Ratios and Proportions

:: Then
- You solved problems by writing and solving equations.

:: Now
1. Write ratios.
2. Write and solve proportions.

:: Why?
- The aspect ratio of a television or computer screen is the screen's width divided by its height. A standard television screen has an aspect ratio of $\frac{4}{3}$ or 4:3, while a high definition television screen (HDTV) has an aspect ratio of 16:9.

NewVocabulary
ratio
extended ratios
proportion
extremes
means
cross products

Common Core State Standards

Content Standards
G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

Mathematical Practices
7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.

1 Write and Use Ratios

A **ratio** is a comparison of two quantities using division. The ratio of quantities a and b can be expressed as a to b, $a:b$, or $\frac{a}{b}$, where $b \neq 0$. Ratios are usually expressed in simplest form.

The aspect ratios 32:18 and 16:9 are equivalent.

$$\frac{\text{width of screen}}{\text{height of screen}} = \frac{32 \text{ in.}}{18 \text{ in.}} \qquad \text{Divide out units.}$$

$$= \frac{32 \div 2}{18 \div 2} \text{ or } \frac{16}{9} \qquad \text{Divide out common factors.}$$

### 🌐 Real-World Example 1 Write and Simplify Ratios

SPORTS A baseball player's batting average is the ratio of the number of base hits to the number of at-bats, not including walks. Minnesota Twins' Joe Mauer had the highest batting average in Major League Baseball in 2006. If he had 521 official at-bats and 181 hits, find his batting average.

Divide the number of hits by the number of at-bats.

$$\frac{\text{number of hits}}{\text{number of at-bats}} = \frac{181}{521}$$

$$\approx \frac{0.347}{1} \qquad \text{A ratio in which the denominator is 1 is called a } \textit{unit ratio.}$$

Joe Mauer's batting average was 0.347.
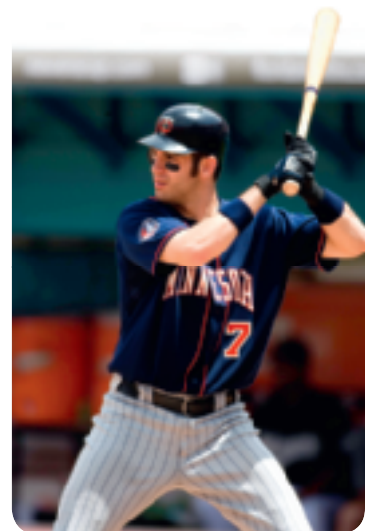
▶ GuidedPractice

1. **SCHOOL** In Logan's high school, there are 190 teachers and 2650 students. What is the approximate student-teacher ratio at his school?

Extended ratios can be used to compare three or more quantities. The expression $a:b:c$ means that the ratio of the first two quantities is $a:b$, the ratio of the last two quantities is $b:c$, and the ratio of the first and last quantities is $a:c$.
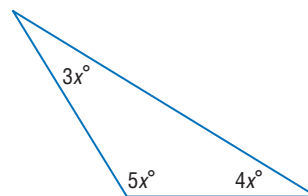
Example 2 Use Extended Ratios

The ratio of the measures of the angles in a triangle is 3:4:5. Find the measures of the angles.

Just as the ratio $\frac{3}{4}$ or 3:4 is equivalent to $\frac{3x}{4x}$ or $3x:4x$, the extended ratio 3:4:5 can be written as $3x:4x:5x$.

Sketch and label the angle measures of the triangle. Then write and solve an equation to find the value of x.

$3x + 4x + 5x = 180$	Triangle Sum Theorem
$12x = 180$	Combine like terms.
$x = 15$	Divide each side by 12.

So the measures of the angles are 3(15) or 45, 4(15) or 60, and 5(15) or 75.

CHECK The sum of the angle measures should be 180.

$$45 + 60 + 75 = 180 ✓$$
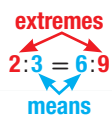
▶ **Guided**Practice

2. In a triangle, the ratio of the measures of the sides is 2:2:3 and the perimeter is 392 inches. Find the length of the longest side of the triangle.

---

2 Use Properties of Proportions An equation stating that two ratios are equal is called a **proportion**. In the proportion $\frac{a}{b} = \frac{c}{d}$, the numbers a and d are called the **extremes** of the proportion, while the numbers b and c are called the **means** of the proportion.

$$\text{extreme} \rightarrow \frac{a}{b} = \frac{c}{d} \leftarrow \text{mean}$$
$$\text{mean} \rightarrow b \qquad d \leftarrow \text{extreme}$$

The product of the extremes ad and the product of the means bc are called **cross products**.

KeyConcept Cross Products Property

Words	In a proportion, the product of the extremes equals the product of the means.
Symbols	If $\frac{a}{b} = \frac{c}{d}$ when $b \neq 0$ and $d \neq 0$, then $ad = bc$.
Example	If $\frac{4}{10} = \frac{6}{15}$, then $4 \cdot 15 = 10 \cdot 6$.

You will prove the Cross Products Property in Exercise 41.

The converse of the Cross Products Property is also true. If $ad = bc$ and $b \neq 0$ and $d \neq 0$, then $\frac{a}{b} = \frac{c}{d}$. That is, $\frac{a}{b}$ and $\frac{c}{d}$ form a proportion. You can use the Cross Products Property to solve a proportion.

Example 3 Use Cross Products to Solve Proportions

Solve each proportion.

a. $\frac{6}{x} = \frac{21}{31.5}$

$\frac{6}{x} = \frac{21}{31.5}$	Original proportion
$6(31.5) = x(21)$	Cross Products Property
$189 = 21x$	Simplify.
$9 = x$	Solve for x.

b. $\frac{x+3}{2} = \frac{4x}{5}$

$$\frac{x+3}{2} = \frac{4x}{5}$$
$$(x+3)5 = 2(4x)$$
$$5x + 15 = 8x$$
$$15 = 3x$$
$$5 = x$$

▶ **Guided**Practice

3A. $\frac{x}{4} = \frac{11}{-6}$ **3B.** $\frac{-4}{7} = \frac{6}{2y+5}$ **3C.** $\frac{7}{z-1} = \frac{9}{z+4}$

Proportions can be used to make predictions.

🌐 Real-World Example 4 Use Proportions to Make Predictions

CAR OWNERSHIP Fernando conducted a survey of 50 students driving to school and found that 28 owned cars. If 755 students drive to his school, predict the total number of students who own cars.

Write and solve a proportion that compares the number of students who own cars to the number who drive to school.

$$\frac{28}{50} = \frac{x}{755}$$ ◀ students owning cars
◀ students driving to school

$28 \cdot 755 = 50 \cdot x$	Cross Products Property
$21{,}140 = 50x$	Simplify.
$422.8 = x$	Divide each side by 50.

Based on Fernando's survey, about 423 students at his school own cars.

▶ **Guided**Practice

4. BIOLOGY In an experiment, students netted butterflies, recorded the number with tags on their wings, and then released them. The students netted 48 butterflies and 3 of those had tagged wings. Predict the number of butterflies that would have tagged wings out of 100 netted.

The proportion shown in Example 4 is not the only correct proportion for that situation. Equivalent forms of a proportion all have identical cross products.

KeyConcept Equivalent Proportions

Symbols   The following proportions are equivalent.

$$\frac{a}{b} = \frac{c}{d}, \quad \frac{b}{a} = \frac{d}{c}, \quad \frac{a}{c} = \frac{b}{d}, \quad \frac{c}{a} = \frac{d}{b}$$

Examples $\frac{28}{50} = \frac{x}{755}, \ \frac{50}{28} = \frac{755}{x}, \ \frac{28}{x} = \frac{50}{755}, \ \frac{x}{28} = \frac{755}{50}$.

Example 1

1. **PETS** Out of a survey of 1000 households, 460 had at least one dog or cat as a pet. What is the ratio of pet owners to households?

2. **SPORTS** Thirty girls tried out for 15 spots on the basketball team. What is the ratio of open spots to the number of girls competing?

Example 2

3. The ratio of the measures of three sides of a triangle is $2:5:4$, and its perimeter is 165 units. Find the measure of each side of the triangle.

4. The ratios of the measures of three angles of a triangle are $4:6:8$. Find the measure of each angle of the triangle.

Example 3

Solve each proportion.

5. $\frac{2}{3} = \frac{x}{24}$

6. $\frac{x}{5} = \frac{28}{100}$

7. $\frac{2.2}{x} = \frac{26.4}{96}$

8. $\frac{x-3}{3} = \frac{5}{8}$

Example 4

9. **CCSS MODELING** Ella is baking apple muffins for the Student Council bake sale. The recipe that she is using calls for 2 eggs per dozen muffins, and she needs to make 108 muffins. How many eggs will she need?

Practice and Problem Solving

Extra Practice is on page R7.

Example 1

MOVIES For Exercises 10 and 11, refer to the graphic below.

Academy Awards®

Nominations / Awards

Movie A   Movie B   Movie C   Movie D
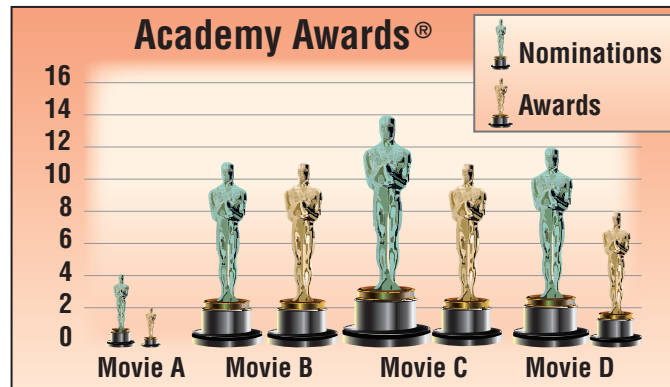
10. Of the films listed, which had the greatest ratio of Academy Awards to number of nominations?

11. Which film listed had the lowest ratio of awards to nominations?

Example 2

12. **GAMES** A video game store has 60 games to choose from, including 40 sports games. What is the ratio of sports games to video games?

13. The ratio of the measures of the three sides of a triangle is $9 : 7 : 5$. Its perimeter is 191.1 inches. Find the measure of each side.

14. The ratio of the measures of the three sides of a triangle is $3:7:5$, and its perimeter is 156.8 meters. Find the measure of each side.

15. The ratio of the measures of the three sides of a triangle is $\frac{1}{4}:\frac{1}{8}:\frac{1}{6}$. Its perimeter is 4.75 feet. Find the length of the longest side.

16. The ratio of the measures of the three sides of a triangle is $\frac{1}{4}:\frac{1}{3}:\frac{1}{6}$, and its perimeter is 31.5 centimeters. Find the length of the shortest side.

Find the measures of the angles of each triangle.

17. The ratio of the measures of the three angles is $3:6:1$.

18. The ratio of the measures of the three angles is $7:5:8$.

19. The ratio of the measures of the three angles is $10:8:6$.

20. The ratio of the measures of the three angles is $5:4:7$.

Solve each proportion.

21. $\dfrac{5}{8} = \dfrac{y}{3}$ **22.** $\dfrac{w}{6.4} = \dfrac{1}{2}$ **23.** $\dfrac{4x}{24} = \dfrac{56}{112}$ **24.** $\dfrac{11}{20} = \dfrac{55}{20x}$

25. $\dfrac{2x+5}{10} = \dfrac{42}{20}$ **26.** $\dfrac{a+2}{a-2} = \dfrac{3}{2}$ **27.** $\dfrac{3x-1}{4} = \dfrac{2x+4}{5}$ **28.** $\dfrac{3x-6}{2} = \dfrac{4x-2}{4}$

29. **NUTRITION** According to a recent study, 7 out of every 500 Americans aged 13 to 17 years are vegetarian. In a group of 350 13- to 17-year-olds, about how many would you expect to be vegetarian?

30. **CURRENCY** Your family is traveling to Mexico on vacation. You have saved $500 to use for spending money. If 269 Mexican pesos is equivalent to 25 United States dollars, how much money will you get when you exchange your $500 for pesos?

ALGEBRA Solve each proportion. Round to the nearest tenth.

31. $\dfrac{2x+3}{3} = \dfrac{6}{x-1}$ **32.** $\dfrac{x^2+4x+4}{40} = \dfrac{x+2}{10}$ **33.** $\dfrac{9x+6}{18} = \dfrac{20x+4}{3x}$

34. The perimeter of a rectangle is 98 feet. The ratio of its length to its width is $5:2$. Find the area of the rectangle.

35. The perimeter of a rectangle is 220 inches. The ratio of its length to its width is $7:3$. Find the area of the rectangle.

36. The ratio of the measures of the side lengths of a quadrilateral is $2:3:5:4$. Its perimeter is 154 feet. Find the length of the shortest side.

37. The ratio of the measures of the angles of a quadrilateral is $2:4:6:3$. Find the measures of the angles of the quadrilateral.

38. **SUMMER JOBS** In June of 2000, 60.2% of American teens 16 to 19 years old had summer jobs. By June of 2006, 51.6% of teens in that age group were a part of the summer work force.

  a. Has the number of 16- to 19-year-olds with summer jobs increased or decreased since 2000? Explain your reasoning.

  b. In June 2006, how many 16- to 19-year-olds would you expect to have jobs out of 700 in that age group? Explain your reasoning.

39. **CCSS MODELING** In a golden rectangle, the ratio of the length to the width is about 1.618. This is known as the *golden ratio*.

  a. Recall from page 461 that a standard television screen has an aspect ratio of $4:3$, while a high-definition television screen has an aspect ratio of $16:9$. Is either type of screen a golden rectangle? Explain.

  b. The golden ratio can also be used to determine column layouts for Web pages. Consider a site with two columns, the left for content and the right as a sidebar. The ratio of the left to right column widths is the golden ratio. Determine the width of each column if the page is 960 pixels wide.

40. **SCHOOL ACTIVITIES** A survey of club involvement showed that, of the 36 students surveyed, the ratio of French Club members to Spanish Club members to Drama Club members was $2:3:7$. How many of those surveyed participate in Spanish Club? Assume that each student is active in only one club.

41. **PROOF** Write an algebraic proof of the Cross Products Property.

42. **SPORTS** Jane jogs the same path every day in the winter to stay in shape for track season. She runs at a constant rate, and she spends a total of 39 minutes jogging. If the ratio of the times of the four legs of the jog is $3:5:1:4$, how long does the second leg of the jog take her?

43. **MULTIPLE REPRESENTATIONS** In this problem, you will explore proportional relationships in triangles.

   a. **Geometric** Draw an isosceles triangle ABC. Measure and label the legs and the vertex angle. Draw a second triangle MNO with a congruent vertex angle and legs twice as long as ABC. Draw a third triangle PQR with a congruent vertex angle and legs half as long as ABC.

   b. **Tabular** Copy and complete the table below using the appropriate measures.

Triangle	ABC	MNO	PQR
Leg length			
Perimeter			

   c. **Verbal** Make a conjecture about the change in the perimeter of an isosceles triangle if the vertex angle is held constant and the leg length is increased or decreased by a factor.

H.O.T. Problems   Use Higher-Order Thinking Skills

44. **ERROR ANALYSIS** Mollie and Eva have solved the proportion $\frac{x-3}{4} = \frac{1}{2}$. Is either of them correct? Explain your reasoning.

   Mollie
   $(x - 3)1 = 4(2)$
   $x - 3 = 8$
   $x = 11$

   Eva
   $x - 3(2) = 4(1)$
   $x - 3 = 4$
   $x = 7$

45. **CHALLENGE** The dimensions of a rectangle are y and $y^2 + 1$ and the perimeter of the rectangle is 14 units. Find the ratio of the longer side of the rectangle to the shorter side of the rectangle.

46. **CCSS REASONING** The ratio of the lengths of the diagonals of a quadrilateral is $1:1$. The ratio of the lengths of the consecutive sides of the quadrilateral is $3:4:3:5$. Classify the quadrilateral. Explain.

47. **WHICH ONE DOESN'T BELONG?** Identify the proportion that does not belong with the other three. Explain your reasoning.

   $\frac{3}{8} = \frac{8.4}{22.4}$ $\frac{2}{3} = \frac{5}{7.5}$ $\frac{5}{6} = \frac{14}{16.8}$ $\frac{7}{9} = \frac{19.6}{25.2}$

48. **OPEN ENDED** Write four ratios that are equivalent to the ratio $2:5$. Explain why all of the ratios are equivalent.

49. **WRITING IN MATH** Compare and contrast a ratio and a proportion. Explain how you use both to solve a problem.
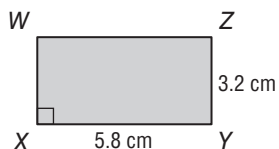
50. Solve the following proportion.

$$\frac{x}{-8} = \frac{12}{6}$$

A -12 **C** -16

B -14 **D** -18

51. What is the area of rectangle $WXYZ$?

F 18.6 cm^2 **H** 21.2 cm^2

G 20.4 cm^2 **J** 22.8 cm^2

52. GRIDDED RESPONSE Mrs. Sullivan's rectangular bedroom measures 12 feet by 10 feet. She wants to purchase carpet for the bedroom that costs $2.56 per square foot, including tax. How much will it cost in dollars to carpet her bedroom?

53. SAT/ACT Kamilah has 5 more than 4 times the number of DVDs that Mercedes has. If Mercedes has x DVDs, then in terms of x, how many DVDs does Kamilah have?
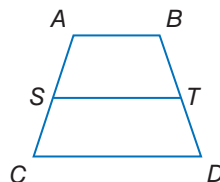
A $4(x + 5)$ **D** $4x + 5$

B $4(x + 3)$ **E** $5x + 4$

C $9x$

Spiral Review

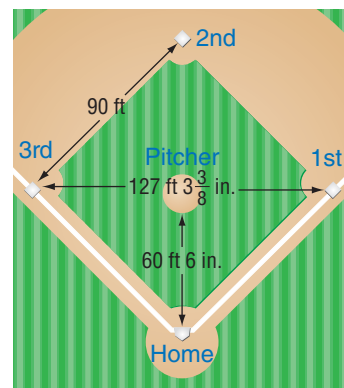**For trapezoid $ABCD$, S and T are midpoints of the legs.** (Lesson 6-6)

54. If $CD = 14$, $ST = 10$, and $AB = 2x$, find x.

55. If $AB = 3x$, $ST = 15$, and $CD = 9x$, find x.

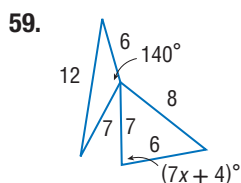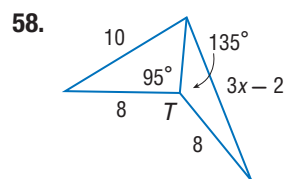**56.** If $AB = x + 4$, $CD = 3x + 2$, and $ST = 9$, find AB.

57. SPORTS The infield of a baseball diamond is a square, as shown at the right. Is the pitcher's mound located in the center of the infield? Explain. (Lesson 6-5)

**Write an inequality for the range of values for x.** (Lesson 5-6)

58.

59.

Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition. (Lesson 5-3)

60. measures less than $m\angle 5$

61. measures greater than $m\angle 6$

62. measures greater than $m\angle 10$

63. measures less than $m\angle 11$

64. REASONING Find a counterexample for the following statement. (Lesson 3-5)
If lines p and m are cut by transversal t so that consecutive interior angles are congruent, then lines p and m are parallel and t is perpendicular to both lines.
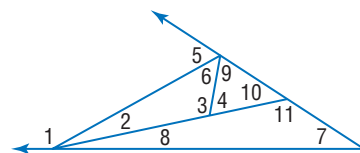
Skills Review

Write a paragraph proof.

65. Given: $\triangle ABC \cong \triangle DEF$; $\triangle DEF \cong \triangle GHI$
    Prove: $\triangle ABC \cong \triangle GHI$