. Find each value or measure.**



7. If $AB = 14$, find BC.

8. If $m\angle BCD = 54$, find $m\angle BAC$.

9. If $AP = 3x - 1$ and $PC = x + 9$, find AC.

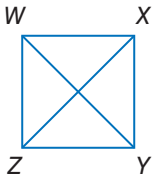
10. If $DB = 2x - 4$ and $PB = 2x - 9$, find PD.

11 If $m\angle ABC = 2x - 7$ and $m\angle BCD = 2x + 3$, find $m\angle DAB$.

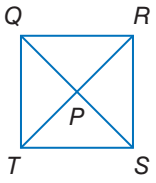
12. If $m\angle DPC = 3x - 15$, find x.

Example 2 CCSS **ARGUMENTS** **Write a two-column proof.**

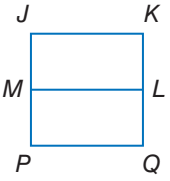
13. **Given:** $\overline{WZ} \parallel \overline{XY}$, $\overline{WX} \parallel \overline{ZY}$
$\overline{WZ} \cong \overline{ZY}$
Prove: *WXYZ* is a rhombus.



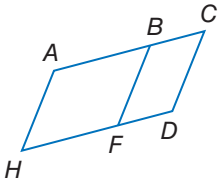
14. **Given:** *QRST* is a parallelogram.
$\overline{TR} \cong \overline{QS}$, $m\angle QPR = 90$
Prove: *QRST* is a square.



15. **Given:** *JKQP* is a square.
$\overline{ML}$ bisects $\overline{JP}$ and $\overline{KQ}$.
Prove: *JKLM* is a parallelogram.

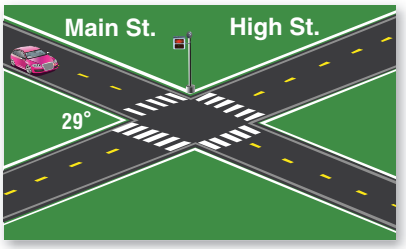


16. **Given:** *ACDH* and *BCDF* are parallelograms; $\overline{BF} \cong \overline{AB}$.
Prove: *ABFH* is a rhombus.

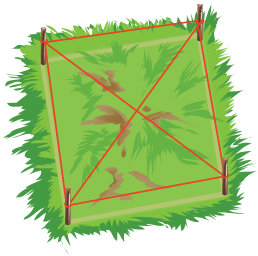


Example 3

17. ROADWAYS Main Street and High Street intersect as shown in the diagram. Each of the crosswalks is the same length. Classify the quadrilateral formed by the crosswalks. Explain your reasoning.



18. CCSS MODELING A landscaper has staked out the area for a square garden as shown. She has confirmed that each side of the quadrilateral formed by the stakes is congruent and that the diagonals are perpendicular. Is this information enough for the landscaper to be sure that the garden is a square? Explain your reasoning.



Example 4

COORDINATE GEOMETRY **Given each set of vertices, determine whether $\square JKLM$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.**

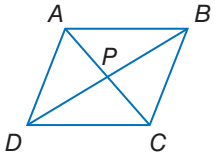
19. $J(-4, -1)$, $K(1, -1)$, $L(4, 3)$, $M(-1, 3)$

20. $J(-3, -2)$, $K(2, -2)$, $L(5, 2)$, $M(0, 2)$

21. $J(-2, -1)$, $K(-4, 3)$, $L(1, 5)$, $M(3, 1)$

22. $J(-1, 1)$, $K(4, 1)$, $L(4, 6)$, $M(-1, 6)$

$ABCD$ is a rhombus. If $PB = 12$, $AB = 15$, and $m\angle ABD = 24$, find each measure.



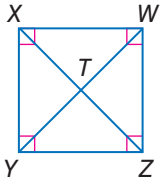
23. AP

24. CP

25. $m\angle BDA$

26. $m\angle ACB$

$WXYZ$ is a square. If $WT = 3$, find each measure.



27. ZX

28. XY

29. $m\angle WTZ$

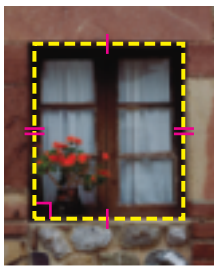
30. $m\angle WYX$

Classify each quadrilateral.

31.

32.

33.



PROOF **Write a paragraph proof.**

34. Theorem 6.16

35. Theorem 6.17

36. Theorem 6.18

37. Theorem 6.19

38. Theorem 6.20

CONSTRUCTION **Use diagonals to construct each figure. Justify each construction.**

39. rhombus

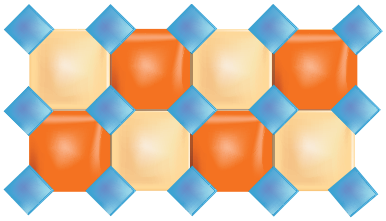
40. square

PROOF **Write a coordinate proof of each statement.**

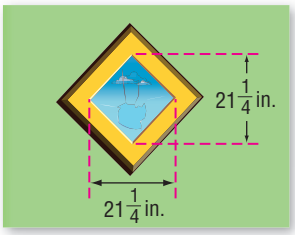
41. The diagonals of a square are perpendicular.

42. The segments joining the midpoints of the sides of a rectangle form a rhombus.

43 **DESIGN** The tile pattern below consists of regular octagons and quadrilaterals. Classify the quadrilaterals in the pattern and explain your reasoning.

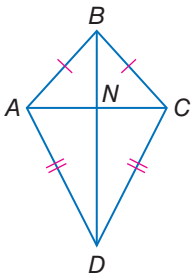


44. **REPAIR** The window pane shown needs to be replaced. What are the dimensions of the replacement pane?



45. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the properties of kites, which are quadrilaterals with exactly two distinct pairs of adjacent congruent sides.

a. Geometric Draw three kites with varying side lengths. Label one kite *ABCD*, one *PQRS*, and one *WXYZ*. Then draw the diagonals of each kite, labeling the point of intersection *N* for each kite.



Kite *ABCD*

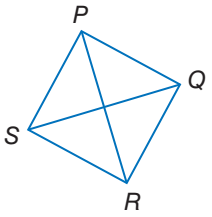
b. Tabular Measure the distance from *N* to each vertex. Record your results in a table like the one shown.

Figure	Distance from *N* to Each Vertex Along Shorter Diagonal		Distance from *N* to Each Vertex Along Longer Diagonal	
ABCD				
PQRS				
WXYZ				

c. Verbal Make a conjecture about the diagonals of a kite.

H.O.T. Problems Use Higher-Order Thinking Skills

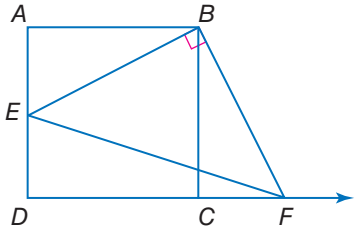
46. **ERROR ANALYSIS** In parallelogram *PQRS*, $\overline{PR} \cong \overline{QS}$. Lola thinks that the parallelogram is a square, and Xavier thinks that it is a rhombus. Is either of them correct? Explain your reasoning.



47. **ARGUMENTS** Determine whether the statement is *true* or *false*. Then write the converse, inverse, and contrapositive of the statement and determine the truth value of each. Explain your reasoning.

If a quadrilateral is a square, then it is a rectangle.

48. **CHALLENGE** The area of square *ABCD* is 36 square units and the area of $\triangle EBF$ is 20 square units. If $\overline{EB} \perp \overline{BF}$ and $\overline{AE} = 2$, find the length of $\overline{CF}$.

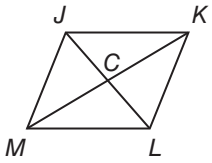


49. **OPEN ENDED** Find the vertices of a square with diagonals that are contained in the lines $y = x$ and $y = -x + 6$. Justify your reasoning.

50. **WRITING IN MATH** Compare all of the properties of the following quadrilaterals: parallelograms, rectangles, rhombi, and squares.

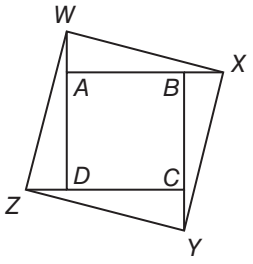
Standardized Test Practice

51. $JKLM$ is a rhombus. If $CK = 8$ and $JK = 10$, find JC.



A 4 **C** 8
B 6 **D** 10

52. EXTENDED RESPONSE The sides of square $ABCD$ are extended by sides of equal length to form square $WXYZ$.

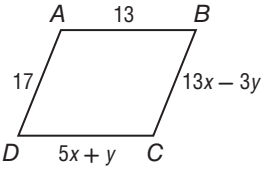


a. If $CY = 3$ cm and the area of $ABCD$ is 81 cm^2, find the area of $WXYZ$.

b. If the areas of $ABCD$ and $WXYZ$ are 49 cm^2 and 169 cm^2 respectively, find DZ.

c. If $AB = 2CY$ and the area of $ABCD = g$ square meters, find the area of $WXYZ$ in square meters.

53. ALGEBRA What values of x and y make quadrilateral $ABCD$ a parallelogram?



F $x = 3, y = 2$
G $x = \frac{3}{2}, y = -1$
H $x = 2, y = 3$
J $x = 3, y = -1$

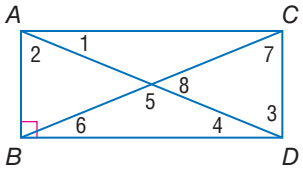
54. SAT/ACT What is 6 more than the product of -3 and a certain number x?

A $-3x - 6$ **D** $-3x + 6$
B $-3x$ **E** $6 + 3x$
C $-x$

Spiral Review

Quadrilateral $ABDC$ is a rectangle. Find each measure if $m\angle 1 = 38$. (Lesson 6-4)

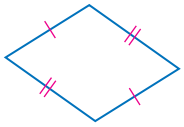
55. $m\angle 2$ **56.** $m\angle 5$ **57.** $m\angle 6$



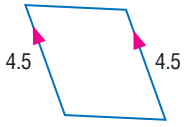
Determine whether each quadrilateral is a parallelogram. Justify your answer. (Lesson 6-3)

58.

59.

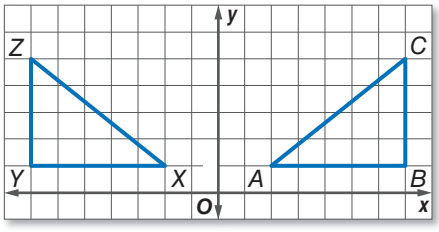


60.



61. MEASUREMENT Monifa says that her backyard is shaped like a triangle and that the lengths of its sides are 22 feet, 23 feet, and 45 feet. Do you think these measurements are correct? Explain your reasoning. (Lesson 5-5)

62. COORDINATE GEOMETRY Identify the transformation and verify that it is a congruence transformation. (Lesson 4-7)



Skills Review

Solve each equation.

63. $\frac{1}{2}(5x + 7x - 1) = 11.5$ **64.** $\frac{1}{2}(10x + 6x + 2) = 7$ **65.** $\frac{1}{2}(12x + 6 - 8x + 7) = 9$