LESSON 7-5 Parts of Similar Triangles

:: Then

- You learned that corresponding sides of similar polygons are proportional.

:: Now

1. Recognize and use proportional relationships of corresponding angle bisectors, altitudes, and medians of similar triangles.
2. Use the Triangle Bisector Theorem.

:: Why?

- The "Rule of Thumb" uses the average ratio of a person's arm length to the distance between his or her eyes and the altitudes of similar triangles to estimate the distance between a person and an object of approximately known width.

CCSS Common Core State Standards

Content Standards

G.SRT.4 Prove theorems about triangles.

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

3 Construct viable arguments and critique the reasoning of others.

1 Special Segments of Similar Triangles

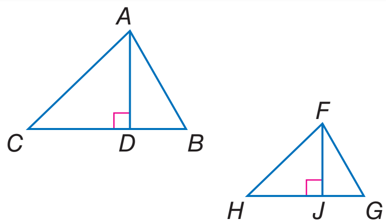
You learned in Lesson 7-2 that the corresponding side lengths of similar polygons, such as triangles, are proportional. This concept can be extended to other segments in triangles.

Theorems — Special Segments of Similar Triangles

7.8 If two triangles are similar, the lengths of corresponding altitudes are proportional to the lengths of corresponding sides.

Abbreviation ~△s *have corr. altitudes proportional to corr. sides.*

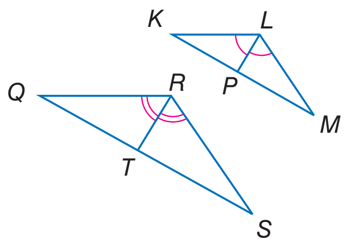
Example If $\triangle ABC \sim \triangle FGH$, then $\frac{AD}{FJ} = \frac{AB}{FG}$.



7.9 If two triangles are similar, the lengths of corresponding angle bisectors are proportional to the lengths of corresponding sides.

Abbreviation ~△s *have corr. ∠ bisectors proportional to corr. sides.*

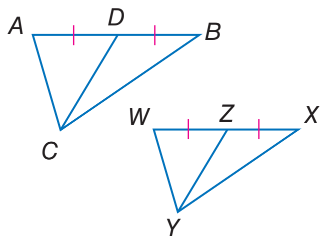
Example If $\triangle KLM \sim \triangle QRS$, then $\frac{LP}{RT} = \frac{LM}{RS}$.



7.10 If two triangles are similar, the lengths of corresponding medians are proportional to the lengths of corresponding sides.

Abbreviation ~△s *have corr. medians proportional to corr. sides.*

Example If $\triangle ABC \sim \triangle WXY$, then $\frac{CD}{YZ} = \frac{AB}{WX}$.



You will prove Theorems 7.9 and 7.10 in Exercises 18 and 19, respectively.

Real-World Career

Athletic Trainer Athletic trainers help prevent and treat sports injuries. They ensure that protective equipment is used properly and that people understand safe practices that prevent injury. An athletic trainer must have a bachelor's degree to be certified. Most also have master's degrees. Refer to Exercise 29.

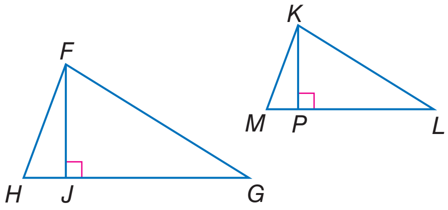
Study Tip

Use Scale Factor Example 1 could also have been solved by first finding the scale factor between $\triangle ABC$ and $\triangle FDG$. The ratio of the angle bisector in $\triangle ABC$ to the angle bisector in $\triangle FDG$ would then be equal to this scale factor.

Proof Theorem 7.8

Given: $\triangle FGH \sim \triangle KLM$
$\overline{FJ}$ and $\overline{KP}$ are altitudes.

Prove: $\frac{FJ}{KP} = \frac{HF}{MK}$



Paragraph Proof:
Since $\triangle FGH \sim \triangle KLM$, $\angle H \cong \angle M$. $\angle FJH \cong \angle KPM$ because they are both right angles created by the altitudes drawn to the opposite side and all right angles are congruent.

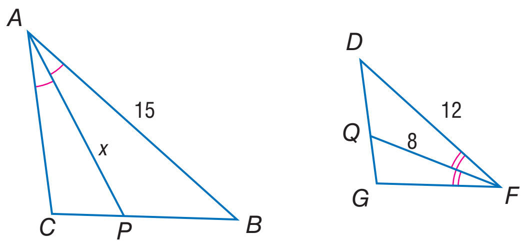
Thus $\triangle HFJ \sim \triangle MKP$ by AA Similarity. So $\frac{FJ}{KP} = \frac{HF}{MK}$ by the definition of similar polygons.

Since the corresponding altitudes are chosen at random, we need not prove Theorem 7.8 for every pair of altitudes.

You can use special segments in similar triangles to find missing measures.

Example 1 Use Special Segments in Similar Triangles

In the figure, $\triangle ABC \sim \triangle FDG$. Find the value of x.



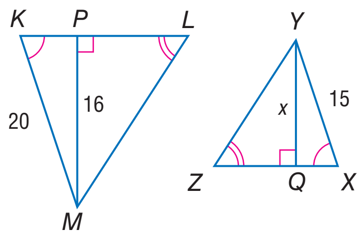
$\overline{AP}$ and $\overline{FQ}$ are corresponding angle bisectors and $\overline{AB}$ and $\overline{FD}$ are corresponding sides of similar triangles ABC and FDG.

$\frac{AP}{FQ} = \frac{AB}{FD}$	$\sim\triangle$s have corr. $\angle$ bisectors proportional to the corr. sides.
$\frac{x}{8} = \frac{15}{12}$	Substitution
$8 \cdot 15 = x \cdot 12$	Cross Products Property
$120 = 12x$	Simplify.
$10 = x$	Divide each side by 12.

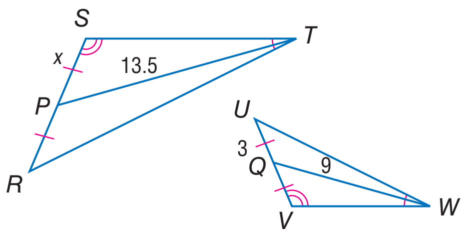
Guided Practice

Find the value of x.

1A.



1B.



You can use special segments in similar triangles to solve real-world problems.

Real-WorldLink

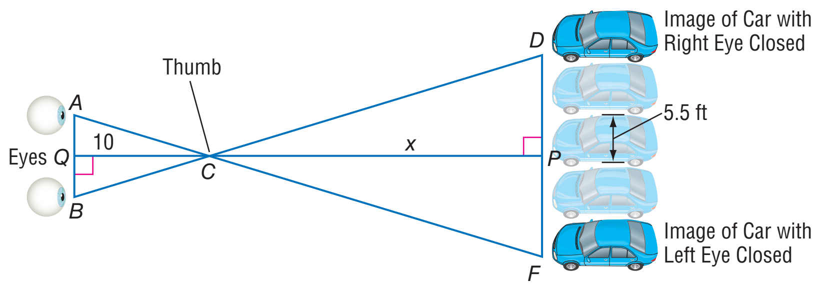
Hold your outstretched hand horizontal at arm's length with your palm facing you; for each hand width the sun is above the horizon, there is one remaining hour of sunlight.

Source: Sail Island Channels

Real-World Example 2 Use Similar Triangles to Solve Problems

ESTIMATING DISTANCES Liliana holds her arm straight out in front of her with her elbow straight and her thumb pointing up. Closing one eye, she aligns one edge of her thumb with a car she is sighting. Next she switches eyes without moving her head or her arm. The car appears to jump 4 car widths. If Liliana's arm is about 10 times longer than the distance between her eyes, and the car is about 5.5 feet wide, estimate the distance from Liliana's thumb to the car.

Understand Make a diagram of the situation labeling the given distances and the distance you need to find as x. Also, label the vertices of the triangles formed.



Note: Not drawn to scale.

We assume that if Liliana's thumb is straight out in front of her, then $\overline{PC}$ is an altitude of $\triangle ABC$. Likewise, $\overline{QC}$ is the corresponding altitude. We assume that $\overline{AB} \parallel \overline{DF}$.

Plan Since $\overline{AB} \parallel \overline{DF}$, $\angle BAC \cong \angle DFC$ and $\angle CBA \cong \angle CDF$ by the Alternate Interior Angles Theorem. Therefore $\triangle ABC \sim \triangle FDC$ by AA Similarity. Write a proportion and solve for x.

Solve

$\frac{PC}{QC} = \frac{AB}{DF}$ Theorem 7.8

$\frac{10}{x} = \frac{1}{5.5 \cdot 4}$ Substitution

$\frac{10}{x} = \frac{1}{22}$ Simplify.

$10 \cdot 22 = x \cdot 1$ Cross Products Property

$220 = x$ Simplify.

So the estimated distance to the car is 220 feet.

Check The ratio of Liliana's arm length to the width between her eyes is 10 to 1. The ratio of the distance to the car to the distance the image of the car jumped is 22 to 220 or 10 to 1. ✓

GuidedPractice

2. Suppose Liliana stands at the back of her classroom and sights a clock on the wall at the front of the room. If the clock is 30 centimeters wide and appears to move 3 clock widths when she switches eyes, estimate the distance from Liliana's thumb to the clock.

2 Triangle Angle Bisector Theorem

Triangle Angle Bisector Theorem An angle bisector of a triangle also divides the side opposite the angle proportionally.

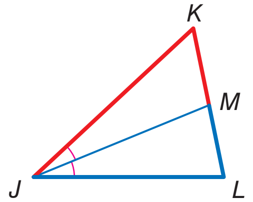
> **StudyTip**
>
> **Proportions** Another proportion that could be written using the Triangle Angle Bisector Theorem is $\frac{KM}{KJ} = \frac{LM}{LJ}$.

Theorem 7.11 Triangle Angle Bisector

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

Example If $\overline{JM}$ is an angle bisector of $\triangle JKL$,

then $\frac{KM}{LM} = \frac{KJ}{LJ}$. ← segments with vertex K ← segments with vertex L

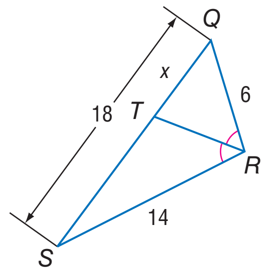


You will prove Theorem 7.11 in Exercise 25.

Example 3 Use the Triangle Angle Bisector Theorem

Find x.

Since $\overline{RT}$ is an angle bisector of $\triangle QRS$, you can use the Triangle Angle Bisector Theorem to write a proportion.

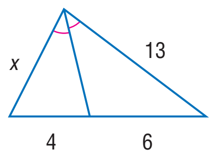


$\frac{QT}{ST} = \frac{QR}{SR}$	Triangle Angle Bisector Theorem
$\frac{x}{18 - x} = \frac{6}{14}$	Substitution
$(18 - x)(6) = x \cdot 14$	Cross Products Property
$108 - 6x = 14x$	Simplify.
$108 = 20x$	Add $6x$ to each side.
$5.4 = x$	Divide each side by 20.

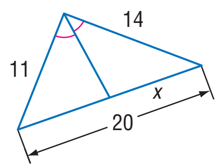
GuidedPractice

Find the value of x.

3A.



3B.

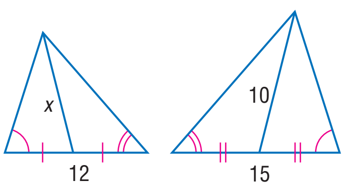


Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 **Find x.**

1



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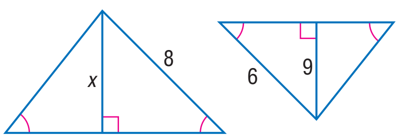
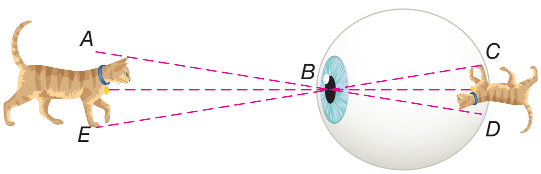
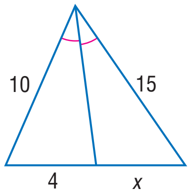


Example 2 **3. VISION** A cat that is 10 inches tall forms a retinal image that is 7 millimeters tall. If $\triangle ABE \sim \triangle DBC$ and the distance from the pupil to the retina is 25 millimeters, how far away from your pupil is the cat?

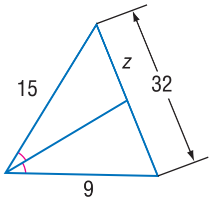


Example 3 Find the value of each variable.

4.



5.

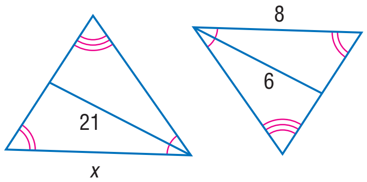


Practice and Problem Solving

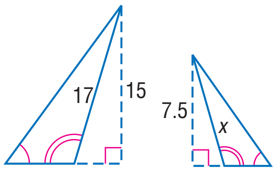
Extra Practice is on page R7.

Example 1 Find x.

6.



7.



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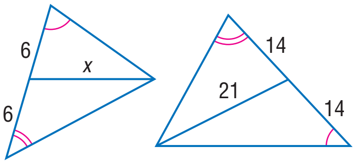

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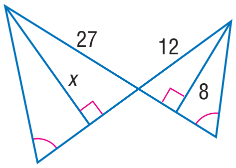
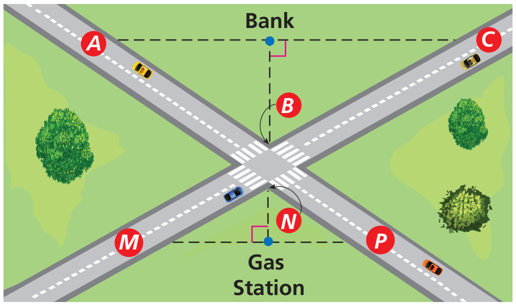
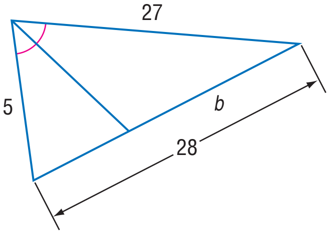


Example 2 **10.** **ROADWAYS** The intersection of the two roads shown forms two similar triangles. If AC is 382 feet, MP is 248 feet, and the gas station is 50 feet from the intersection, how far from the intersection is the bank?



Example 3 CCSS **SENSE-MAKING** Find the value of each variable.

11.



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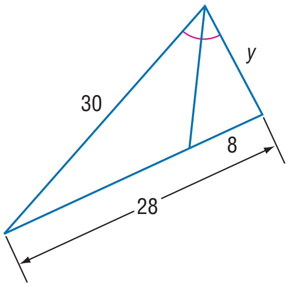

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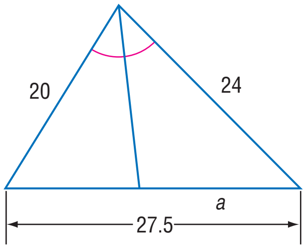
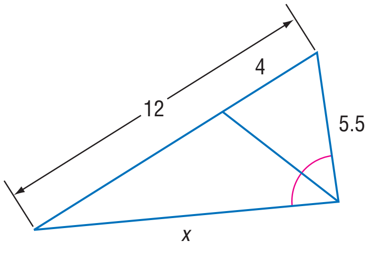
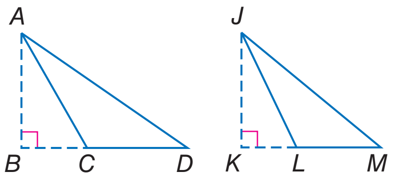


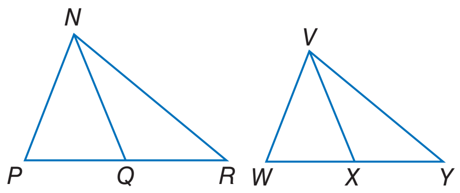
14.



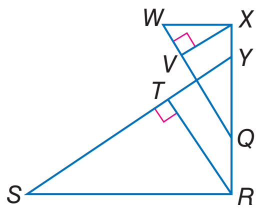
15. **ALGEBRA** If $\overline{AB}$ and $\overline{JK}$ are altitudes, $\triangle DAC \sim \triangle MJL$, $AB = 9$, $AD = 4x - 8$, $JK = 21$, and $JM = 5x + 3$, find x.



16. **ALGEBRA** If $\overline{NQ}$ and $\overline{VX}$ are medians, $\triangle PNR \sim \triangle WVY$, $NQ = 8$, $PR = 12$, $WY = 7x - 1$, and $VX = 4x + 2$, find x.



17. If $\triangle SRY \sim \triangle WXQ$, $\overline{RT}$ is an altitude of $\triangle SRY$, $\overline{XV}$ is an altitude of $\triangle WXQ$, $RT = 5$, $RQ = 4$, $QY = 6$, and $YX = 2$, find XV.

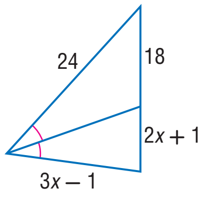


18. **PROOF** Write a paragraph proof of Theorem 7.9.

19. **PROOF** Write a two-column proof of Theorem 7.10.

ALGEBRA **Find x.**

20.



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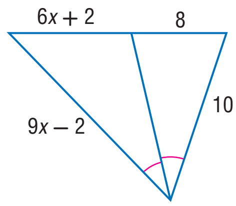

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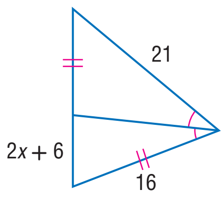

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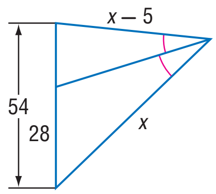
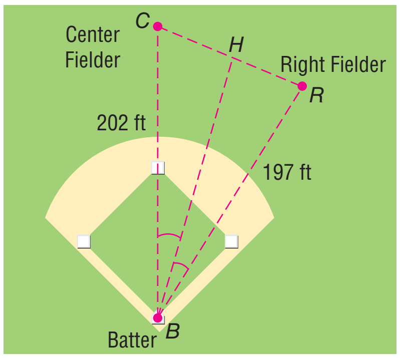


24. **SPORTS** Consider the triangle formed by the path between a batter, center fielder, and right fielder as shown. If the batter gets a hit that bisects the triangle at $\angle B$, is the center fielder or the right fielder closer to the ball? Explain your reasoning.

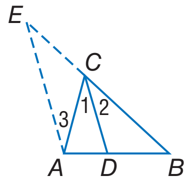


CCSS ARGUMENTS **Write a two-column proof.**

25. Theorem 7.11

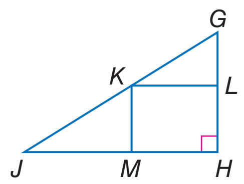
Given: $\overline{CD}$ bisects $\angle ACB$.
By construction, $\overline{AE} \parallel \overline{CD}$.

Prove: $\frac{AD}{DB} = \frac{AC}{BC}$



26. **Given:** $\angle H$ is a right angle.
L, K, and M are midpoints.

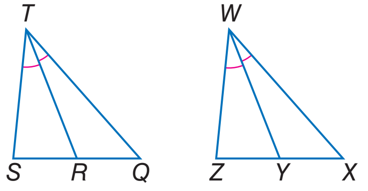
Prove: $\angle LKM$ is a right angle.



PROOF **Write a two-column proof.**

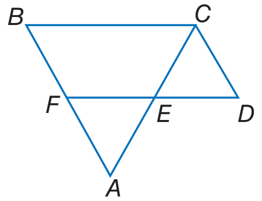
27. **Given:** $\triangle QTS \sim \triangle XWZ$, $\overline{TR}$ and $\overline{WY}$ are angle bisectors.

Prove: $\frac{TR}{WY} = \frac{QT}{XW}$

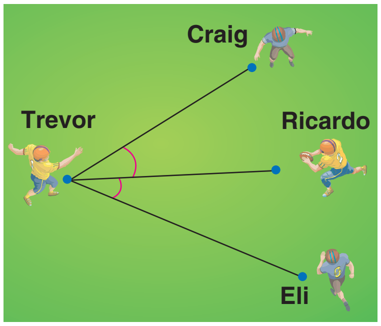


28. **Given:** $\overline{FD} \parallel \overline{BC}$, $\overline{BF} \parallel \overline{CD}$, $\overline{AC}$ bisects $\angle C$.

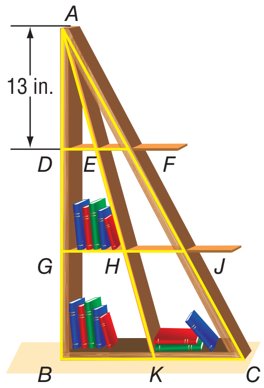
Prove: $\frac{DE}{EC} = \frac{BA}{AC}$



29. **SPORTS** During football practice, Trevor threw a pass to Ricardo as shown below. If Eli is farther from Trevor when he completes the pass to Ricardo and Craig and Eli move at the same speed, who will reach Ricardo to tackle him first?

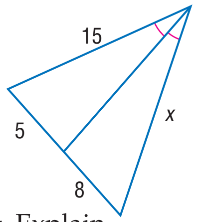


30. **SHELVING** In the bookshelf shown, the distance between each shelf is 13 inches and $\overline{AK}$ is a median of $\triangle ABC$. If EF is $3\frac{1}{3}$ inches, what is BK?



H.O.T. Problems Use Higher-Order Thinking Skills

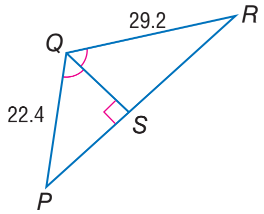
31. **ERROR ANALYSIS** Chun and Traci are determining the value of x in the figure. Chun says to find x, solve the proportion $\frac{5}{8} = \frac{15}{x}$, but Traci says to find x, the proportion $\frac{5}{x} = \frac{8}{15}$ should be solved. Is either of them correct? Explain.



32. **CCSS ARGUMENTS** Find a counterexample to the following statement. Explain.

If the measure of an altitude and side of a triangle are proportional to the corresponding altitude and corresponding side of another triangle, then the triangles are similar.

33. **CHALLENGE** The perimeter of $\triangle PQR$ is 94 units. $\overline{QS}$ bisects $\angle PQR$. Find PS and RS.



34. **OPEN ENDED** Draw two triangles so that the measures of corresponding medians and a corresponding side are proportional, but the triangles are not similar.

35. **WRITING IN MATH** Compare and contrast Theorem 7.9 and the Triangle Angle Bisector Theorem.

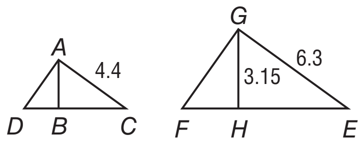
Standardized Test Practice

36. ALGEBRA Which shows 0.00234 written in scientific notation?

A 2.34×10^{5} **C** 2.34×10^{-2}

B 2.34×10^{3} **D** 2.34×10^{-3}

37. SHORT RESPONSE In the figures below, $\overline{AB} \perp \overline{DC}$ and $\overline{GH} \perp \overline{FE}$.



If $\triangle ACD \sim \triangle GEF$, find AB.

38. Quadrilateral *HJKL* is a parallelogram. If the diagonals are perpendicular, which statement must be true?

F Quadrilateral *HJKL* is a square.

G Quadrilateral *HJKL* is a rectangle.

H Quadrilateral *HJKL* is a rhombus.

J Quadrilateral *HJKL* is an isosceles trapezoid.

39. SAT/ACT The sum of three numbers is 180. Two of the numbers are the same, and each of them is one third of the greatest number. What is the least number?

A 15 **D** 45

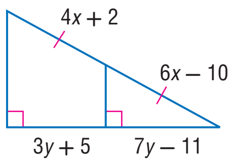
B 30 **E** 60

C 36

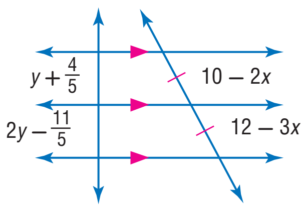
Spiral Review

ALGEBRA **Find x and y.** (Lesson 7-4)

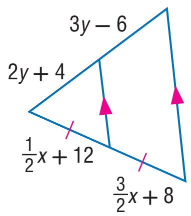
40.



41.

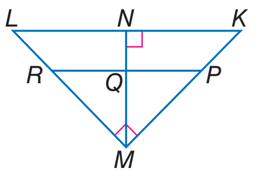


42.

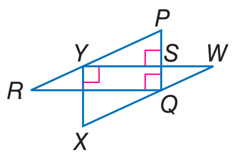


Find the indicated measure(s). (Lesson 7-3)

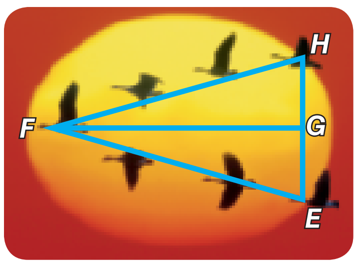
43. If $\overline{PR} \parallel \overline{KL}$, $KN = 9$, $LN = 16$, and $PM = 2(KP)$, find KP, KM, MR, ML, MN, and PR.



44. If $\overline{PR} \parallel \overline{WX}$, $WX = 10$, $XY = 6$, $WY = 8$, $RY = 5$, and $PS = 3$, find PY, SY, and PQ.



45. GEESE A flock of geese flies in formation. Prove that $\triangle EFG \cong \triangle HFG$ if $\overline{EF} \cong \overline{HF}$ and that G is the midpoint of $\overline{EH}$. (Lesson 4-4)



Skills Review

Find the distance between each pair of points.

46. $E(-3, -2)$, $F(5, 8)$

47. $A(2, 3)$, $B(5, 7)$

48. $C(-2, 0)$, $D(6, 4)$

49. $W(7, 3)$, $Z(-4, -1)$

50. $J(-4, -5)$, $K(2, 9)$

51. $R(-6, 10)$, $S(8, -2)$