LESSON 7-2

Similar Polygons

Then
- You used proportions to solve problems.

Now
1. Use proportions to identify similar polygons.
2. Solve problems using the properties of similar polygons.

Why?
- People often customize their computer desktops using photos, centering the images at their original size or stretching them to fit the screen. This second method distorts the image, because the original and new images are not geometrically similar.

NewVocabulary
- similar polygons
- scale factor

Common Core State Standards

Content Standards

G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Mathematical Practices

7 Look for and make use of structure.

3 Construct viable arguments and critique the reasoning of others.

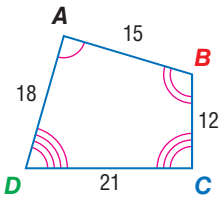
1 Identify Similar Polygons

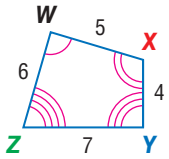
Similar polygons have the same shape but not necessarily the same size.

KeyConcept Similar Polygons

Two polygons are similar if and only if their corresponding angles are congruent and corresponding side lengths are proportional.

Example In the diagram below, *ABCD* is similar to *WXYZ*.





Corresponding angles

$\angle A \cong \angle W$, $\angle B \cong \angle X$, $\angle C \cong \angle Y$, and $\angle D \cong \angle Z$

Corresponding sides

$$\frac{AB}{WX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{DA}{ZW} = \frac{3}{1}$$

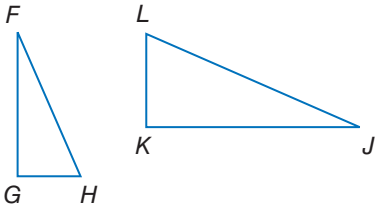
Symbols $ABCD \sim WXYZ$

As with congruence statements, the order of vertices in a similarity statement like $ABCD \sim WXYZ$ is important. It identifies the corresponding angles and sides.

Example 1 Use a Similarity Statement

If $\triangle FGH \sim \triangle JKL$, list all pairs of congruent angles, and write a proportion that relates the corresponding sides.

Use the similarity statement.



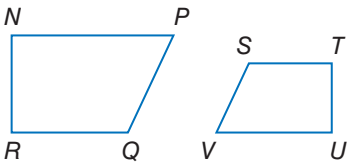
$\triangle FGH \sim \triangle JKL$

Congruent angles: $\angle F \cong \angle J$, $\angle G \cong \angle K$, $\angle H \cong \angle L$

Proportion: $\frac{FG}{JK} = \frac{GH}{KL} = \frac{HF}{LJ}$

GuidedPractice

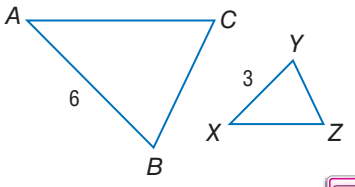
1. In the diagram, $NPQR \sim UVST$. List all pairs of congruent angles, and write a proportion that relates the corresponding sides.



StudyTip
Similarity Ratio The scale factor between two similar polygons is sometimes called the *similarity ratio*.

The ratio of the lengths of the corresponding sides of two similar polygons is called the **scale factor**. The scale factor depends on the order of comparison.

In the diagram, $\triangle ABC \sim \triangle XYZ$.

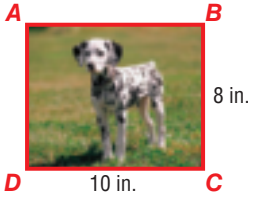


The scale factor of $\triangle ABC$ to $\triangle XYZ$ is $\frac{6}{3}$ or 2.

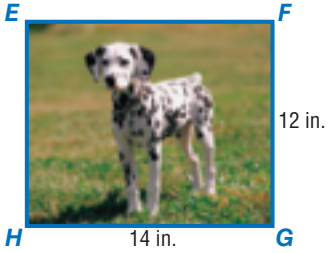
The scale factor of $\triangle XYZ$ to $\triangle ABC$ is $\frac{3}{6}$ or $\frac{1}{2}$.

Real-World Example 2 Identify Similar Polygons

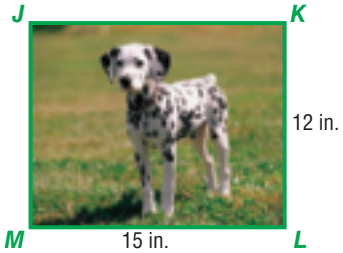
PHOTO EDITING **Kuma wants to use the rectangular photo shown as the background for her computer's desktop, but she needs to resize it. Determine whether the following rectangular images are similar. If so, write the similarity statement and scale factor. Explain your reasoning.**



a.



b.



a. **Step 1** Compare corresponding angles.

Since all angles of a rectangle are right angles and right angles are congruent, corresponding angles are congruent.

Step 2 Compare corresponding sides.

$\frac{DC}{HG} = \frac{10}{14}$ or $\frac{5}{7}$ $\quad \frac{BC}{FG} = \frac{8}{12}$ or $\frac{2}{3}$ $\quad \frac{5}{7} \neq \frac{2}{3}$

Since corresponding sides are not proportional, $ABCD \not\sim EFGH$. So the photos are not similar.

ReadingMath
Similarity Symbol The symbol $\not\sim$ is read as *is not similar to*.

b. **Step 1** Since $ABCD$ and $JKLM$ are both rectangles, corresponding angles are congruent.

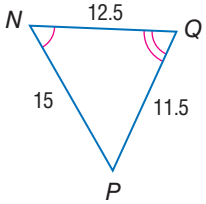
Step 2 Compare corresponding sides.

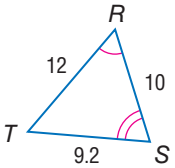
$\frac{DC}{ML} = \frac{10}{15}$ or $\frac{2}{3}$ $\quad \frac{BC}{KL} = \frac{8}{12}$ or $\frac{2}{3}$ $\quad \frac{2}{3} = \frac{2}{3}$

Since corresponding sides are proportional, $ABCD \sim JKLM$. So the rectangles are similar with a scale factor of $\frac{2}{3}$.

GuidedPractice

2. Determine whether the triangles shown are similar. If so, write the similarity statement and scale factor. Explain your reasoning.





Stockbyte/Getty Images

StudyTip

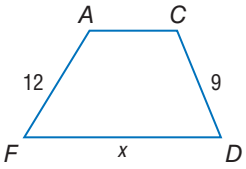
Similarity and Congruence If two polygons are congruent, they are also similar. All of the corresponding angles are congruent, and the lengths of the corresponding sides have a ratio of 1:1.

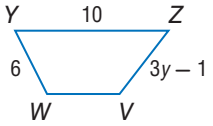
2 Use Similar Figures

You can use scale factors and proportions to solve problems involving similar figures.

Example 3 Use Similar Figures to Find Missing Measures

In the diagram, $ACDF \sim VWYZ$.





a. Find x.

Use the corresponding side lengths to write a proportion.

$\frac{CD}{WY} = \frac{DF}{YZ}$ Similarity proportion

$\frac{9}{6} = \frac{x}{10}$ $CD = 9$, $WY = 6$, $DF = x$, $YZ = 10$

$9(10) = 6(x)$ Cross Products Property

$90 = 6x$ Multiply.

$15 = x$ Divide each side by 6.

b. Find y.

$\frac{CD}{WY} = \frac{FA}{ZV}$ Similarity proportion

$\frac{9}{6} = \frac{12}{3y - 1}$ $CD = 9$, $WY = 6$, $FA = 12$, $ZV = 3y - 1$

$9(3y - 1) = 6(12)$ Cross Products Property

$27y - 9 = 72$ Multiply.

$27y = 81$ Add 9 to each side.

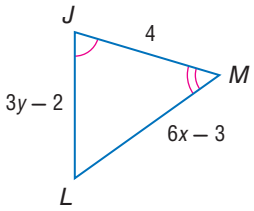
$y = 3$ Divide each side by 27.

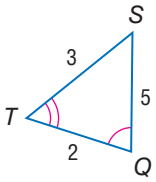
StudyTip

Identifying Similar Triangles When only two congruent angles of a triangle are given, remember that you can use the Third Angles Theorem to establish that the remaining corresponding angles are also congruent.

GuidedPractice

Find the value of each variable if $\triangle JLM \sim \triangle QST$.





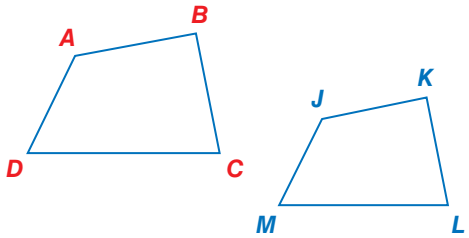
3A. x

3B. y

In similar polygons, the ratio of any two corresponding lengths is proportional to the scale factor between them. This leads to the following theorem about the perimeters of two similar polygons.

Theorem 7.1 Perimeters of Similar Polygons

If two polygons are similar, then their perimeters are proportional to the scale factor between them.

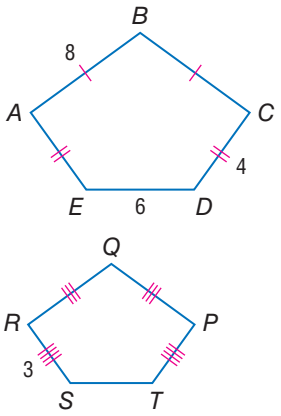


Example If $ABCD \sim JKLM$, then

$$\frac{AB + BC + CD + DA}{JK + KL + LM + MJ} = \frac{AB}{JK} = \frac{BC}{KL} = \frac{CD}{LM} = \frac{DA}{MJ}.$$

Example 4 Use a Scale Factor to Find Perimeter

If $ABCDE \sim PQRST$, find the scale factor of $ABCDE$ to $PQRST$ and the perimeter of each polygon.



The scale factor of $ABCDE$ to $PQRST$ is $\frac{CD}{RS}$ or $\frac{4}{3}$.

Since $\overline{BC} \cong \overline{AB}$ and $\overline{AE} \cong \overline{CD}$, the perimeter of $ABCDE$ is $8 + 8 + 4 + 6 + 4$ or 30.

Use the perimeter of $ABCDE$ and the scale factor to write a proportion. Let x represent the perimeter of $PQRST$.

$$\frac{4}{3} = \frac{\text{perimeter of } ABCDE}{\text{perimeter of } PQRST} \qquad \text{Theorem 7.1}$$

$$\frac{4}{3} = \frac{30}{x} \qquad \text{Substitution}$$

$$(3)(30) = 4x \qquad \text{Cross Products Property}$$

$$22.5 = x \qquad \text{Solve.}$$

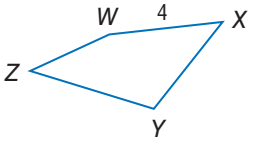
So, the perimeter of $PQRST$ is 22.5.

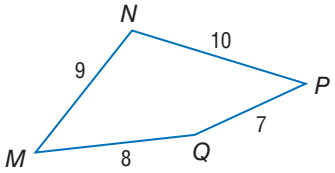
WatchOut!

Perimeter Remember that perimeter is the distance around a figure. Be sure to find the sum of all side lengths when finding the perimeter of a polygon. You may need to use other markings or geometric principles to find the length of unmarked sides.

GuidedPractice

4. If $MNPQ \sim XYZW$, find the scale factor of $MNPQ$ to $XYZW$ and the perimeter of each polygon.



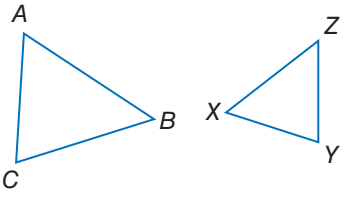


Check Your Understanding

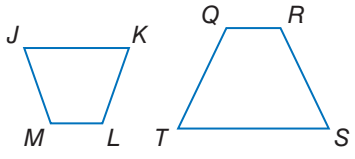
= Step-by-Step Solutions begin on page R14.

Example 1 **List all pairs of congruent angles, and write a proportion that relates the corresponding sides for each pair of similar polygons.**

1 $\triangle ABC \sim \triangle ZYX$

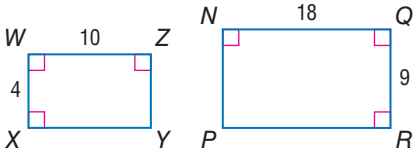


2. $JKLM \sim TSRQ$



Example 2 **Determine whether each pair of figures is similar. If so, write the similarity statement and scale factor. If not, explain your reasoning.**

3.



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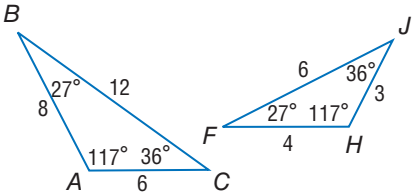
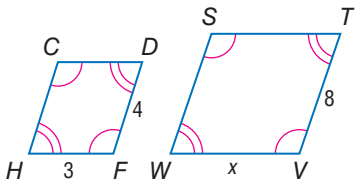


Example 3 Each pair of polygons is similar. Find the value of x.

5.



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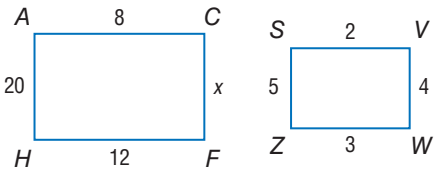
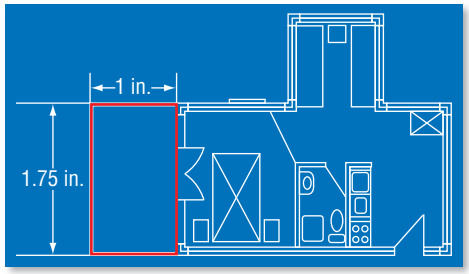


Example 4

7. **DESIGN** On the blueprint of the apartment shown, the balcony measures 1 inch wide by 1.75 inches long. If the actual length of the balcony is 7 feet, what is the perimeter of the balcony?

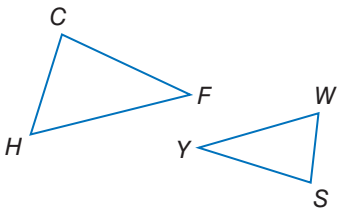


Practice and Problem Solving

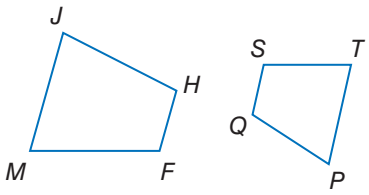
Extra Practice is on page R7.

Example 1 List all pairs of congruent angles, and write a proportion that relates the corresponding sides for each pair of similar polygons.

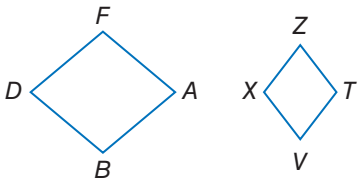
8. $\triangle CHF \sim \triangle YWS$



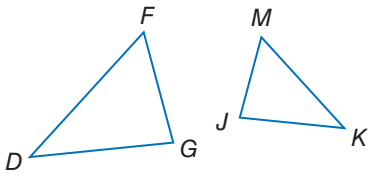
9. $JHFM \sim PQST$



10. $ABDF \sim VXZT$

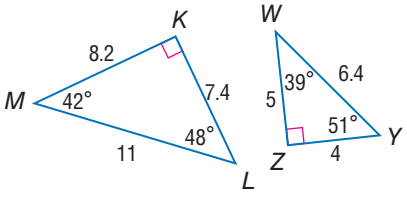


11. $\triangle DFG \sim \triangle KMJ$



Example 2 CCSS **ARGUMENTS** Determine whether each pair of figures is similar. If so, write the similarity statement and scale factor. If not, explain your reasoning.

12.



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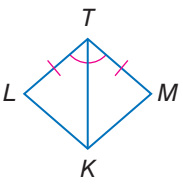

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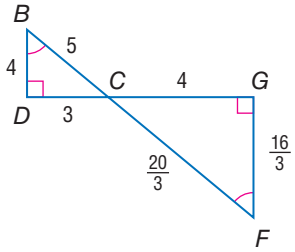

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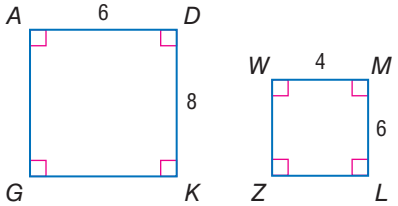
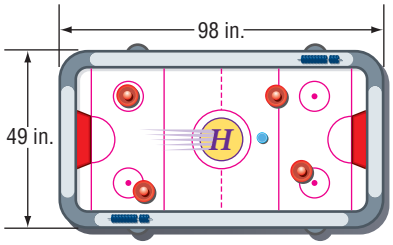


16. GAMES The dimensions of a hockey rink are 200 feet by 85 feet. Are the hockey rink and the air hockey table shown similar? Explain your reasoning.

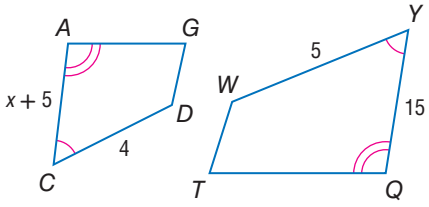


17. COMPUTERS The dimensions of a 17-inch flat panel computer screen are approximately $13\frac{1}{4}$ by $10\frac{3}{4}$ inches. The dimensions of a 19-inch flat panel computer screen are approximately $14\frac{1}{2}$ by 12 inches. To the nearest tenth, are the computer screens similar? Explain your reasoning.

Example 3

REGULARITY **Each pair of polygons is similar. Find the value of x.**

18.



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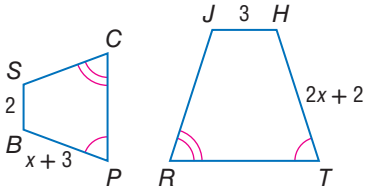

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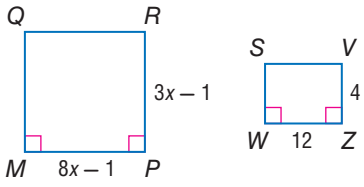

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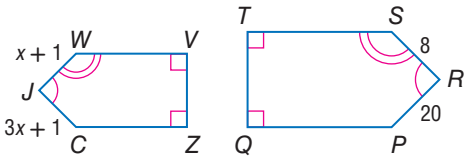
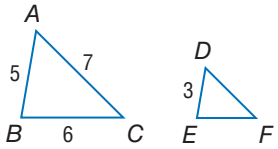


Example 4

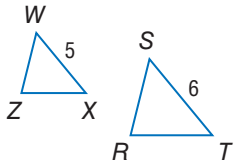
22. Rectangle $ABCD$ has a width of 8 yards and a length of 20 yards. Rectangle $QRST$, which is similar to rectangle $ABCD$, has a length of 40 yards. Find the scale factor of rectangle $ABCD$ to rectangle $QRST$ and the perimeter of each rectangle.

Find the perimeter of the given triangle.

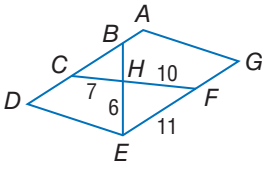
23. $\triangle DEF$, if $\triangle ABC \sim \triangle DEF$, $AB = 5$, $BC = 6$, $AC = 7$, and and $DE = 3$



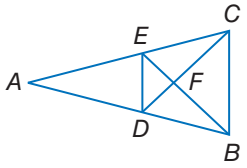
24. $\triangle WZX$, if $\triangle WZX \sim \triangle SRT$, $ST = 6$, $WX = 5$, and the perimeter of $\triangle SRT = 15$



25. $\triangle CBH$, if $\triangle CBH \sim \triangle FEH$, $ADEG$ is a parallelogram, $CH = 7$, $FH = 10$, $FE = 11$, and $EH = 6$



26. $\triangle DEF$, if $\triangle DEF \sim \triangle CBF$, perimeter of $\triangle CBF = 27$, $DF = 6$, $FC = 8$

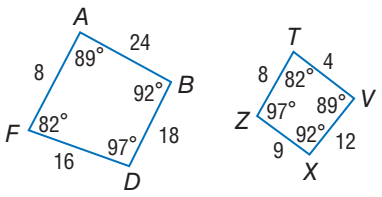


27. Two similar rectangles have a scale factor of 2:4. The perimeter of the large rectangle is 80 meters. Find the perimeter of the small rectangle.

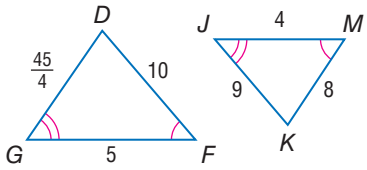
28. Two similar rectangles have a scale factor of 3:2. The perimeter of the small rectangle is 50 feet. Find the perimeter of the large rectangle.

List all pairs of congruent angles, and write a proportion that relates the corresponding sides.

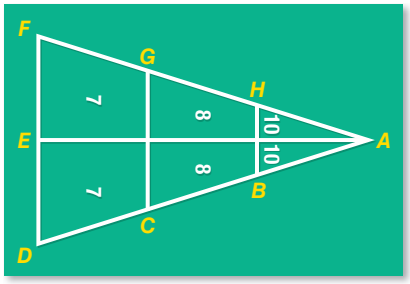
29.



30.



SHUFFLEBOARD **A shuffleboard court forms three similar triangles in which $\angle AHB \cong \angle AGC \cong \angle AFD$. For the given sides or angles, find the corresponding side(s) or angle(s) that are congruent.**



31. $\overline{AB}$

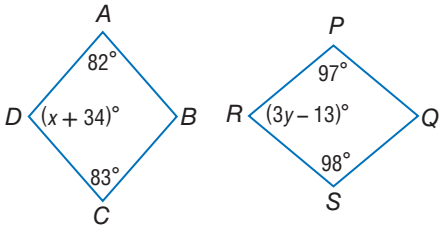
32. $\overline{FD}$

33. $\angle ACG$

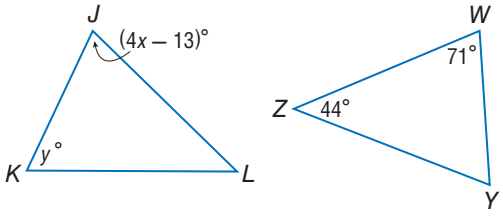
34. $\angle A$

Find the value of each variable.

35 $ABCD \sim QSRP$



36. $\triangle JKL \sim \triangle WYZ$



37. SLIDE SHOW You are using a digital projector for a slide show. The photos are 13 inches by $9\frac{1}{4}$ inches on the computer screen, and the scale factor of the computer image to the projected image is 1:4. What are the dimensions of the projected image?

COORDINATE GEOMETRY **For the given vertices, determine whether rectangle *ABCD* is similar to rectangle *WXYZ*. Justify your answer.**

38. $A(-1, 5)$, $B(7, 5)$, $C(7, -1)$, $D(-1, -1)$; $W(-2, 10)$, $X(14, 10)$, $Y(14, -2)$, $Z(-2, -2)$

39. $A(5, 5)$, $B(0, 0)$, $C(5, -5)$, $D(10, 0)$; $W(1, 6)$, $X(-3, 2)$, $Y(2, -3)$, $Z(6, 1)$

CCSS **ARGUMENTS** **Determine whether the polygons are *always*, *sometimes*, or *never* similar. Explain your reasoning.**

40. two obtuse triangles

41. a trapezoid and a parallelogram

42. two right triangles

43. two isosceles triangles

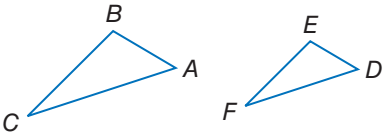
44. a scalene triangle and an isosceles triangle

45. two equilateral triangles

46. PROOF Write a paragraph proof of Theorem 7.1.

Given: $\triangle ABC \sim \triangle DEF$ and $\frac{AB}{DE} = \frac{m}{n}$

Prove: $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{m}{n}$



47 **PHOTOS** You are enlarging the photo shown at the right for your school yearbook. If the dimensions of the original photo are $2\frac{1}{3}$ inches by $1\frac{2}{3}$ inches and the scale factor of the old photo to the new photo is 2:3, what are the dimensions of the new photo?

48. **CHANGING DIMENSIONS** Rectangle $QRST$ is similar to rectangle $JKLM$ with sides in a ratio of 4:1.

   a. What is the ratio of the areas of the two rectangles?

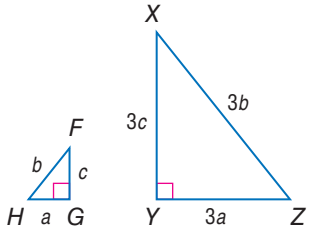
   b. Suppose the dimension of each rectangle is tripled. What is the new ratio of the sides of the rectangles?

   c. What is the ratio of the areas of these larger rectangles?

49. **CHANGING DIMENSIONS** In the figure shown, $\triangle FGH \sim \triangle XYZ$.

   a. Show that the perimeters of $\triangle FGH$ and $\triangle XYZ$ have the same ratio as their corresponding sides.

   b. If 6 units are added to the lengths of each side, are the new triangles similar? Explain.



50. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate similarity in squares.

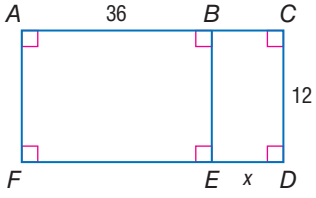
   a. Geometric Draw three different-sized squares. Label them $ABCD$, $PQRS$, and $WXYZ$. Measure and label each square with its side length.

   b. Tabular Calculate and record in a table the ratios of corresponding sides for each pair of squares: $ABCD$ and $PQRS$, $PQRS$ and $WXYZ$, and $WXYZ$ and $ABCD$. Is each pair of squares similar?

   c. Verbal Make a conjecture about the similarity of all squares.

H.O.T. Problems Use Higher-Order Thinking Skills

51. **CHALLENGE** For what value(s) of x is $BEFA \sim EDCB$?



52. **REASONING** Recall that an *equivalence relation* is any relationship that satisfies the Reflexive, Symmetric, and Transitive Properties. Is similarity an equivalence relation? Explain.

53. **OPEN ENDED** Find a counterexample for the following statement.

All rectangles are similar.

54. **REASONING** Draw two regular pentagons of different sizes. Are the pentagons similar? Will any two regular polygons with the same number of sides be similar? Explain.

55. **WRITING IN MATH** How can you describe the relationship between two figures?

Standardized Test Practice

56. ALGEBRA If the arithmetic mean of $4x$, $3x$, and 12 is 18, then what is the value of x?

A 6 **C** 4
B 5 **D** 3

57. Two similar rectangles have a scale factor of 3:5. The perimeter of the large rectangle is 65 meters. What is the perimeter of the small rectangle?

F 29 m **H** 49 m
G 39 m **J** 59 m

58. SHORT RESPONSE If a jar contains 25 dimes and 7 quarters, what is the probability that a coin selected from the jar at random will be a dime?

59. SAT/ACT If the side of a square is $x + 3$, then what is the diagonal of the square?

A $x^2 + 3$ **D** $x\sqrt{3} + 3\sqrt{3}$
B $3x + 3$ **E** $x\sqrt{2} + 3\sqrt{2}$
C $2x + 6$

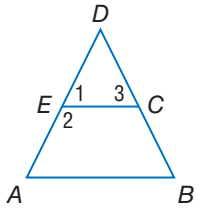
Spiral Review

60. COMPUTERS In a survey of 5000 households, 4200 had at least one computer. What is the ratio of computers to households? (Lesson 7-1)

61. PROOF Write a flow proof. (Lesson 6-6)

Given: E and C are midpoints of $\overline{AD}$ and $\overline{DB}$, $\overline{AD} \cong \overline{DB}$, $\angle A \cong \angle 1$.

Prove: $ABCE$ is an isosceles trapezoid.



62. COORDINATE GEOMETRY Determine the coordinates of the intersection of the diagonals of $\square JKLM$ with vertices $J(2, 5)$, $K(6, 6)$, $L(4, 0)$, and $M(0, -1)$. (Lesson 6-2)

State the assumption you would make to start an indirect proof of each statement. (Lesson 5-4)

63. If $3x > 12$, then $x > 4$.

64. $\overline{PQ} \cong \overline{ST}$

65. The angle bisector of the vertex angle of an isosceles triangle is also an altitude of the triangle.

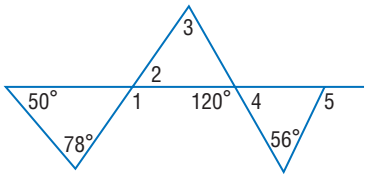
66. If a rational number is any number that can be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$, then 6 is a rational number.

Find the measures of each numbered angle. (Lesson 4-2)

67. $m\angle 1$

68. $m\angle 2$

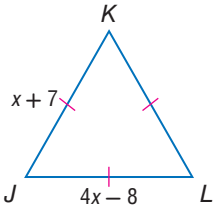
69. $m\angle 3$



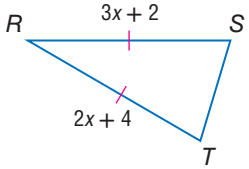
Skills Review

ALGEBRA Find x and the unknown side measures of each triangle.

70.



71.



72.