LESSON 7-3

Similar Triangles

Then

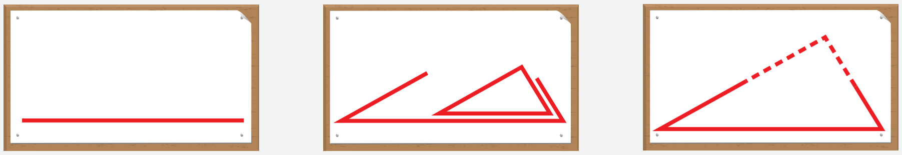
- You used the AAS, SSS, and SAS Congruence Theorems to prove triangles congruent.

Now

- 1 Identify similar triangles using the AA Similarity Postulate and the SSS and SAS Similarity Theorems.
- 2 Use similar triangles to solve problems.

Why?

- Julian wants to draw a similar version of his skate club's logo on a poster. He first draws a line at the bottom of the poster. Next, he uses a cutout of the original triangle to copy the two bottom angles. Finally, he extends the noncommon sides of the two angles.



Common Core State Standards

Content Standards

G.SRT.4 Prove theorems about triangles.

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Mathematical Practices

4 Model with mathematics.

7 Look for and make use of structure.

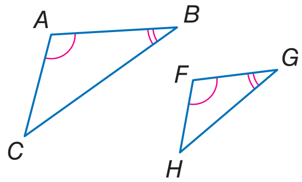
1 Identify Similar Triangles

The example suggests that two triangles are similar if two pairs of corresponding angles are congruent.

Postulate 7.1 Angle-Angle (AA) Similarity

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

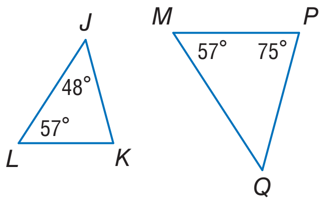
Example If $\angle A \cong \angle F$ and $\angle B \cong \angle G$, then $\triangle ABC \sim \triangle FGH$.



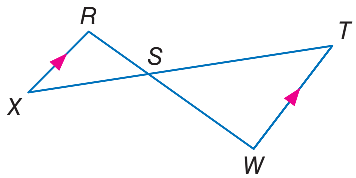
Example 1 Use the AA Similarity Postulate

Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.

a.



b.

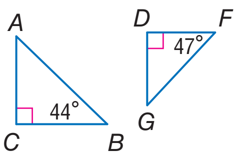


a. Since $m\angle L = m\angle M$, $\angle L \cong \angle M$. By the Triangle Sum Theorem, $57 + 48 + m\angle K = 180$, so $m\angle K = 75$. Since $m\angle P = 75$, $\angle K \cong \angle P$. So, $\triangle LJK \sim \triangle MQP$ by AA Similarity.

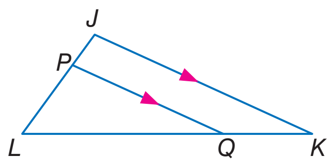
b. $\angle RSX \cong \angle WST$ by the Vertical Angles Theorem. Since $\overline{RX} \parallel \overline{TW}$, $\angle R \cong \angle W$. So, $\triangle RSX \sim \triangle WST$ by AA Similarity.

Guided Practice

1A.



1B.

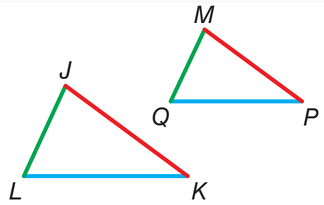


You can use the AA Similarity Postulate to prove the following two theorems.

Theorems Triangle Similarity

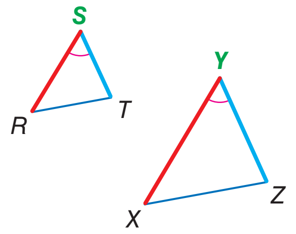
7.2 Side-Side-Side (SSS) Similarity
If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

Example If $\frac{JK}{MP} = \frac{KL}{PQ} = \frac{LJ}{QM}$, then $\triangle JKL \sim \triangle MPQ$.



7.3 Side-Angle-Side (SAS) Similarity
If the lengths of two sides of one triangle are proportional to the lengths of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

Example If $\frac{RS}{XY} = \frac{ST}{YZ}$ and $\angle S \cong \angle Y$, then $\triangle RST \sim \triangle XYZ$.

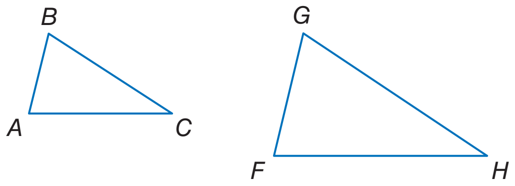


You will prove Theorem 7.3 in Exercise 25.

Proof Theorem 7.2

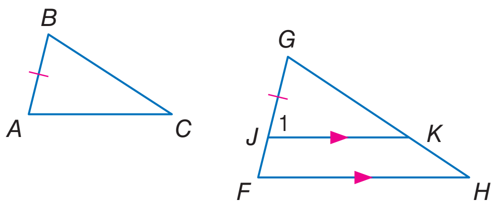
Given: $\frac{AB}{FG} = \frac{BC}{GH} = \frac{AC}{FH}$

Prove: $\triangle ABC \sim \triangle FGH$



Paragraph Proof:

Locate J on $\overline{FG}$ so that $JG = AB$. Draw $\overline{JK}$ so that $\overline{JK} \parallel \overline{FH}$. Label $\angle GJK$ as $\angle 1$.



Since $\angle G \cong \angle G$ by the Reflexive Property and $\angle 1 \cong \angle F$ by the Corresponding Angles Postulate, $\triangle GJK \sim \triangle GFH$ by the AA Similarity Postulate.

By the definition of similar polygons, $\frac{JG}{FG} = \frac{GK}{GH} = \frac{JK}{FH}$. By substitution, $\frac{AB}{FG} = \frac{GK}{GH} = \frac{JK}{FH}$.

Since we are also given that $\frac{AB}{FG} = \frac{BC}{GH} = \frac{AC}{FH}$, we can say that $\frac{GK}{GH} = \frac{BC}{GH}$ and $\frac{JK}{FH} = \frac{AC}{FH}$. This means that $GK = BC$ and $JK = AC$, so $\overline{GK} \cong \overline{BC}$ and $\overline{JK} \cong \overline{AC}$.

By SSS, $\triangle ABC \cong \triangle JGK$.

By CPCTC, $\angle B \cong \angle G$ and $\angle A \cong \angle 1$. Since $\angle 1 \cong \angle F$, $\angle A \cong \angle F$ by the Transitive Property. By AA Similarity, $\triangle ABC \sim \triangle FGH$.

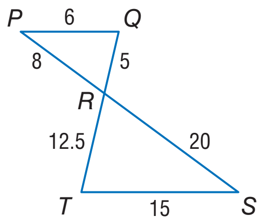
Study Tip

Corresponding Sides To determine which sides of two triangles correspond, begin by comparing the longest sides, then the next longest sides, and finish by comparing the shortest sides.

Example 2 Use the SSS and SAS Similarity Theorems

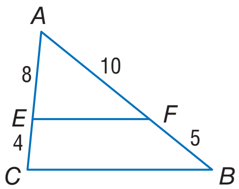
Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.

a.



$\frac{PR}{SR} = \frac{8}{20}$ or $\frac{2}{5}$, $\frac{PQ}{ST} = \frac{6}{15}$ or $\frac{2}{5}$, and $\frac{QR}{TR} = \frac{5}{12.5} = \frac{50}{125}$ or $\frac{2}{5}$. So, $\triangle PQR \sim \triangle STR$ by the SSS Similarity Theorem.

b.



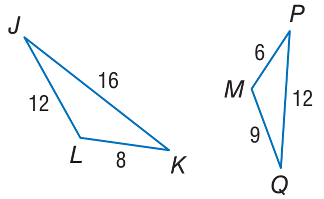
By the Reflexive Property, $\angle A \cong \angle A$.

$\frac{AF}{AB} = \frac{10}{10+5} = \frac{10}{15}$ or $\frac{2}{3}$ and $\frac{AE}{AC} = \frac{8}{8+4} = \frac{8}{12}$ or $\frac{2}{3}$.

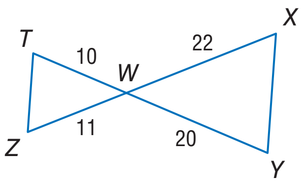
Since the lengths of the sides that include $\angle A$ are proportional, $\triangle AEF \sim \triangle ACB$ by the SAS Similarity Theorem.

GuidedPractice

2A.



2B.



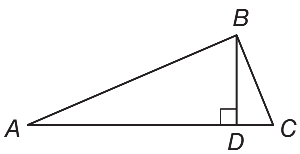
StudyTip

Draw Diagrams It is helpful to redraw similar triangles so that the corresponding side lengths have the same orientation.

You can decide what is sufficient to prove that two triangles are similar.

Standardized Test Example 3 Sufficient Conditions

In the figure, $\angle ADB$ is a right angle. Which of the following would *not* be sufficient to prove that $\triangle ADB \sim \triangle CDB$?



A $\frac{AD}{BD} = \frac{BD}{CD}$

B $\frac{AB}{BC} = \frac{BD}{CD}$

C $\angle ABD \cong \angle C$

D $\frac{AD}{BD} = \frac{BD}{CD} = \frac{AB}{BC}$

Read the Test Item

You are given that $\angle ADB$ is a right angle and asked to identify which additional information would not be enough to prove that $\triangle ADB \sim \triangle CDB$.

Solve the Test Item

Since $\angle ADB$ is a right angle, $\angle CDB$ is also a right angle. Since all right angles are congruent, $\angle ADB \cong \angle CDB$. Check each answer choice until you find one that does not supply a sufficient additional condition to prove that $\triangle ADB \sim \triangle CDB$.

Choice A: If $\frac{AD}{BD} = \frac{BD}{CD}$ and $\angle ADB \cong \angle CDB$, then $\triangle ADB \sim \triangle CDB$ by SAS Similarity.

Choice B: If $\frac{AB}{BC} = \frac{BD}{CD}$ and $\angle ADB \cong \angle CDB$, then we cannot conclude that $\triangle ADB \sim \triangle CDB$ because the included angle of side $\overline{AB}$ and $\overline{BD}$ is not $\angle ADB$. So the answer is B.

Test-TakingTip

Identifying Nonexamples Sometimes test questions require you to find a nonexample, as in this case. You must check each option until you find a valid nonexample. If you would like to check your answer, confirm that each additional option is correct.

Guided Practice

3. If $\triangle JKL$ and $\triangle FGH$ are two triangles such that $\angle J \cong \angle F$, which of the following would be sufficient to prove that the triangles are similar?

F $\frac{KL}{GH} = \frac{JL}{FH}$ **G** $\frac{JL}{JK} = \frac{FH}{FG}$ **H** $\frac{JK}{FG} = \frac{KL}{GH}$ **J** $\frac{JL}{JK} = \frac{GH}{FG}$

2 Use Similar Triangles

Like the congruence of triangles, similarity of triangles is reflexive, symmetric, and transitive.

Theorem 7.4 Properties of Similarity

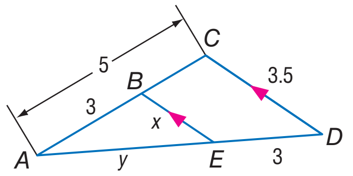
Reflexive Property of Similarity	$\triangle ABC \sim \triangle ABC$
Symmetric Property of Similarity	If $\triangle ABC \sim \triangle DEF$, then $\triangle DEF \sim \triangle ABC$.
Transitive Property of Similarity	If $\triangle ABC \sim \triangle DEF$, and $\triangle DEF \sim \triangle XYZ$, then $\triangle ABC \sim \triangle XYZ$.

You will prove Theorem 7.4 in Exercise 26.

PT

Example 4 Parts of Similar Triangles

Find *BE* and *AD*.



Since $\overline{BE} \parallel \overline{CD}$, $\angle ABE \cong \angle BCD$, and $\angle AEB \cong \angle EDC$ because they are corresponding angles. By AA Similarity, $\triangle ABE \sim \triangle ACD$.

$\frac{AB}{AC} = \frac{BE}{CD}$ Definition of Similar Polygons

$\frac{3}{5} = \frac{x}{3.5}$ $AC = 5$, $CD = 3.5$, $AB = 3$, $BE = x$

$3.5 \cdot 3 = 5 \cdot x$ Cross Products Property

$2.1 = x$ *BE* is 2.1.

$\frac{AC}{AB} = \frac{AD}{AE}$ Definition of Similar Polygons

$\frac{5}{3} = \frac{y+3}{y}$ $AC = 5$, $AB = 3$, $AD = y + 3$, $AE = y$

$5 \cdot y = 3(y + 3)$ Cross Products Property

$5y = 3y + 9$ Distributive Property

$2y = 9$ Subtract $3y$ from each side.

$y = 4.5$ *AD* is $y + 3$ or 7.5.

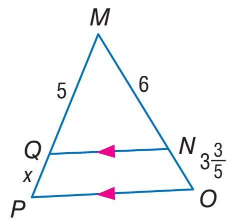
Study Tip

Proportions An additional proportion that is true for Example 4 is $\frac{AC}{CD} = \frac{AB}{BE}$.

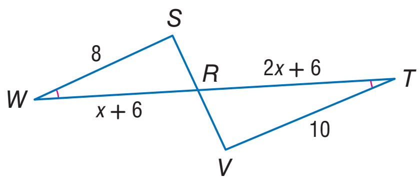
Guided Practice

Find each measure.

4A. *QP* and *MP*



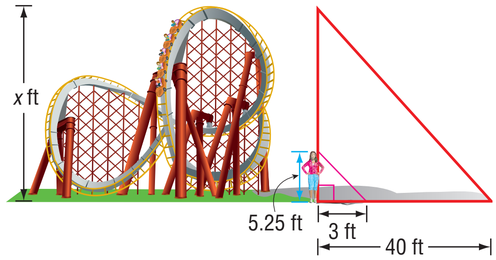
4B. *WR* and *RT*



Real-World Example 5 Indirect Measurement

ROLLER COASTERS **Hallie is estimating the height of the Superman roller coaster in Mitchellville, Maryland. She is 5 feet 3 inches tall and her shadow is 3 feet long. If the length of the shadow of the roller coaster is 40 feet, how tall is the roller coaster?**

Understand Make a sketch of the situation. 5 feet 3 inches is equivalent to 5.25 feet.



Plan In shadow problems, you can assume that the angles formed by the Sun's rays with any two objects are congruent and that the two objects form the sides of two right triangles.

Since two pairs of angles are congruent, the right triangles are similar by the AA Similarity Postulate. So, the following proportion can be written.

$$\frac{\text{Hallie's height}}{\text{coaster's height}} = \frac{\text{Hallie's shadow length}}{\text{coaster's shadow length}}$$

Solve Substitute the known values and let x = roller coaster's height.

$\frac{5.25}{x} = \frac{3}{40}$	Substitution
$3 \cdot x = 40(5.25)$	Cross Products Property
$3x = 210$	Simplify.
$x = 70$	Divide each side by 3.

The roller coaster is 70 feet tall.

Check The roller coaster's shadow length is $\frac{40 \text{ ft}}{3 \text{ ft}}$ or about 13.3 times Hallie's shadow length. Check to see that the roller coaster's height is about 13.3 times Hallie's height. $\frac{70 \text{ ft}}{5.25 \text{ ft}} \approx 13.3$ ✓

Problem-Solving Tip

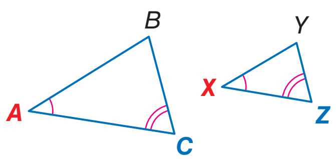
Reasonable Answers When you have solved a problem, check your answer for reasonableness. In this example, Hallie's shadow is a little more than half her height. The coaster's shadow is also a little more than half of the height you calculated. Therefore, the answer is reasonable.

Guided Practice

5. BUILDINGS Adam is standing next to the Palmetto Building in Columbia, South Carolina. He is 6 feet tall and the length of his shadow is 9 feet. If the length of the shadow of the building is 322.5 feet, how tall is the building?

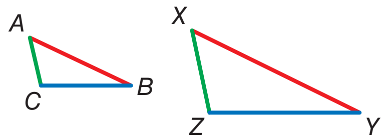
Concept Summary Triangle Similarity

AA Similarity Postulate



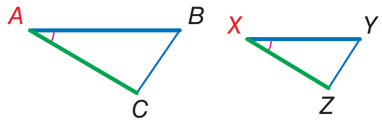
If $\angle A \cong \angle X$ and $\angle C \cong \angle Z$, then $\triangle ABC \sim \triangle XYZ$.

SSS Similarity Theorem



If $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX}$, then $\triangle ABC \sim \triangle XYZ$.

SAS Similarity Theorem



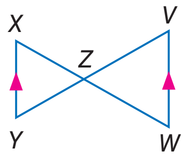
If $\angle A \cong \angle X$ and $\frac{AB}{XY} = \frac{CA}{ZX}$, then $\triangle ABC \sim \triangle XYZ$.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Examples 1–2 **Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.**

1.



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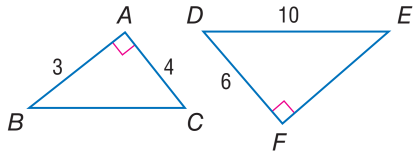
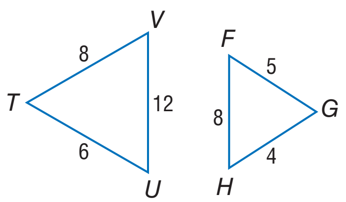
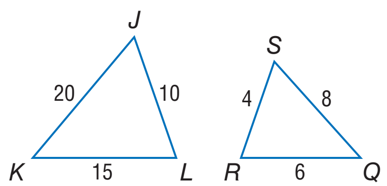


3.



4.



Example 3

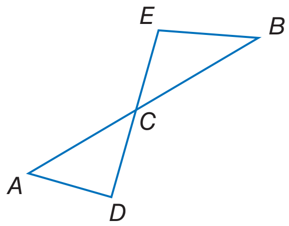
5. MULTIPLE CHOICE In the figure, $\overline{AB}$ intersects $\overline{DE}$ at point C. Which additional information would be enough to prove that $\triangle ADC \sim \triangle BEC$?

A $\angle DAC$ and $\angle ECB$ are congruent.

B $\overline{AC}$ and $\overline{BC}$ are congruent.

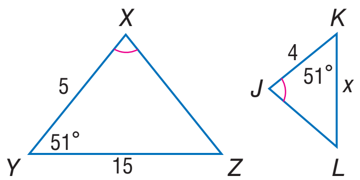
C $\overline{AD}$ and $\overline{EB}$ are parallel.

D $\angle CBE$ is a right angle.

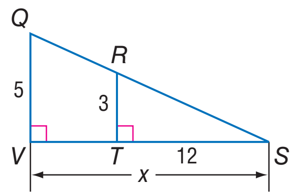


Example 4 **CCSS STRUCTURE** **Identify the similar triangles. Find each measure.**

6. KL



7. VS



Example 5

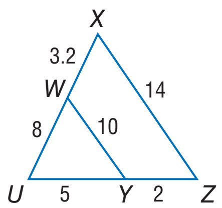
8. COMMUNICATION A cell phone tower casts a 100-foot shadow. At the same time, a 4-foot 6-inch post near the tower casts a shadow of 3 feet 4 inches. Find the height of the tower.

Practice and Problem Solving

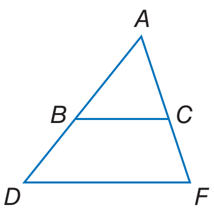
Extra Practice is on page R7.

Examples 1–3 **Determine whether the triangles are similar. If so, write a similarity statement. If not, what would be sufficient to prove the triangles similar? Explain your reasoning.**

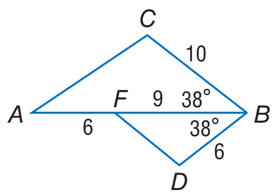
9.



10.

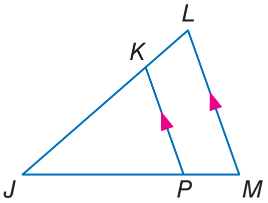


11.

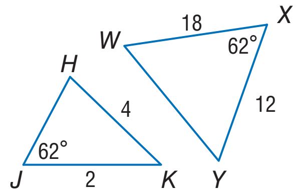


Examples 1–3 **Determine whether the triangles are similar. If so, write a similarity statement. If not, what would be sufficient to prove the triangles similar? Explain your reasoning.**

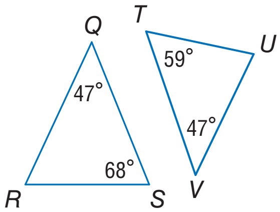
12.



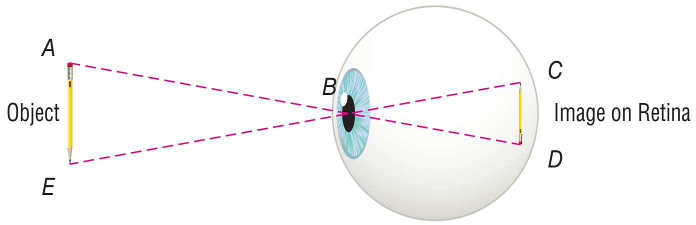
13.



14.

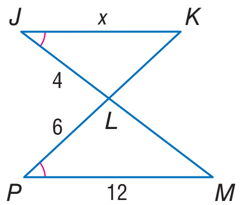


15. CCSS **MODELING** When we look at an object, it is projected on the retina through the pupil. The distances from the pupil to the top and bottom of the object are congruent and the distances from the pupil to the top and bottom of the image on the retina are congruent. Are the triangles formed between the object and the pupil and the object and the image similar? Explain your reasoning.

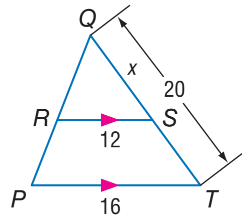


Example 4 **ALGEBRA** **Identify the similar triangles. Then find each measure.**

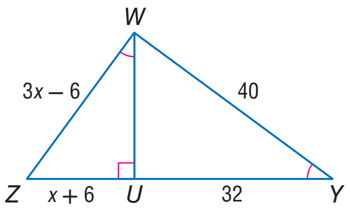
16. *JK*



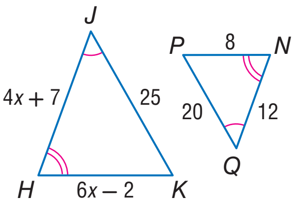
17 *ST*



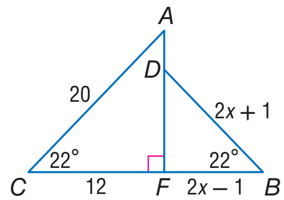
18. *WZ*, *UZ*



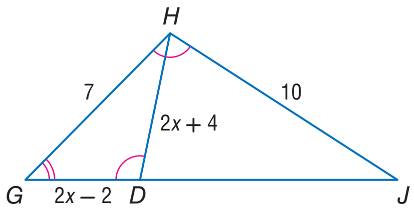
19. *HJ*, *HK*



20. *DB*, *CB*



21. *GD*, *DH*

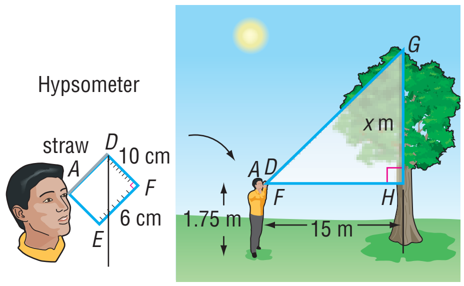


Example 5

22. **STATUES** Mei is standing next to a statue in the park. If Mei is 5 feet tall, her shadow is 3 feet long, and the statue's shadow is $10\frac{1}{2}$ feet long, how tall is the statue?

23. **SPORTS** When Alonzo, who is 5′11″ tall, stands next to a basketball goal, his shadow is 2′ long, and the basketball goal's shadow is 4′4″ long. About how tall is the basketball goal?

24. **FORESTRY** A hypsometer, as shown, can be used to estimate the height of a tree. Bartolo looks through the straw to the top of the tree and obtains the readings given. Find the height of the tree.



PROOF **Write a two-column proof.**

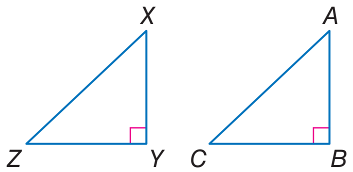
25. Theorem 7.3

26. Theorem 7.4

PROOF **Write a two-column proof.**

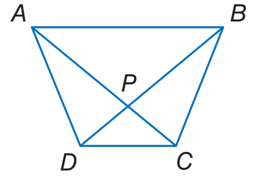
27. **Given:** $\triangle XYZ$ and $\triangle ABC$ are right triangles; $\frac{XY}{AB} = \frac{YZ}{BC}$.

Prove: $\triangle YXZ \sim \triangle BAC$

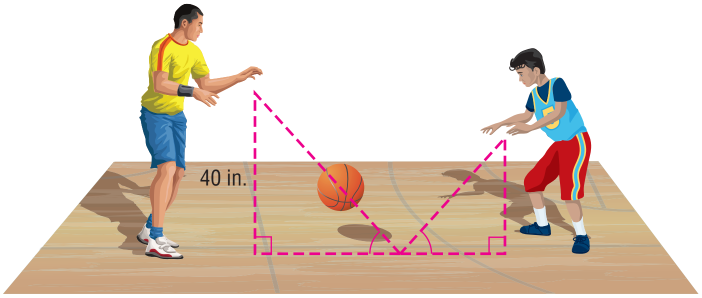


28. **Given:** $ABCD$ is a trapezoid.

Prove: $\frac{DP}{PB} = \frac{CP}{PA}$



29. **CCSS MODELING** When Luis's dad threw a bounce pass to him, the angles formed by the basketball's path were congruent. The ball landed $\frac{2}{3}$ of the way between them before it bounced back up. If Luis's dad released the ball 40 inches above the floor, at what height did Luis catch the ball?

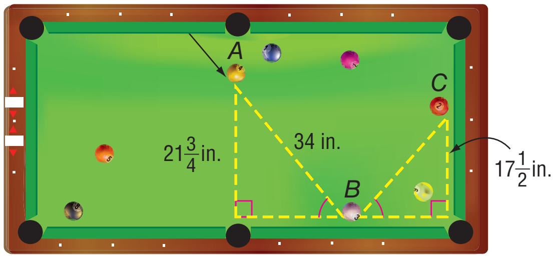


COORDINATE GEOMETRY **$\triangle XYZ$ and $\triangle WYV$ have vertices $X(-1, -9)$, $Y(5, 3)$, $Z(-1, 6)$, $W(1, -5)$, and $V(1, 5)$.**

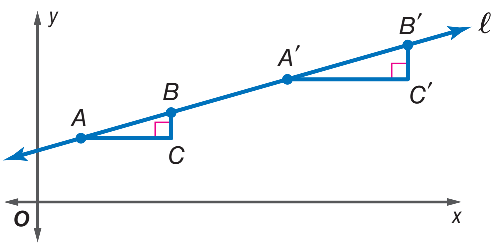
30. Graph the triangles, and prove that $\triangle XYZ \sim \triangle WYV$.

31. Find the ratio of the perimeters of the two triangles.

32. **BILLIARDS** When a ball is deflected off a smooth surface, the angles formed by the path are congruent. Booker hit the orange ball and it followed the path from A to B to C as shown below. What was the total distance traveled by the ball from the time Booker hit it until it came to rest at the end of the table?



33. **PROOF** Use similar triangles to show that the slope of the line through any two points on that line is constant. That is, if points A, B, A' and B' are on line ℓ, use similar triangles to show that the slope of the line from A to B is equal to the slope of the line from A' to B'.

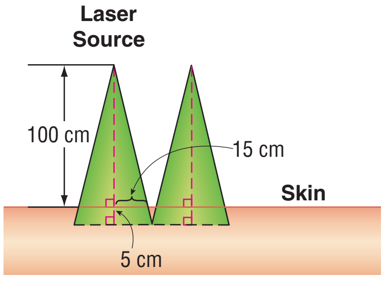


34. **CHANGING DIMENSIONS** Assume that $\triangle ABC \sim \triangle JKL$.

a. If the lengths of the sides of $\triangle JKL$ are half the length of the sides of $\triangle ABC$, and the area of $\triangle ABC$ is 40 square inches, what is the area of $\triangle JKL$? How is the area related to the scale factor of $\triangle ABC$ to $\triangle JKL$?

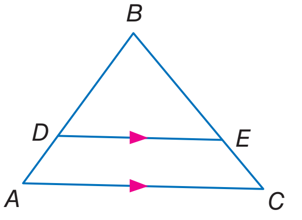
b. If the lengths of the sides of $\triangle ABC$ are three times the length of the sides of $\triangle JKL$, and the area of $\triangle ABC$ is 63 square inches, what is the area of $\triangle JKL$? How is the area related to the scale factor of $\triangle ABC$ to $\triangle JKL$?

35 **MEDICINE** Certain medical treatments involve laser beams that contact and penetrate the skin, forming similar triangles. Refer to the diagram at the right. How far apart should the laser sources be placed to ensure that the areas treated by each source do not overlap?



36. **MULTIPLE REPRESENTATIONS** In this problem, you will explore proportional parts of triangles.

a. **Geometric** Draw $\triangle ABC$ with $\overline{DE}$ parallel to $\overline{AC}$ as shown at the right.



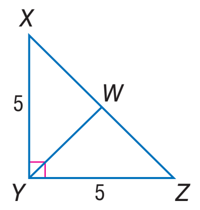
b. **Tabular** Measure and record the lengths AD, DB, CD, and EB and the ratios $\frac{AD}{DB}$ and $\frac{CE}{EB}$ in a table.

c. **Verbal** Make a conjecture about the segments created by a line parallel to one side of a triangle and intersecting the other two sides.

H.O.T. Problems Use Higher-Order Thinking Skills

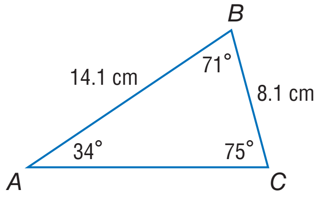
37. **WRITING IN MATH** Compare and contrast the AA Similarity Postulate, the SSS Similarity Theorem, and the SAS similarity theorem.

38. **CHALLENGE** $\overline{YW}$ is an altitude of $\triangle XYZ$. Find YW.



39. **CCSS** **REASONING** A pair of similar triangles has angle measures of 50°, 85°, and 45°. The sides of one triangle measure 3, 3.25, and 4.23 units, and the sides of the second triangle measure $x - 0.46$, x, and $x + 1.81$ units. Find the value of x.

40. **OPEN ENDED** Draw a triangle that is similar to $\triangle ABC$ shown. Explain how you know that it is similar.



41. **WRITING IN MATH** How can you choose an appropriate scale?

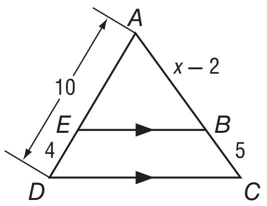
Standardized Test Practice

42. PROBABILITY $\frac{x!}{(x-3)!} =$

A 3.0 **C** $x^2 - 3x + 2$

B 0.33 **D** $x^3 - 3x^2 + 2x$

43. EXTENDED RESPONSE In the figure below, $\overline{EB} \parallel \overline{DC}$.



a. Write a proportion that could be used to find x.

b. Find the value of x and the measure of $\overline{AB}$.

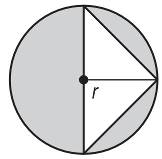
44. ALGEBRA Which polynomial represents the area of the shaded region?

F πr^2

G $\pi r^2 + r^2$

H $\pi r^2 + r$

J $\pi r^2 - r^2$



45. SAT/ACT The volume of a certain rectangular solid is $16x$ cubic units. If the dimensions of the solid are integers x, y, and z units, what is the greatest possible value of z?

A 32 **D** 4

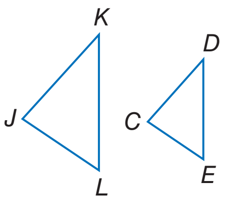
B 16 **E** 2

C 8

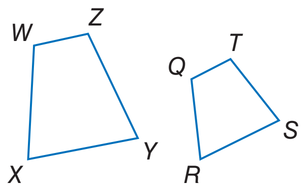
Spiral Review

List all pairs of congruent angles, and write a proportion that relates the corresponding sides for each pair of similar polygons. (Lesson 7-2)

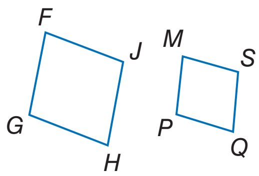
46. $\triangle JKL \sim \triangle CDE$



47. $WXYZ \sim QRST$



48. $FGHJ \sim MPQS$



Solve each proportion. (Lesson 7-1)

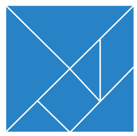
49. $\frac{3}{4} = \frac{x}{16}$

50. $\frac{x}{10} = \frac{22}{50}$

51. $\frac{20.2}{88} = \frac{12}{x}$

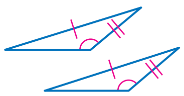
52. $\frac{x-2}{2} = \frac{3}{8}$

53. TANGRAMS A tangram set consists of seven pieces: a small square, two small congruent right triangles, two large congruent right triangles, a medium-sized right triangle, and a quadrilateral. How can you determine the shape of the quadrilateral? Explain. (Lesson 6-3)



Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write *not possible*. (Lesson 4-4)

54.



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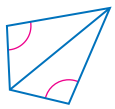
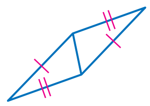


56.



Skills Review

Write a two-column proof.

57. Given: $r \parallel t$; $\angle 5 \cong \angle 6$

Prove: $\ell \parallel m$