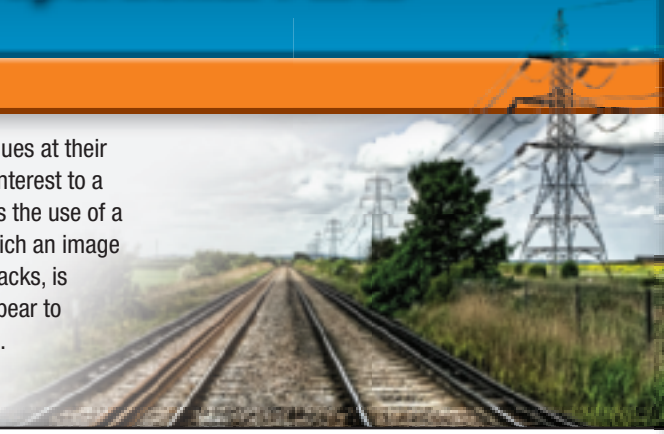
LESSON 7-4

Parallel Lines and Proportional Parts

Then
- You used proportions to solve problems between similar triangles.

Now
1. Use proportional parts within triangles.
2. Use proportional parts with parallel lines.

Why?
- Photographers have many techniques at their disposal that can be used to add interest to a photograph. One such technique is the use of a vanishing point perspective, in which an image with parallel lines, such as train tracks, is photographed so that the lines appear to converge at a point on the horizon.



NewVocabulary
midsegment of a triangle

Common Core State Standards

Content Standards
G.SRT.4 Prove theorems about triangles.
G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

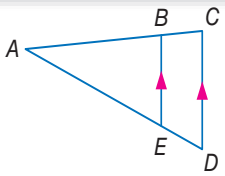
Mathematical Practices
1 Make sense of problems and persevere in solving them.
3 Construct viable arguments and critique the reasoning of others.

1 Proportional Parts Within Triangles

When a triangle contains a line that is parallel to one of its sides, the two triangles formed can be proved similar using the Angle-Angle Similarity Postulate. Since the triangles are similar, their sides are proportional.

Theorem 7.5 Triangle Proportionality Theorem

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.



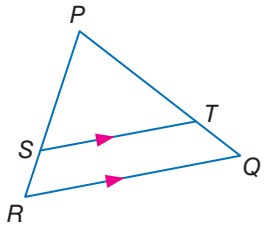
Example If $\overline{BE} \parallel \overline{CD}$, then $\frac{AB}{BC} = \frac{AE}{ED}$.

You will prove Theorem 7.5 in Exercise 30.

Example 1 Find the Length of a Side

In $\triangle PQR$, $\overline{ST} \parallel \overline{RQ}$. If $PT = 7.5$, $TQ = 3$, and $SR = 2.5$, find PS.

Use the Triangle Proportionality Theorem.



$\frac{PS}{SR} = \frac{PT}{TQ}$ Triangle Proportionality Theorem

$\frac{PS}{2.5} = \frac{7.5}{3}$ Substitute.

$PS \cdot 3 = (2.5)(7.5)$ Cross Products Property

$3PS = 18.75$ Multiply.

$PS = 6.25$ Divide each side by 3.

GuidedPractice

1. If $PS = 12.5$, $SR = 5$, and $PT = 15$, find TQ.

Gregory Warran/Flickr/Getty Images

Math History Link

Galileo Galilei (1564–1642) Galileo was born in Pisa, Italy. He studied philosophy, astronomy, and mathematics. Galileo made essential contributions to all three disciplines. Refer to Exercise 39.

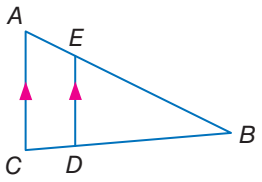
Source: *Encyclopaedia Britannica*

The converse of Theorem 7.5 is also true and can be proved using the proportional parts of a triangle.

Theorem 7.6 Converse of Triangle Proportionality Theorem

If a line intersects two sides of a triangle and separates the sides into proportional corresponding segments, then the line is parallel to the third side of the triangle.

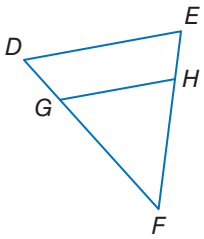
Example If $\frac{AE}{EB} = \frac{CD}{DB}$, then $\overline{AC} \parallel \overline{ED}$.



You will prove Theorem 7.6 in Exercise 31.

Example 2 Determine if Lines are Parallel

In $\triangle DEF$, $EH = 3$, $HF = 9$, and DG is one-third the length of $\overline{GF}$. Is $\overline{DE} \parallel \overline{GH}$?



Using the converse of the Triangle Proportionality Theorem, in order to show that $\overline{DE} \parallel \overline{GH}$, we must show that $\frac{DG}{GF} = \frac{EH}{HF}$.

Find and simplify each ratio. Let $DG = x$.
Since DG is one-third of GF, $GF = 3x$.

$\frac{DG}{GF} = \frac{x}{3x}$ or $\frac{1}{3}$ $\qquad$ $\frac{EH}{HF} = \frac{3}{9}$ or $\frac{1}{3}$

Since $\frac{1}{3} = \frac{1}{3}$, the sides are proportional, so $\overline{DE} \parallel \overline{GH}$.

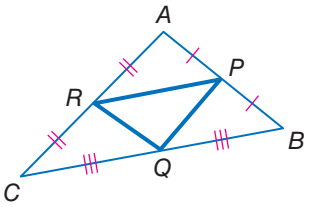
GuidedPractice

2. DG is half the length of $\overline{GF}$, $EH = 6$, and $HF = 10$. Is $\overline{DE} \parallel \overline{GH}$?

StudyTip

Midsegment Triangle
The three midsegments of a triangle form the *midsegment triangle.*

A **midsegment of a triangle** is a segment with endpoints that are the midpoints of two sides of the triangle. Every triangle has three midsegments. The midsegments of $\triangle ABC$ are $\overline{RP}$, $\overline{PQ}$, $\overline{RQ}$.

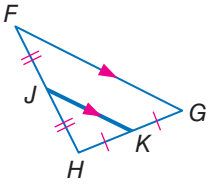


A special case of the Triangle Proportionality Theorem is the Triangle Midsegment Theorem.

Theorem 7.7 Triangle Midsegment Theorem

A midsegment of a triangle is parallel to one side of the triangle, and its length is one half the length of that side.

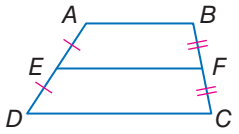
Example If J and K are midpoints of $\overline{FH}$ and $\overline{HG}$, respectively, then $\overline{JK} \parallel \overline{FG}$ and $JK = \frac{1}{2}FG$.



You will prove Theorem 7.7 in Exercise 32.

StudyTip

Midsegment The Triangle Midsegment Theorem is similar to the Trapezoid Midsegment Theorem, which states that the midsegment of a trapezoid is parallel to the bases and its length is one half the sum of the measures of the bases. (Lesson 6-6)

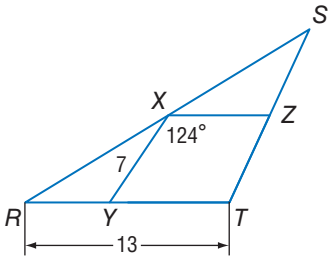


$\overline{EF} \parallel \overline{AB} \parallel \overline{DC}$

$EF = \frac{1}{2}(AB + DC)$

Example 3 Use the Triangle Midsegment Theorem

In the figure, $\overline{XY}$ and $\overline{XZ}$ are midsegments of $\triangle RST$. Find each measure.



a. XZ

$XZ = \frac{1}{2}RT$ Triangle Midsegment Theorem

$XZ = \frac{1}{2}(13)$ Substitution

$XZ = 6.5$ Simplify.

b. ST

$XY = \frac{1}{2}ST$ Triangle Midsegment Theorem

$7 = \frac{1}{2}ST$ Substitution

$14 = ST$ Multiply each side by 2.

c. $m\angle RYX$

By the Triangle Midsegment Theorem, $\overline{XZ} \parallel \overline{RT}$.

$\angle RYX \cong \angle YXZ$ Alternate Interior Angles Theorem

$m\angle RYX = m\angle YXZ$ Definition of congruence

$m\angle RYX = 124$ Substitution

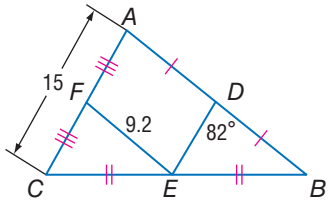
GuidedPractice

Find each measure.

3A. DE

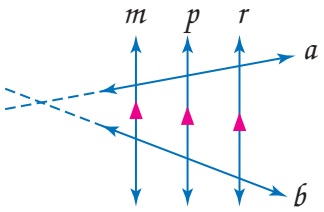
3B. DB

3C. $m\angle FED$



2 Proportional Parts with Parallel Lines

Another special case of the Triangle Proportionality Theorem involves three or more parallel lines cut by two transversals. Notice that if transversals a and b are extended, they form triangles with the parallel lines.



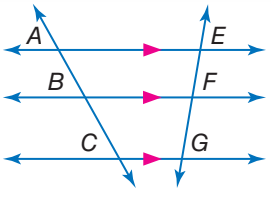
StudyTip

Other Proportions Two other proportions can be written for the example in Corollary 7.1. $\frac{AB}{EF} = \frac{BC}{FG}$ and $\frac{AC}{BC} = \frac{EG}{FG}$

Corollary 7.1 Proportional Parts of Parallel Lines

If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

Example If $\overline{AE} \parallel \overline{BF} \parallel \overline{CG}$, then $\frac{AB}{BC} = \frac{EF}{FG}$.



You will prove Corollary 7.1 in Exercise 28.

Real-WorldLink

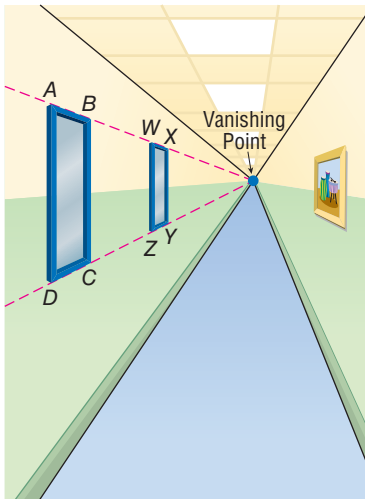
To make a two-dimensional drawing appear three-dimensional, an artist provides several perceptual cues.

- *size* - faraway items look smaller
- *clarity* - closer objects appear more in focus
- *detail* - nearby objects have texture, while distant ones are roughly outlined

Source: Center for Media Literacy

Real-World Example 4 Use Proportional Segments of Transversals

ART **Megan is drawing a hallway in one-point perspective. She uses the guidelines shown to draw two windows on the left wall. If segments $\overline{AD}$, $\overline{BC}$, $\overline{WZ}$, and $\overline{XY}$ are all parallel, $AB = 8$ centimeters, $DC = 9$ centimeters, and $ZY = 5$ centimeters, find WX.**



By Corollary 7.1, if $\overline{AD} \parallel \overline{BC} \parallel \overline{WZ} \parallel \overline{XY}$,

then $\frac{AB}{WX} = \frac{DC}{ZY}$.

$\frac{AB}{WX} = \frac{DC}{ZY}$ Corollary 7.1

$\frac{8}{WX} = \frac{9}{5}$ Substitute.

$WX \cdot 9 = 8 \cdot 5$ Cross Products Property

$9WX = 40$ Simplify.

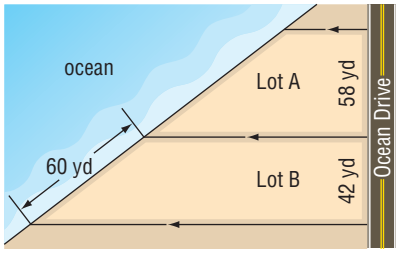
$WX = \frac{40}{9}$ Divide each side by 4.

The distance between W and X should be $\frac{40}{9}$ or about 4.4 centimeters.

CHECK The ratio of DC to ZY is 9 to 5, which is about 10 to 5 or 2 to 1. The ratio of AB to WX is 8 to 4.4 or about 8 to 4 or 2 to 1 as well, so the answer is reasonable. ✓

GuidedPractice

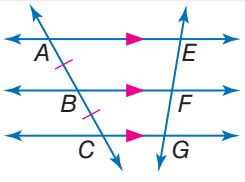
4. REAL ESTATE *Frontage* is the measurement of a property's boundary that runs along the side of a particular feature such as a street, lake, ocean, or river. Find the ocean frontage for Lot A to the nearest tenth of a yard.



If the scale factor of the proportional segments is 1, they separate the transversals into congruent parts.

Corollary 7.2 Congruent Parts of Parallel Lines

If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

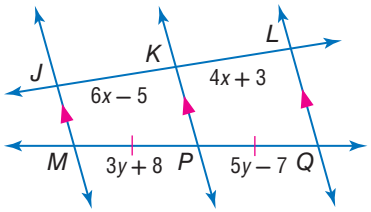


Example If $\overline{AE} \parallel \overline{BF} \parallel \overline{CG}$, and $\overline{AB} \cong \overline{BC}$, then $\overline{EF} \cong \overline{FG}$.

You will prove Corollary 7.2 in Exercise 29.

Real-World Example 5 Use Congruent Segments of Transversals

ALGEBRA Find x and y.

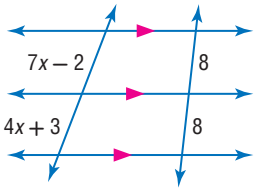


Since $\overleftrightarrow{JM} \parallel \overleftrightarrow{KP} \parallel \overleftrightarrow{LQ}$ and $\overline{MP} \cong \overline{PQ}$, then $\overline{JK} \cong \overline{KL}$ by Corollary 7.2.

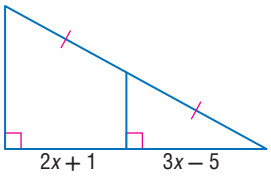
$JK = KL$	Definition of congruence
$6x - 5 = 4x + 3$	Substitution
$2x - 5 = 3$	Subtract $4x$ from each side.
$2x = 8$	Add 5 to each side.
$x = 4$	Divide each side by 2.
$MP = PQ$	Definition of congruence
$3y + 8 = 5y - 7$	Substitution
$8 = 2y - 7$	Subtract $3y$ from each side.
$15 = 2y$	Add 7 to each side.
$7.5 = y$	Divide each side by 2.

GuidedPractice

5A.



5B.

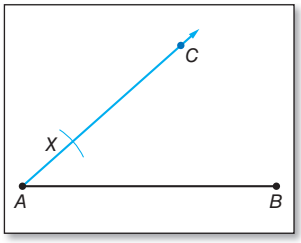


It is possible to separate a segment into two congruent parts by constructing the perpendicular bisector of a segment. However, a segment cannot be separated into three congruent parts by constructing perpendicular bisectors. To do this, you must use parallel lines and Corollary 7.2.

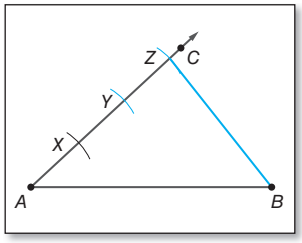
Construction Trisect a Segment

Draw a segment $\overline{AB}$. Then use Corollary 7.2 to trisect $\overline{AB}$.

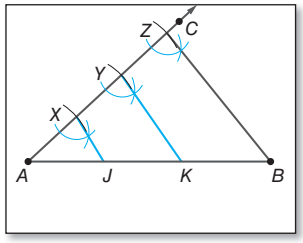
Step 1 Draw $\overline{AC}$. Then with the compass at A, mark off an arc that intersects $\overline{AC}$ at X.



Step 2 Use the same compass setting to mark off Y and Z such that $\overline{AX} \cong \overline{XY} \cong \overline{YZ}$. Then draw ZB.



Step 3 Construct lines through Y and X that are parallel to $\overline{ZB}$. Label the intersection points on $\overline{AB}$ as J and K.



Conclusion: Since parallel lines cut off congruent segments on transversals, $\overline{AJ} \cong \overline{JK} \cong \overline{KB}$.

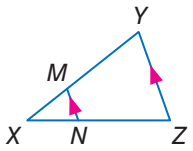
Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1

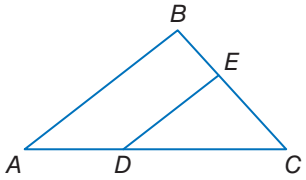
1. If $XM = 4$, $XN = 6$, and $NZ = 9$, find XY.

2. If $XN = 6$, $XM = 2$, and $XY = 10$, find NZ.

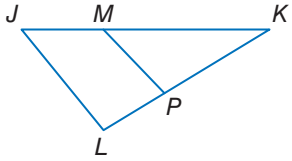


Example 2

3. In $\triangle ABC$, $BC = 15$, $BE = 6$, $DC = 12$, and $AD = 8$. Determine whether $\overline{DE} \parallel \overline{AB}$. Justify your answer.



4. In $\triangle JKL$, $JK = 15$, $JM = 5$, $LK = 13$, and $PK = 9$. Determine whether $\overline{JL} \parallel \overline{MP}$. Justify your answer.



Example 3 $\overline{JH}$ **is a midsegment of** $\triangle KLM$**. Find the value of** x**.**

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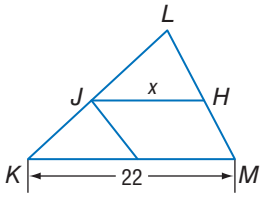

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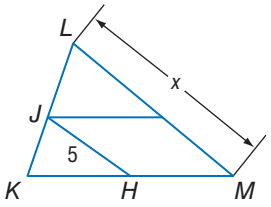
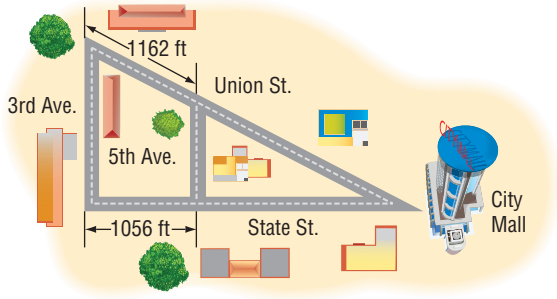


Example 4

7. MAPS Refer to the map at the right. 3rd Avenue and 5th Avenue are parallel. If the distance from 3rd Avenue to City Mall along State Street is 3201 feet, find the distance between 5th Avenue and City Mall along Union Street. Round to the nearest tenth.



Example 5 **ALGEBRA Find** x **and** y**.**

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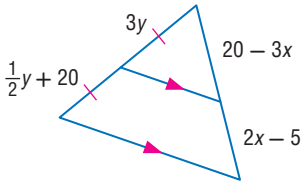

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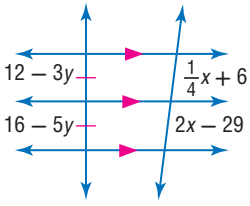


Practice and Problem Solving

Extra Practice is on page R7.

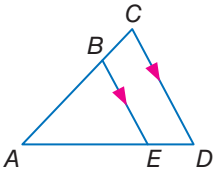
Example 1

10. If $AB = 6$, $BC = 4$, and $AE = 9$, find ED.

11. If $AB = 12$, $AC = 16$, and $ED = 5$, find AE.

12. If $AC = 14$, $BC = 8$, and $AD = 21$, find ED.

13. If $AD = 27$, $AB = 8$, and $AE = 12$, find BC.



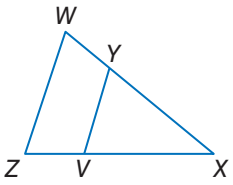
Example 2 **Determine whether $\overline{VY} \parallel \overline{ZW}$. Justify your answer.**

14. $ZX = 18$, $ZV = 6$, $WX = 24$, and $YX = 16$

15. $VX = 7.5$, $ZX = 24$, $WY = 27.5$, and $WX = 40$

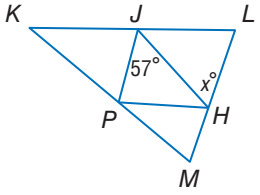
16. $ZV = 8$, $VX = 2$, and $YX = \frac{1}{2}WY$

17. $WX = 31$, $YX = 21$, and $ZX = 4ZV$



Example 3 **$\overline{JH}$, $\overline{JP}$, and $\overline{PH}$ are midsegments of $\triangle KLM$. Find the value of x.**

18.



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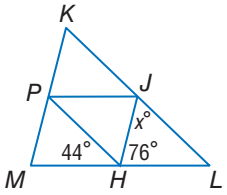

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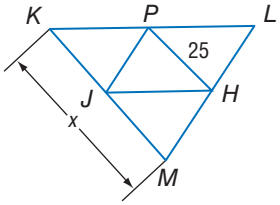

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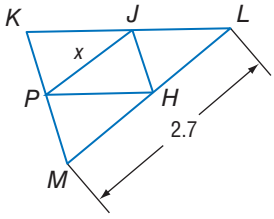
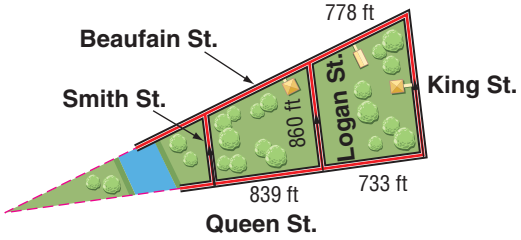
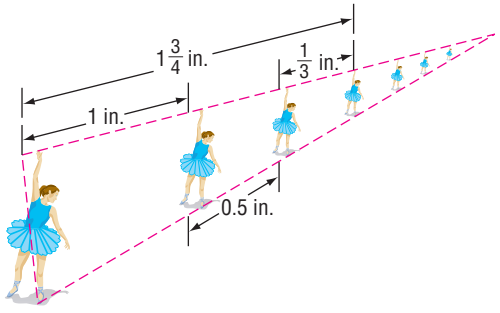


Example 4

22. CCSS **MODELING** In Charleston, South Carolina, Logan Street is parallel to both King Street and Smith Street between Beaufain Street and Queen Street. What is the distance from Smith to Logan along Beaufain? Round to the nearest foot.

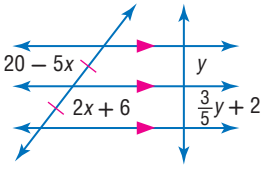


23. **ART** Tonisha drew the line of dancers shown below for her perspective project in art class. Each of the dancers is parallel. Find the lower distance between the first two dancers.

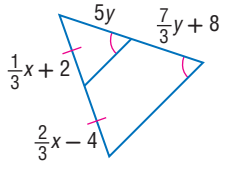


Example 5 **ALGEBRA** **Find x and y.**

24.

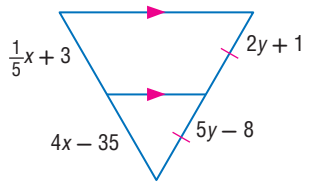


25.

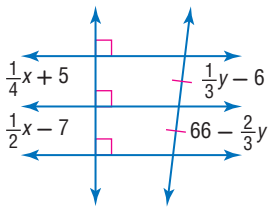


ALGEBRA **Find x and y.**

26.



27.



CCSS **ARGUMENTS** **Write a paragraph proof.**

28. Corollary 7.1

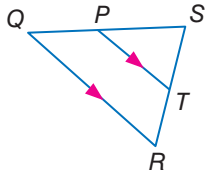
29. Corollary 7.2

30. Theorem 7.5

CCSS **ARGUMENTS** **Write a two-column proof.**

31. Theorem 7.6

32. Theorem 7.7

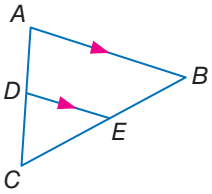


Refer to $\triangle QRS$.

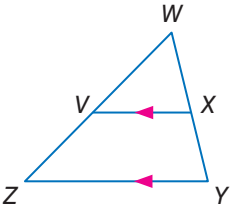
33. If $ST = 8$, $TR = 4$, and $PT = 6$, find QR.

34. If $SP = 4$, $PT = 6$, and $QR = 12$, find SQ.

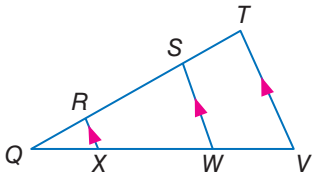
35 If $CE = t - 2$, $EB = t + 1$, $CD = 2$, and $CA = 10$, find t and CE.



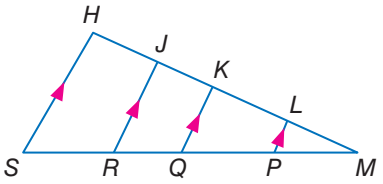
36. If $WX = 7$, $WY = a$, $WV = 6$, and $VZ = a - 9$, find WY.



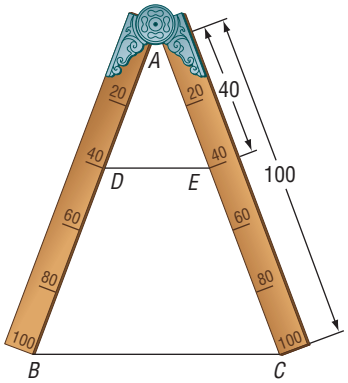
37. If $QR = 2$, $XW = 12$, $QW = 15$, and $ST = 5$, find RS and WV.



38. If $LK = 4$, $MP = 3$, $PQ = 6$, $KJ = 2$, $RS = 6$, and $LP = 2$, find ML, QR, QK, and JH.



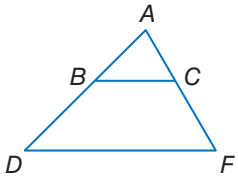
39. **MATH HISTORY** The sector compass was a tool perfected by Galileo in the sixteenth century for measurement. To draw a segment two-fifths the length of a given segment, align the ends of the arms with the given segment. Then draw a segment at the 40 mark. Write a justification that explains why the sector compass works for proportional measurement.



Determine the value of x so that $\overline{BC} \parallel \overline{DF}$.

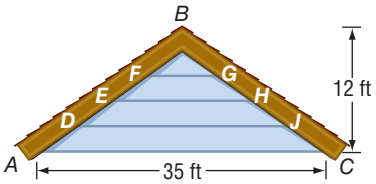
40. $AB = x + 5$, $BD = 12$, $AC = 3x + 1$, and $CF = 15$

41. $AC = 15$, $BD = 3x - 2$, $CF = 3x + 2$, and $AB = 12$



42. COORDINATE GEOMETRY $\triangle ABC$ has vertices $A(-8, 7)$, $B(0, 1)$, and $C(7, 5)$. Draw $\triangle ABC$. Determine the coordinates of the midsegment of $\triangle ABC$ that is parallel to $\overline{BC}$. Justify your answer.

43. HOUSES Refer to the diagram of the gable at the right. Each piece of siding is a uniform width. Find the lengths of $\overline{FG}$, $\overline{EH}$, and $\overline{DJ}$.



CONSTRUCTIONS **Construct each segment as directed.**

44. a segment separated into five congruent segments

45. a segment separated into two segments in which their lengths have a ratio of 1 to 3

46. a segment 3 inches long, separated into four congruent segments

47. MULTIPLE REPRESENTATIONS In this problem, you will explore angle bisectors and proportions.

a. Geometric Draw three triangles, one acute, one right, and one obtuse. Label one triangle ABC and draw angle bisector $\overrightarrow{BD}$. Label the second MNP with angle bisector $\overrightarrow{NQ}$ and the third WXY with angle bisector $\overrightarrow{XZ}$.

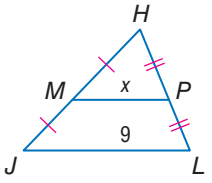
b. Tabular Copy and complete the table at the right with the appropriate values.

c. Verbal Make a conjecture about the segments of a triangle created by an angle bisector.

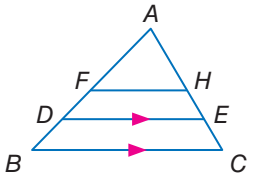
Triangle	Length		Ratio	
ABC	AD		$\frac{AD}{CD}$	
	CD			
	AB		$\frac{AB}{CB}$	
	CB			
MNP	MQ		$\frac{MQ}{PQ}$	
	PQ			
	MN		$\frac{MN}{PN}$	
	PN			
WXY	WZ		$\frac{WZ}{YZ}$	
	YZ			
	WX		$\frac{WX}{YX}$	
	YX			

H.O.T. Problems Use Higher-Order Thinking Skills

48. CCSS CRITIQUE Jacob and Sebastian are finding the value of x in $\triangle JHL$. Jacob says that MP is one half of JL, so x is 4.5. Sebastian says that JL is one half of MP, so x is 18. Is either of them correct? Explain.



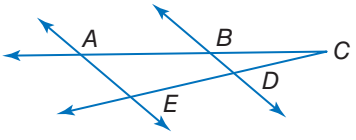
49. REASONING In $\triangle ABC$, $AF = FB$ and $AH = HC$. If D is $\frac{3}{4}$ of the way from A to B and E is $\frac{3}{4}$ of the way from A to C, is DE *always, sometimes,* or *never* $\frac{3}{4}$ of BC? Explain.



50. CHALLENGE Write a two-column proof.

Given: $AB = 4$, $BC = 4$, and $CD = DE$

Prove: $\overline{BD} \parallel \overline{AE}$

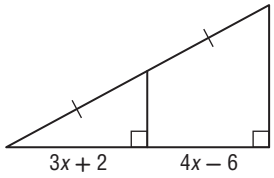


51. OPEN ENDED Draw three segments, a, b, and c, of all different lengths. Draw a fourth segment, d, such that $\frac{a}{b} = \frac{c}{d}$.

52. WRITING IN MATH Compare the Triangle Proportionality Theorem and the Triangle Midsegment Theorem.

Standardized Test Practice

53. SHORT RESPONSE What is the value of x?



54. If the vertices of triangle JKL are (0, 0), (0, 10) and (10, 10), then the area of triangle JKL is

A 20 units^2 **C** 40 units^2
B 30 units^2 **D** 50 units^2

55. ALGEBRA A breakfast cereal contains wheat, rice, and oats in the ratio 2:4:1. If the manufacturer makes a mixture using 110 pounds of wheat, how many pounds of rice will be used?

F 120 lb **H** 240 lb
G 220 lb **J** 440 lb

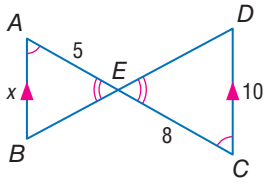
56. SAT/ACT If the area of a circle is 16 square meters, what is its radius in meters?

A $\frac{4\sqrt{\pi}}{\pi}$ **D** 12π
B $\frac{8}{\pi}$ **E** 16π
C $\frac{16}{\pi}$

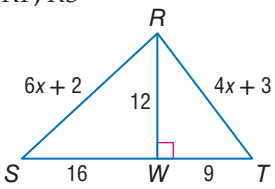
Spiral Review

ALGEBRA Identify the similar triangles. Then find the measure(s) of the indicated segment(s). (Lesson 7-3)

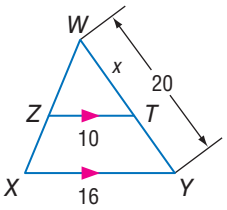
57. $\overline{AB}$



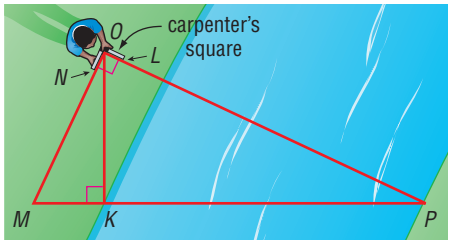
58. $\overline{RT}$, $\overline{RS}$



59. $\overline{TY}$



60. SURVEYING Mr. Turner uses a carpenter's square to find the distance across a stream. The carpenter's square models right angle NOL. He puts the square on top of a pole that is high enough to sight along $\overline{OL}$ to point P across the river. Then he sights along $\overline{ON}$ to point M. If MK is 1.5 feet and OK is 4.5 feet, find the distance KP across the stream. (Lesson 7-2)



COORDINATE GEOMETRY For each quadrilateral with the given vertices, verify that the quadrilateral is a trapezoid and determine whether the figure is an isosceles trapezoid. (Lesson 6-6)

61. $Q(-12, 1)$, $R(-9, 4)$, $S(-4, 3)$, $T(-11, -4)$

62. $A(-3, 3)$, $B(-4, -1)$, $C(5, -1)$, $D(2, 3)$

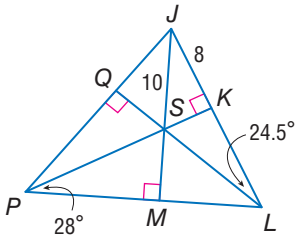
Point S is the incenter of $\triangle JPL$. Find each measure. (Lesson 5-1)

63. SQ

64. QJ

65. $m\angle MPQ$

66. $m\angle SJP$



Skills Review

Solve each proportion.

67. $\frac{1}{3} = \frac{x}{2}$ **68.** $\frac{3}{4} = \frac{5}{x}$ **69.** $\frac{2.3}{4} = \frac{x}{3.7}$ **70.** $\frac{x-2}{2} = \frac{4}{5}$ **71.** $\frac{x}{12-x} = \frac{8}{3}$