LESSON 6-6

Trapezoids and Kites

Then
- You used properties of special parallelograms.

Now
1. Apply properties of trapezoids.
2. Apply properties of kites.

Why?
- In gymnastics, vaulting boxes made out of high compression foam are used as spotting platforms, vaulting horses, and steps. The left and right side of each section is a *trapezoid*.

amana images inc/Alamy

NewVocabulary

trapezoid
bases
legs of a trapezoid
base angles
isosceles trapezoid
midsegment of a trapezoid
kite

Common Core State Standards

Content Standards

G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

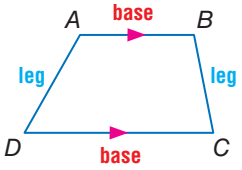
Mathematical Practices

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

1 Properties of Trapezoids

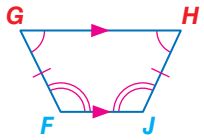
A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called **bases**. The nonparallel sides are called **legs**. The **base angles** are formed by the base and one of the legs. In trapezoid $ABCD$, $\angle A$ and $\angle B$ are one pair of base angles and $\angle C$ and $\angle D$ are the other pair. If the legs of a trapezoid are congruent, then it is an **isosceles trapezoid**.



Theorems Isosceles Trapezoids

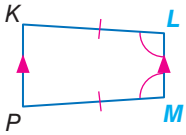
6.21 If a trapezoid is isosceles, then each pair of base angles is congruent.

Example If trapezoid $FGHJ$ is isosceles, then $\angle G \cong \angle H$ and $\angle F \cong \angle J$.



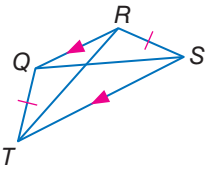
6.22 If a trapezoid has one pair of congruent base angles, then it is an isosceles trapezoid.

Example If $\angle L \cong \angle M$, then trapezoid $KLMP$ is isosceles.



6.23 A trapezoid is isosceles if and only if its diagonals are congruent.

Example If trapezoid $QRST$ is isosceles, then $\overline{QS} \cong \overline{RT}$. Likewise, if $\overline{QS} \cong \overline{RT}$, then trapezoid $QRST$ is isosceles.

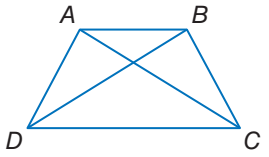


You will prove Theorem 6.21, Theorem 6.22, and the other part of Theorem 6.23 in Exercises 28, 29, and 30.

Proof Part of Theorem 6.23

Given: $ABCD$ is an isosceles trapezoid.

Prove: $\overline{AC} \cong \overline{BD}$



$ABCD$ is an isosceles trapezoid. — Given

- $\overline{DC} \cong \overline{CD}$ — Reflexive Property
- $\overline{AD} \cong \overline{BC}$ — Def. Isos. Trapezoid
- $\angle ADC \cong \angle BCD$ — Base $\measuredangle$ of trapezoid are $\cong$.

$\triangle ADC \cong \triangle BCD$ — SAS

$\overline{AC} \cong \overline{BD}$ — CPCTC

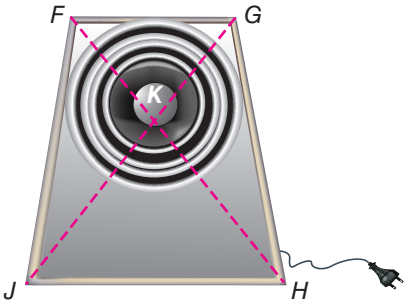
Real-WorldLink

Speakers are amplifiers that intensify sound waves so that they are audible to the unaided ear. Amplifiers exist in devices such as televisions, stereos, and computers.

Source: How Stuff Works

Real-World Example 1 Use Properties of Isosceles Trapezoids

MUSIC **The speaker shown is an isosceles trapezoid. If $m\angle FJH = 85$, $FK = 8$ inches, and $JG = 19$ inches, find each measure.**



a. $m\angle FGH$

Since $FGHJ$ is an isosceles trapezoid, $\angle FJH$ and $\angle GHJ$ are congruent base angles. So, $m\angle GHJ = m\angle FJH = 85$.

Since $FGHJ$ is a trapezoid, $\overline{FG} \parallel \overline{JH}$.

$m\angle FGH + m\angle GHJ = 180$	Consecutive Interior Angles Theorem
$m\angle FGH + 85 = 180$	Substitution
$m\angle FGH = 95$	Subtract 85 from each side.

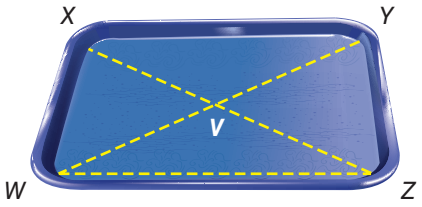
b. KH

Since $FGHJ$ is an isosceles trapezoid, diagonals $\overline{FH}$ and $\overline{JG}$ are congruent.

$FH = JG$	Definition of congruent
$FK + KH = JG$	Segment Addition
$8 + KH = 19$	Substitution
$KH = 11$ cm	Subtract 8 from each side.

GuidedPractice

1. CAFETERIA TRAYS To save space at a square table, cafeteria trays often incorporate trapezoids into their design. If $WXYZ$ is an isosceles trapezoid and $m\angle YZW = 45$, $WV = 15$ centimeters, and $VY = 10$ centimeters, find each measure.



A. $m\angle XWZ$ **B.** $m\angle WXY$

C. XZ **D.** XV

StudyTip

Isosceles Trapezoids

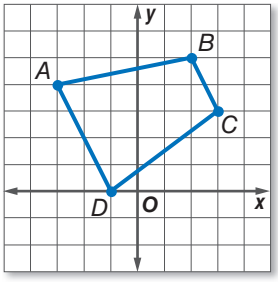
The base angles of a trapezoid are only congruent if the trapezoid is isosceles.

You can use coordinate geometry to determine whether a trapezoid is an isosceles trapezoid.

Example 2 Isosceles Trapezoids and Coordinate Geometry

COORDINATE GEOMETRY **Quadrilateral $ABCD$ has vertices $A(-3, 4)$, $B(2, 5)$, $C(3, 3)$, and $D(-1, 0)$. Show that $ABCD$ is a trapezoid and determine whether it is an isosceles trapezoid.**

Graph and connect the vertices of $ABCD$.



Step 1 Use the Slope Formula to compare the slopes of opposite sides $\overline{BC}$ and $\overline{AD}$ and of opposite sides $\overline{AB}$ and $\overline{DC}$. A quadrilateral is a trapezoid if exactly one pair of opposite sides are parallel.

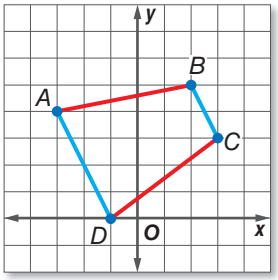
Jorg Greuel/Photographer's Choice/Getty Images

Opposite sides $\overline{BC}$ and $\overline{AD}$:

slope of $\overline{BC} = 3 - \frac{5}{3} - 2 = -\frac{2}{1}$ or -2

slope of $\overline{AD} = \frac{0-4}{-1-(-3)} = \frac{-4}{2}$ or -2

Since the slopes of $\overline{BC}$ and $\overline{AD}$ are equal, $\overline{BC} \parallel \overline{AD}$.



Opposite sides $\overline{AB}$ and $\overline{DC}$:

slope of $\overline{AB} = \frac{5-4}{2-(-3)} = \frac{1}{5}$ slope of $\overline{DC} = \frac{0-3}{-1-3} = \frac{-3}{-4}$ or $\frac{3}{4}$

ReadingMath
Symbols Recall that the symbol $\nparallel$ means *is not parallel to*.

Since the slopes of $\overline{AB}$ and $\overline{DC}$ are *not* equal, $\overline{BC} \nparallel \overline{AD}$. Since quadrilateral $ABCD$ has only one pair of opposite sides that are parallel, quadrilateral $ABCD$ is a trapezoid.

Step 2 Use the Distance Formula to compare the lengths of legs $\overline{AB}$ and $\overline{DC}$. A trapezoid is isosceles if its legs are congruent.

$AB = \sqrt{(-3-2)^2 + (4-5)^2}$ or $\sqrt{26}$

$DC = \sqrt{(-1-3)^2 + (0-3)^2} = \sqrt{25}$ or 5

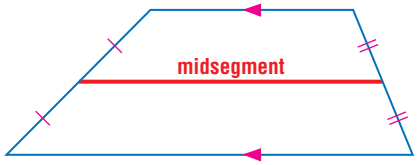
Since $AB \neq DC$, legs $\overline{AB}$ abd $\overline{DC}$ are *not* congruent. Therefore, trapezoid $ABCD$ is not isosceles.

GuidedPractice

2. Quadrilateral $QRST$ has vertices $Q(-8, -4)$, $R(0, 8)$, $S(6, 8)$, and $T(-6, -10)$. Show that $QRST$ is a trapezoid and determine whether $QRST$ is an isosceles trapezoid.

ReadingMath
Midsegment A midsegment of a trapezoid can also be called a *median*.

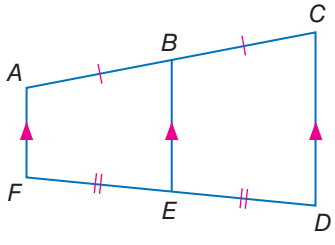
The **midsegment of a trapezoid** is the segment that connects the midpoints of the legs of the trapezoid.



The theorem below relates the midsegment and the bases of a trapezoid.

Theorem 6.24 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

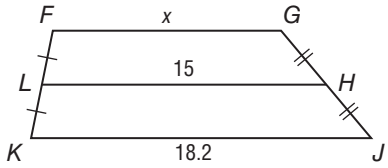


Example If $\overline{BE}$ is the midsegment of trapezoid $ACDF$, then $\overline{AF} \parallel \overline{BE}$, $\overline{CD} \parallel \overline{BE}$, and $BE = \frac{1}{2}(AF + CD)$.

You will prove Theorem 6.24 in Exercise 33.

Standardized Test Example 3 Midsegment of a Trapezoid

GRIDDED RESPONSE In the figure, $\overline{LH}$ is the midsegment of trapezoid $FGJK$. What is the value of x?



Note: The figure is not drawn to scale.

Read the Test Item

You are given the measure of the midsegment of a trapezoid and the measure of one of its bases. You are asked to find the measure of the other base.

Solve the Test Item

$LH = \frac{1}{2}(FG + KJ)$ Trapezoid Midsegment Theorem

$5 = \frac{1}{2}(x + 18.2)$ Substitution

$30 = x + 18.2$ Multiply each side by 2.

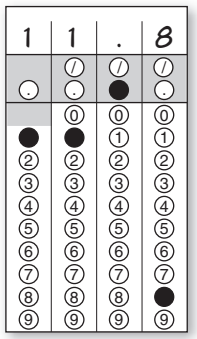
$11.8 = x$ Subtract 18.2 from each side.

Test-Taking Tip

Gridded Responses

Rational answers can often be gridded in more than one way. An answer such as $\frac{8}{5}$ could be gridded as 8/5 or 1.6, but not as 1 3/5.

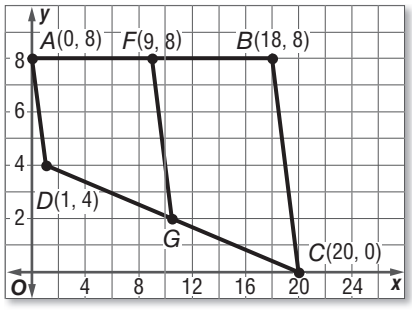
Grid In Your Answer



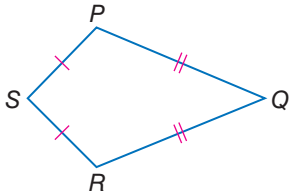
- You can align the numerical answer by placing the first digit in the left answer box or by putting the last digit in the right answer box.
- Do not leave blank boxes in the middle of an answer.
- Fill in **one** bubble for each filled answer box. Do not fill more than one bubble for an answer box. Do not fill in a bubble for blank answer boxes.

Guided Practice

3. GRIDDED RESPONSE Trapezoid $ABCD$ is shown below. If $\overline{FG}$ is parallel to $\overline{AD}$, what is the x-coordinate of point G?



2 Properties of Kites A kite is a quadrilateral with exactly two pairs of consecutive congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.



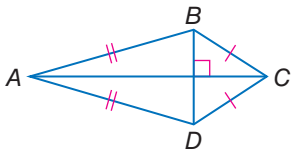
StudyTip

Kites The congruent angles of a kite are included by the non-congruent adjacent sides.

Theorems Kites

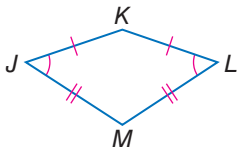
6.25 If a quadrilateral is a kite, then its diagonals are perpendicular.

Example If quadrilateral *ABCD* is a kite, then $\overline{AC} \perp \overline{BD}$.



6.26 If a quadrilateral is a kite, then exactly one pair of opposite angles is congruent.

Example If quadrilateral *JKLM* is a kite, $\overline{JK} \cong \overline{KL}$, and $\overline{JM} \cong \overline{LM}$, then $\angle J \cong \angle L$ and $\angle K \ncong \angle M$.



You will prove Theorems 6.25 and 6.26 in Exercises 31 and 32, respectively.

You can use the theorems above, the Pythagorean Theorem, and the Polygon Interior Angles Sum Theorem to find missing measures in kites.

Real-WorldLink

The fastest recorded speed of a kite is over 120 miles per hour. The record for the highest single kite flown is 12,471 feet.

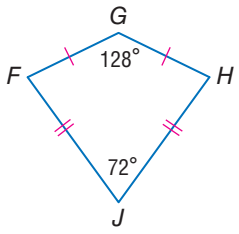
Source: Borealis Kites

Guy Grenier/Masterfile

Example 4 Use Properties of Kites

a. If *FGHJ* is a kite, find $m\angle GFJ$.

Since a kite can only have one pair of opposite congruent angles and $\angle G \ncong \angle J$, then $\angle F \cong \angle H$. So, $m\angle F = m\angle H$. Write and solve an equation to find $m\angle F$.



$m\angle F + m\angle G + m\angle H + m\angle J = 360$ Polygon Interior Angles Sum Theorem

$m\angle F + 128 + m\angle F + 72 = 360$ Substitution

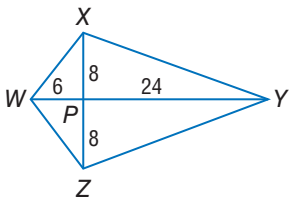
$2m\angle F + 200 = 360$ Simplify.

$2m\angle F = 160$ Subtract 200 from each side.

$m\angle F = 80$ Divide each side by 2.

b. If *WXYZ* is a kite, find *ZY*.

Since the diagonals of a kite are perpendicular, they divide *WXYZ* into four right triangles. Use the Pythagorean Theorem to find *ZY*, the length of the hypotenuse of right $\triangle YPZ$.



$PZ^2 + PY^2 = ZY^2$ Pythagorean Theorem

$8^2 + 24^2 = ZY^2$ Substitution

$640 = ZY^2$ Simplify.

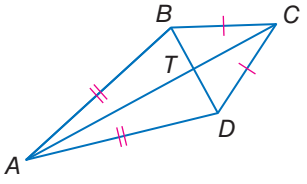
$\sqrt{640} = ZY$ Take the square root of each side.

$8\sqrt{10} = ZY$ Simplify.

GuidedPractice

4A. If $m\angle BAD = 38$ and $m\angle BCD = 50$, find $m\angle ADC$.

4B. If $BT = 5$ and $TC = 8$, find CD.

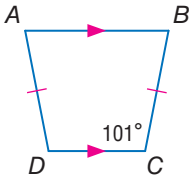


Check Your Understanding

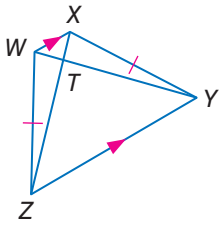
= Step-by-Step Solutions begin on page R14.

Example 1 **Find each measure.**

1. $m\angle D$



2. WT, if $ZX = 20$ and $TY = 15$



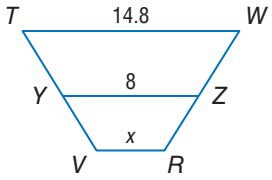
Example 2 **COORDINATE GEOMETRY** **Quadrilateral $ABCD$ has vertices $A(-4, -1)$, $B(-2, 3)$, $C(3, 3)$, and $D(5, -1)$.**

3. Verify that $ABCD$ is a trapezoid.

4. Determine whether $ABCD$ is an isosceles trapezoid. Explain.

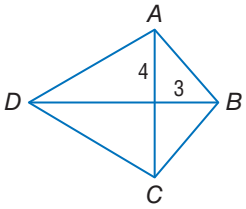
Example 3

5. **GRIDDED RESPONSE** In the figure at the right, $\overline{YZ}$ is the midsegment of trapezoid $TWRV$. Determine the value of x.

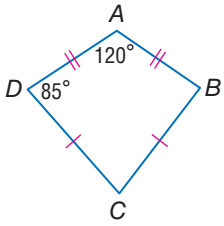


Example 4 **SENSE-MAKING** **If $ABCD$ is a kite, find each measure.**

6. AB



7. $m\angle C$

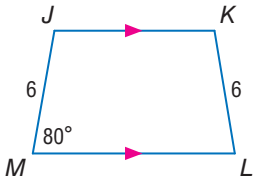


Practice and Problem Solving

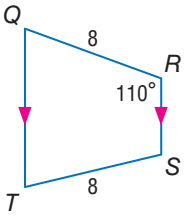
Extra Practice is on page R6.

Example 1 **Find each measure.**

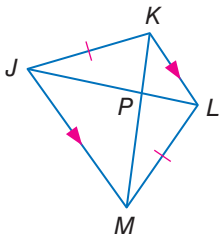
8. $m\angle K$



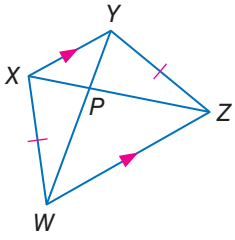
9. $m\angle Q$



10. JL, if $KP = 4$ and $PM = 7$



11 PW, if $XZ = 18$ and $PY = 3$



Example 2 **COORDINATE GEOMETRY** **For each quadrilateral with the given vertices, verify that the quadrilateral is a trapezoid and determine whether the figure is an isosceles trapezoid.**

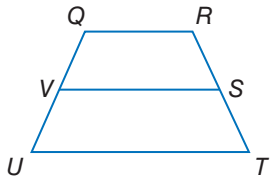
12. $A(-2, 5)$, $B(-3, 1)$, $C(6, 1)$, $D(3, 5)$

13. $J(-4, -6)$, $K(6, 2)$, $L(1, 3)$, $M(-4, -1)$

14. $Q(2, 5)$, $R(-2, 1)$, $S(-1, -6)$, $T(9, 4)$

15. $W(-5, -1)$, $X(-2, 2)$, $Y(3, 1)$, $Z(5, -3)$

Example 3 **For trapezoid *QRTU*, *V* and *S* are midpoints of the legs.**



16. If $QR = 12$ and $UT = 22$, find VS.

17. If $QR = 4$ and $UT = 16$, find VS.

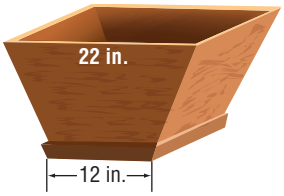
18. If $VS = 9$ and $UT = 12$, find QR.

19. If $TU = 26$ and $SV = 17$, find QR.

20. If $QR = 2$ and $VS = 7$, find UT.

21. If $RQ = 5$ and $VS = 11$, find UT.

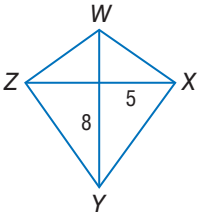
22. **DESIGN** Juana is designing a window box. She wants the end of the box to be a trapezoid with the dimensions shown. If she wants to put a shelf in the middle for the plants to rest on, about how wide should she make the shelf?



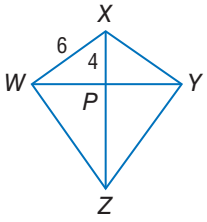
23 **MUSIC** The keys of the xylophone shown form a trapezoid. If the length of the lower pitched C is 6 inches long, and the higher pitched D is 1.8 inches long, how long is the G key?

Example 4 **SENSE-MAKING** **If *WXYZ* is a kite, find each measure.**

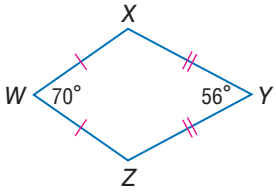
24. YZ



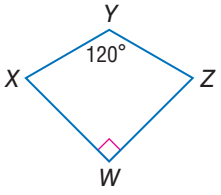
25. WP



26. $m\angle X$



27. $m\angle Z$



PROOF **Write a paragraph proof for each theorem.**

28. Theorem 6.21

29. Theorem 6.22

30. Theorem 6.23

31. Theorem 6.25

32. Theorem 6.26

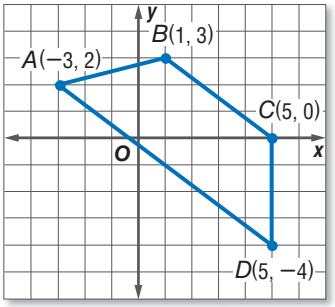
33. **PROOF** Write a coordinate proof for Theorem 6.24.

34. **COORDINATE GEOMETRY** Refer to quadrilateral *ABCD*.

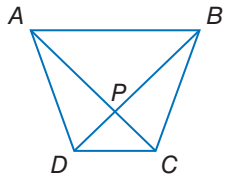
a. Determine whether the figure is a trapezoid. If so, is it isosceles? Explain.

b. Is the midsegment contained in the line with equation $y = -x + 1$? Justify your answer.

c. Find the length of the midsegment.



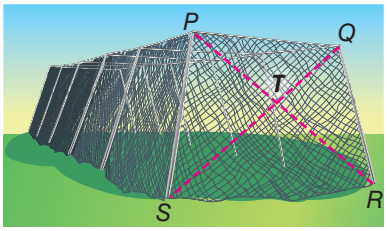
ALGEBRA ***ABCD* is a trapezoid.**



35. If $AC = 3x - 7$ and $BD = 2x + 8$, find the value of x so that $ABCD$ is isosceles.

36. If $m\angle ABC = 4x + 11$ and $m\angle DAB = 2x + 33$, find the value of x so that $ABCD$ is isosceles.

SPORTS **The end of the batting cage shown is an isosceles trapezoid. If $PT = 12$ feet, $ST = 28$ feet, and $m\angle PQR = 110$, find each measure.**



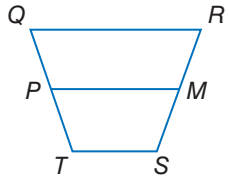
37. TR

38. SQ

39. $m\angle QRS$

40. $m\angle QPS$

ALGEBRA **For trapezoid $QRST$, M and P are midpoints of the legs.**



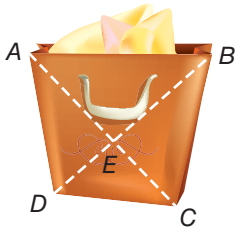
41. If $QR = 16$, $PM = 12$, and $TS = 4x$, find x.

42. If $TS = 2x$, $PM = 20$, and $QR = 6x$, find x.

43. If $PM = 2x$, $QR = 3x$, and $TS = 10$, find PM.

44. If $TS = 2x + 2$, $QR = 5x + 3$, and $PM = 13$, find TS.

SHOPPING **The side of the shopping bag shown is an isosceles trapezoid. If $EC = 9$ inches, $DB = 19$ inches, $m\angle ABE = 40$, and $m\angle EBC = 35$, find each measure.**



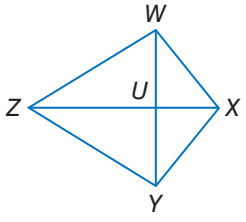
45. AE

46. AC

47. $m\angle BCD$

48. $m\angle EDC$

ALGEBRA ***WXYZ* is a kite.**



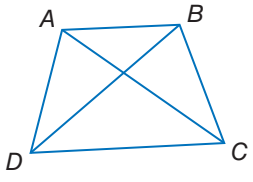
49 If $m\angle WXY = 120$, $m\angle WZY = 4x$, and $m\angle ZWX = 10x$, find $m\angle ZYX$.

50. If $m\angle WXY = 13x + 24$, $m\angle WZY = 35$, and $m\angle ZWX = 13x + 14$, find $m\angle ZYX$.

CCSS **ARGUMENTS** **Write a two-column proof.**

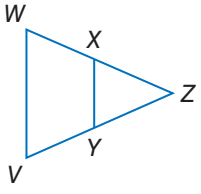
51. **Given:** $ABCD$ is an isosceles trapezoid.

Prove: $\angle DAC \cong \angle CBD$



52. **Given:** $\overline{WZ} \cong \overline{ZV}$, $\overline{XY}$ bisects $\overline{WZ}$ and $\overline{ZV}$, and $\angle W \cong \angle ZXY$.

Prove: $WXYV$ is an isosceles trapezoid.



Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

53. The opposite angles of a trapezoid are supplementary.

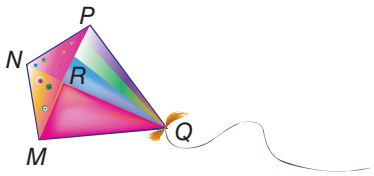
54. One pair of opposite sides are parallel in a kite.

55. A square is a rhombus.

56. A rectangle is a square.

57. A parallelogram is a rectangle.

58. KITES Refer to the kite at the right. Using the properties of kites, write a two-column proof to show that $\triangle MNR$ is congruent to $\triangle PNR$.



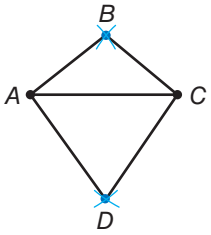
59. VENN DIAGRAM Create a Venn diagram that incorporates all quadrilaterals, including trapezoids, isosceles trapezoids, kites, and quadrilaterals that cannot be classified as anything other than quadrilaterals.

COORDINATE GEOMETRY Determine whether each figure is a *trapezoid*, a *parallelogram*, a *square*, a *rhombus*, or a *quadrilateral* given the coordinates of the vertices. Choose the most specific term. Explain.

60. $A(-1, 4)$, $B(2, 6)$, $C(3, 3)$, $D(0, 1)$

61 $W(-3, 4)$, $X(3, 4)$, $Y(5, 3)$, $Z(-5, 1)$

62. MULTIPLE REPRESENTATIONS In this problem, you will explore proportions in kites.



a. Geometric Draw a segment. Construct a noncongruent segment that perpendicularly bisects the first segment. Connect the endpoints of the segments to form a quadrilateral $ABCD$. Repeat the process two times. Name the additional quadrilaterals $PQRS$ and $WXYZ$.

b. Tabular Copy and complete the table below.

Figure	Side	Length	Side	Length	Side	Length	Side	Length
$ABCD$	AB		BC		CD		DA	
$PQRS$	PQ		QR		RS		SP	
$WXYZ$	WX		XY		YZ		ZW	

c. Verbal Make a conjecture about a quadrilateral in which the diagonals are perpendicular, exactly one diagonal is bisected, and the diagonals are not congruent.

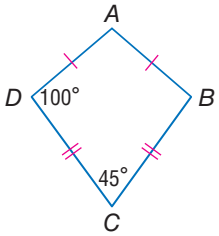
PROOF Write a coordinate proof of each statement.

63. The diagonals of an isosceles trapezoid are congruent.

64. The median of an isosceles trapezoid is parallel to the bases.

H.O.T. Problems Use Higher-Order Thinking Skills

65. ERROR ANALYSIS Bedagi and Belinda are trying to determine $m\angle A$ in kite $ABCD$ shown. Is either of them correct? Explain.



Bedagi	Belinda
$m\angle A = 45$	$m\angle A = 115$

66. CHALLENGE If the parallel sides of a trapezoid are contained by the lines $y = x + 4$ and $y = x - 8$, what equation represents the line contained by the midsegment?

67. CCSS ARGUMENTS Is it *sometimes*, *always*, or *never* true that a square is also a kite? Explain.

68. OPEN ENDED Sketch two noncongruent trapezoids $ABCD$ and $FGHJ$ in which $\overline{AC} \cong \overline{FH}$ and $\overline{BD} \cong \overline{GJ}$.

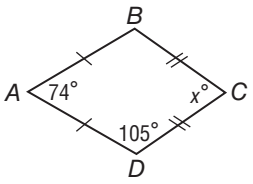
69. WRITING IN MATH Describe the properties a quadrilateral must possess in order for the quadrilateral to be classified as a trapezoid, an isosceles trapezoid, or a kite. Compare the properties of all three quadrilaterals.

Standardized Test Practice

70. ALGEBRA All of the items on a breakfast menu cost the same whether ordered with something else or alone. Two pancakes and one order of bacon costs $4.92. If two orders of bacon cost $3.96, what does one pancake cost?

A $0.96 **C** $1.98
B $1.47 **D** $2.94

71. GRIDDED RESPONSE If quadrilateral *ABCD* is a kite, what is $m\angle C$?

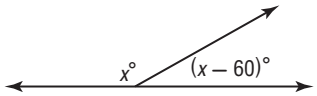


72. Which figure can serve as a counterexample to the conjecture below?

If the diagonals of a quadrilateral are congruent, then the quadrilateral is a rectangle.

F square **H** parallelogram
G rhombus **J** isosceles trapezoid

73. SAT/ACT In the figure below, what is the value of x?



A 60 **D** 240
B 120 **E** 300
C 180

Spiral Review

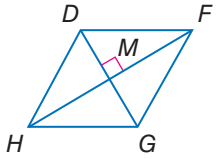
ALGEBRA Quadrilateral *DFGH* is a rhombus. Find each value or measure. (Lesson 6-5)

74. If $m\angle FGH = 118$, find $m\angle MHG$.

75. If $DM = 4x - 3$ and $MG = x + 6$, find DG.

76. If $DF = 10$, find FG.

77. If $HM = 12$ and $HD = 15$, find MG.

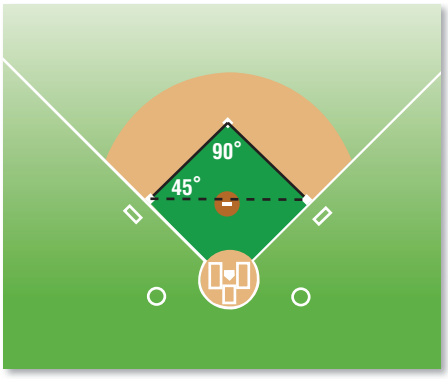


COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula. (Lesson 6-4)

78. $A(4, 2)$, $B(-4, 1)$, $C(-3, -5)$, $D(5, -4)$; Distance Formula

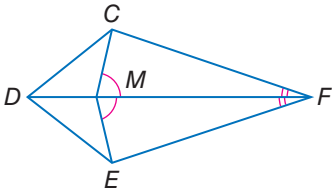
79. $J(0, 7)$, $K(-8, 6)$, $L(-7, 0)$, $M(1, 1)$; Slope Formula

80. BASEBALL A batter hits the ball to the third baseman and begins to run toward first base. At the same time, the runner on first base runs toward second base. If the third baseman wants to throw the ball to the nearest base, to which base should he throw? Explain. (Lesson 5-3)



81. PROOF Write a two-column proof. (Lesson 4-5)

Given: $\angle CMF \cong \angle EMF$, $\angle CFM \cong \angle EFM$
Prove: $\triangle DMC \cong \triangle DME$



Skills Review

Write an expression for the slope of each segment given the coordinates and endpoints.

82. $(x, 4y)$, $(-x, 4y)$ **83.** $(-x, 5x)$, $(0, 6x)$ **84.** (y, x), (y, y)