LESSON 7-6

Similarity Transformations

Then

- You identified congruence transformations.

Now

1. Identify similarity transformations.
2. Verify similarity after a similarity transformation.

Why?

- Adriana uses a copier to enlarge a movie ticket to use as the background for a page in her movie ticket scrapbook. She places the ticket on the glass of the copier. Then she must decide what percentage to input in order to create an image that is three times as big as her original ticket.

NewVocabulary

dilation
similarity transformation
center of dilation
scale factor of a dilation
enlargement
reduction

Common Core State Standards

Content Standards

G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Mathematical Practices

6 Attend to precision.

4 Model with mathematics.

1 Identify Similarity Transformations

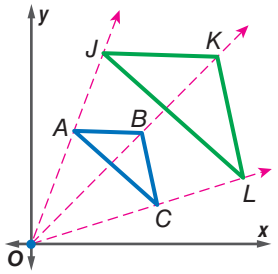
Recall from Lesson 4-7 that a *transformation* is an operation that maps an original figure, the *preimage*, onto a new figure called the *image*.

A **dilation** is a transformation that enlarges or reduces the original figure proportionally. Since a dilation produces a similar figure, a dilation is a type of **similarity transformation**.

Dilations are performed with respect to a fixed point called the **center of dilation**.

The **scale factor of a dilation** describes the extent of the dilation. The scale factor is the ratio of a length on the image to a corresponding length on the preimage.

The letter k usually represents the scale factor of a dilation. The value of k determines whether the dilation is an enlargement or a reduction.



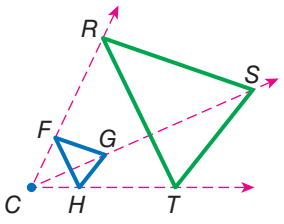
$\triangle JKL$ is a dilation of $\triangle ABC$.
Center of dilation: (0, 0)
Scale factor: $\frac{JK}{AB}$

ConceptSummary Types of Dilations

A dilation with a scale factor greater than 1 produces an **enlargement**, or an image that is larger than the original figure.

Symbols If $k > 1$, the dilation is an enlargement.

Example $\triangle FGH$ is dilated by a scale factor of 3 to produce $\triangle RST$. Since $3 > 1$, $\triangle RST$ is an enlargement of $\triangle FGH$.

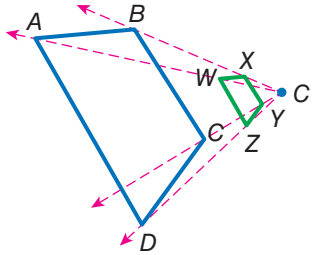


$k < 1$

A dilation with a scale factor between 0 and 1 produces a **reduction**, an image that is smaller than the original figure.

Symbols If $0 < k < 1$, the dilation is a reduction.

Example $ABCD$ is dilated by a scale factor of $\frac{1}{4}$ to produce $WXYZ$. Since $0 < \frac{1}{4} < 1$, $WXYZ$ is a reduction of $ABCD$.



$0 < k < 1$

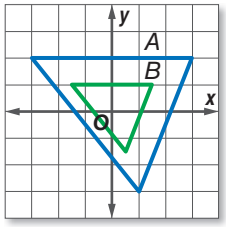
StudyTip

Multiple Representations
The scale factor of a dilation can be represented as a fraction, a decimal, or as a percent. For example, a scale factor of $\frac{2}{5}$ can also be written as 0.4 or as 40%.

Example 1 Identify a Dilation and Find Its Scale Factor

Determine whether the dilation from A to B is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.

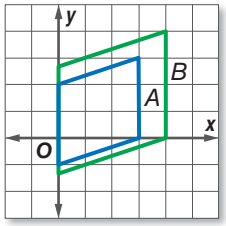
a.



B is smaller than A, so the dilation is a reduction.

The distance between the vertices at (−3, 2) and (3, 2) for A is 6 and from the vertices at (−1.5, 1) and (1.5, 1) for B is 3. So the scale factor is $\frac{3}{6}$ or $\frac{1}{2}$.

b.

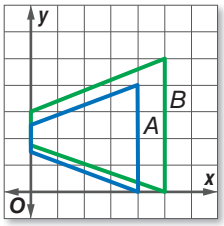


B is larger than A, so the dilation is an enlargement.

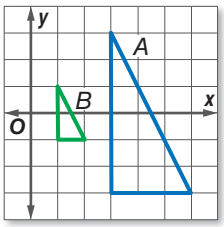
The distance between the vertices at (3, 3) and (3, 0) for A is 3 and between the vertices at (4, 4) and (4, 0) for B is 4. So the scale factor is $\frac{4}{3}$.

GuidedPractice

1A.



1B.



Dilations and their scale factors are used in many real-world situations.

Real-WorldLink

Hew Weng Fatt accepted a contest challenge to collect the most movie stubs from a certain popular fantasy movie. He collected 6561 movie stubs in 38 days!

Source: *Youth2, Star Publications*

Real-World Example 2 Find and Use a Scale Factor

COLLECTING Refer to the beginning of the lesson. By what percent should Adriana enlarge the ticket stub so that the dimensions of its image are 3 times that of her original? What will be the dimensions of the enlarged image?

Adriana wants to create a dilated image of her ticket stub using the copier. The scale factor of her enlargement is 3. Written as a percent, the scale factor is (3 • 100)% or 300%. Now find the dimension of the enlarged image using the scale factor.

width: 5 cm • 300% = 15 cm length: 6.4 cm • 300% = 19.2 cm

The enlarged ticket stub image will be 15 centimeters by 19.2 centimeters.

GuidedPractice

2. If the resulting ticket stub image was 1.5 centimeters wide by about 1.9 centimeters long instead, what percent did Adriana mistakenly use to dilate the original image? Explain your reasoning.

2 Verify Similarity You can verify that a dilation produces a similar figure by comparing corresponding sides and angles. For triangles, you can also use SAS Similarity.

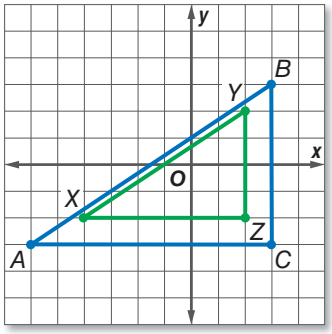
Example 3 Verify Similarity after a Dilation

Graph the original figure and its dilated image. Then verify that the dilation is a similarity transformation.

StudyTip
Center of Dilation
Unless otherwise stated, all dilations on the coordinate plane use the origin as their center of dilation.

a. original: $A(-6, -3)$, $B(3, 3)$, $C(3, -3)$; image: $X(-4, -2)$, $Y(2, 2)$, $Z(2, -2)$

Graph each figure. Since $\angle C$ and $\angle Z$ are both right angles, $\angle C \cong \angle Z$. Show that the lengths of the sides that include $\angle C$ and $\angle Z$ are proportional.



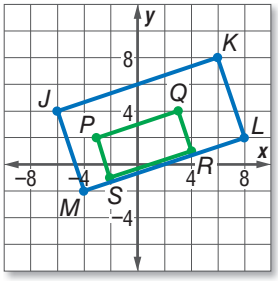
Use the coordinate grid to find the side lengths.

$\frac{XZ}{AC} = \frac{6}{9}$ or $\frac{2}{3}$, and $\frac{YZ}{BC} = \frac{4}{6}$ or $\frac{2}{3}$, so $\frac{XZ}{AC} = \frac{YZ}{BC}$.

Since the lengths of the sides that include $\angle C$ and $\angle Z$ are proportional, $\triangle XYZ \sim \triangle ABC$ by SAS Similarity.

b. original: $J(-6, 4)$, $K(6, 8)$, $L(8, 2)$, $M(-4, -2)$; image: $P(-3, 2)$, $Q(3, 4)$, $R(4, 1)$, $S(-2, -1)$

Use the Distance Formula to find the length of each side.



$JK = \sqrt{[6 - (-6)]^2 + (8 - 4)^2} = \sqrt{160}$ or $4\sqrt{10}$

$PQ = \sqrt{[3 - (-3)]^2 + (4 - 2)^2} = \sqrt{40}$ or $2\sqrt{10}$

$KL = \sqrt{(8 - 6)^2 + (2 - 8)^2} = \sqrt{40}$ or $2\sqrt{10}$

$QR = \sqrt{(4 - 3)^2 + (1 - 4)^2} = \sqrt{10}$

$LM = \sqrt{(-4 - 8)^2 + (-2 - 2)^2} = \sqrt{160}$ or $4\sqrt{10}$

$RS = \sqrt{(-2 - 4)^2 + (-1 - 1)^2} = \sqrt{40}$ or $2\sqrt{10}$

$MJ = \sqrt{[-6 - (-4)]^2 + [4 - (-2)]^2} = \sqrt{40}$ or $2\sqrt{10}$

$SP = \sqrt{[-3 - (-2)]^2 + [2 - (-1)]^2} = \sqrt{10}$

Find and compare the ratios of corresponding sides.

$\frac{PQ}{JK} = \frac{2\sqrt{10}}{4\sqrt{10}}$ or $\frac{1}{2}$ $\quad \frac{QR}{KL} = \frac{\sqrt{10}}{2\sqrt{10}}$ or $\frac{1}{2}$ $\quad \frac{RS}{LM} = \frac{2\sqrt{10}}{4\sqrt{10}}$ or $\frac{1}{2}$ $\quad \frac{SP}{MJ} = \frac{\sqrt{10}}{2\sqrt{10}}$ or $\frac{1}{2}$

$PQRS$ and $JKLM$ are both rectangles. This can be proved by showing that diagonals $\overline{PR} \cong \overline{SQ}$ and $\overline{JL} \cong \overline{KM}$ are congruent using the Distance Formula. Since they are both rectangles, their corresponding angles are congruent.

Since $\frac{PQ}{JK} = \frac{QR}{KL} = \frac{RS}{LM} = \frac{SP}{MJ}$ and corresponding angles are congruent, $PQRS \sim JKLM$.

GuidedPractice

3A. original: $A(2, 3)$, $B(0, 1)$, $C(3, 0)$
image: $D(4, 6)$, $F(0, 2)$, $G(6, 0)$

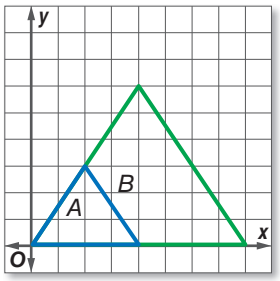
3B. original: $H(0, 0)$, $J(6, 0)$, $K(6, 4)$, $L(0, 4)$
image: $W(0, 0)$, $X(3, 0)$, $Y(3, 2)$, $Z(0, 2)$

Check Your Understanding

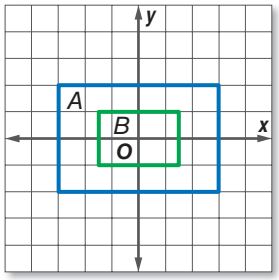
= Step-by-Step Solutions begin on page R14.

Example 1 Determine whether the dilation from A to B is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.

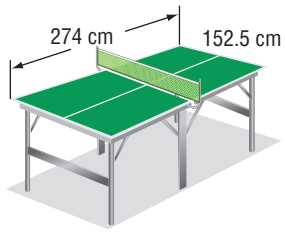
1.



2.

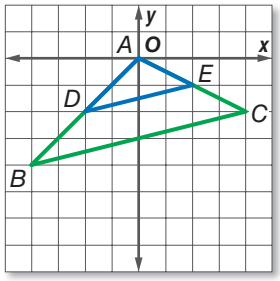


Example 2 3 **GAMES** The dimensions of a regulation tennis court are 27 feet by 78 feet. The dimensions of a table tennis table are 152.5 centimeters by 274 centimeters. Is a table tennis table a dilation of a tennis court? If so, what is the scale factor? Explain.

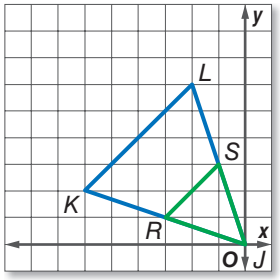


Example 3 CCSS **ARGUMENTS** Verify that the dilation is a similarity transformation.

4.



5.

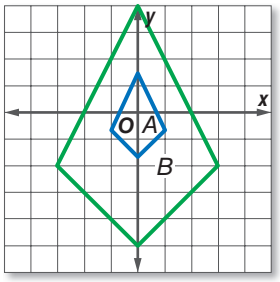


Practice and Problem Solving

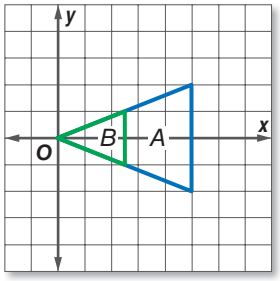
Extra Practice is on page R7.

Example 1 Determine whether the dilation from A to B is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.

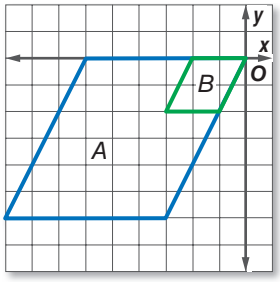
6.



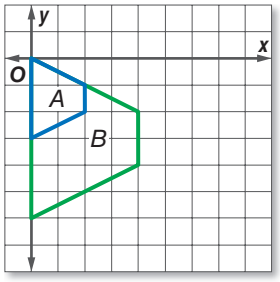
7.



8.

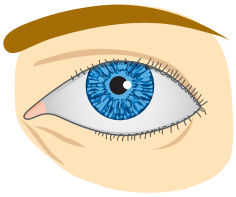


9.

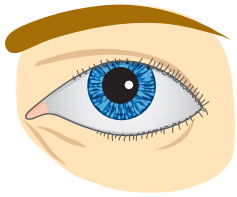


Determine whether each dilation is an *enlargement* or *reduction*.

10. Before



After



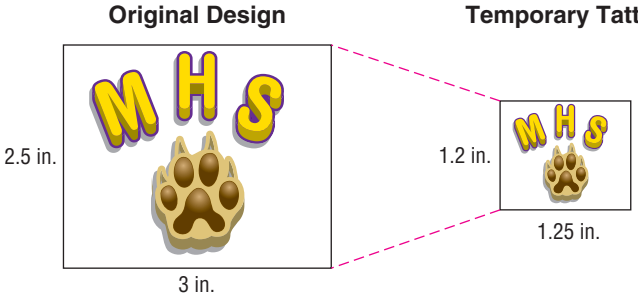
11. Painting

Postcard

Example 2

12. **YEARBOOK** Jordan is putting a photo of the lacrosse team in a full-page layout in the yearbook. The original photo is 4 inches by 6 inches. If the photo in the yearbook is $6\frac{2}{3}$ inches by 10 inches, is the yearbook photo a dilation of the original photo? If so, what is the scale factor? Explain.

13. CCSS **MODELING** Candace created a design to be made into temporary tattoos for a homecoming game as shown. Is the temporary tattoo a dilation of the original design? If so, what is the scale factor? Explain.



Example 3

Graph the original figure and its dilated image. Then verify that the dilation is a similarity transformation.

14. $M(1, 4)$, $P(2, 2)$, $Q(5, 5)$; $S(-3, 6)$, $T(0, 0)$, $U(9, 9)$

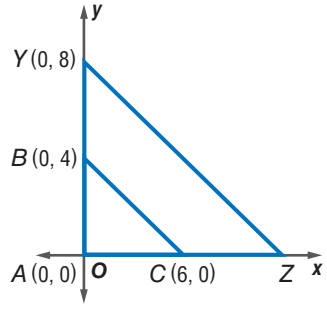
15. $A(1, 3)$, $B(-1, 2)$, $C(1, 1)$; $D(-7, -1)$, $E(1, -5)$

16. $V(-3, 4)$, $W(-5, 0)$, $X(1, 2)$; $Y(-6, -2)$, $Z(3, 1)$

17. $J(-6, 8)$, $K(6, 6)$, $L(-2, 4)$; $D(-12, 16)$, $G(12, 12)$, $H(-4, 8)$

If $\triangle ABC \sim \triangle AYZ$, find the missing coordinate.

18.



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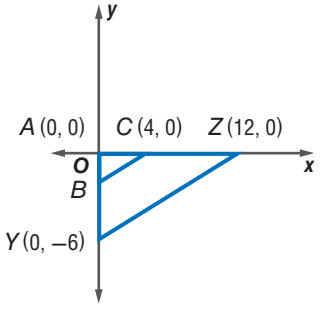
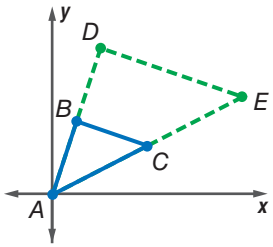


20. GRAPHIC ART Aimee painted the sample sign shown using $\frac{1}{2}$ bottle of glass paint. The actual sign she will paint in a shop window is to be 3 feet by $7\frac{1}{2}$ feet.

a. Explain why the actual sign is a dilation of her sample.

b. How many bottles of paint will Aimee need to complete the actual sign?

21. MULTIPLE REPRESENTATIONS In this problem, you will investigate similarity of triangles on the coordinate plane.



a. Geometric Draw a triangle with vertex A at the origin. Make sure that the two additional vertices B and C have whole-number coordinates. Draw a similar triangle that is twice as large as $\triangle ABC$ with its vertex also located at the origin. Label the triangle ADE.

b. Geometric Repeat the process in part **a** two times. Label the second pair of triangles MNP and MQR and the third pair TWX and TYZ. Use different scale factors than part **a**.

c. Tabular Copy and complete the table below with the appropriate values.

Coordinates												
$\triangle ABC$		$\triangle ADE$		$\triangle MNP$		$\triangle MQR$		$\triangle TWX$		$\triangle TYZ$	
A		A		M		M		T		T	
B		D		N		Q		W		Y	
C		E		P		R		X		Z	

d. Verbal Make a conjecture about how you could predict the coordinates of a dilated triangle with a scale factor of n if the two similar triangles share a corresponding vertex at the origin.

H.O.T. Problems Use Higher-Order Thinking Skills

22. CHALLENGE $MNOP$ is a dilation of $ABCD$. How is the scale factor of the dilation related to the similarity ratio of $ABCD$ to $MNOP$? Explain your reasoning.

23. CCSS REASONING The coordinates of two triangles are provided in the table at the right. Is $\triangle XYZ$ a dilation of $\triangle PQR$? Explain.

$\triangle PQR$		$\triangle XYZ$	
P	(a, b)	X	$(3a, 2b)$
Q	(c, d)	Y	$(3c, 2d)$
R	(e, f)	Z	$(3e, 2f)$

OPEN ENDED **Describe a real-world example of each transformation other than those given in this lesson.**

24. enlargement

25. reduction

26. congruence transformation

27. WRITING IN MATH Explain how you can use scale factor to determine whether a transformation is an enlargement, a reduction, or a congruence transformation.

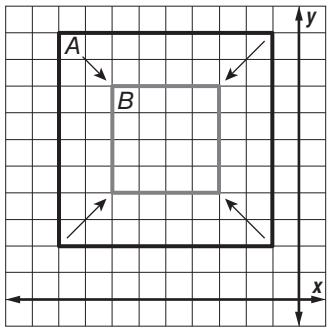
Standardized Test Practice

28. ALGEBRA Which equation describes the line that passes through $(-3, 4)$ and is perpendicular to $3x - y = 6$?

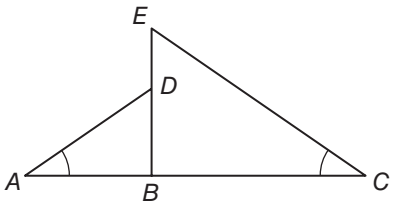
A $y = -\frac{1}{3}x + 4$ **C** $y = 3x + 4$

B $y = -\frac{1}{3}x + 3$ **D** $y = 3x + 3$

29. SHORT RESPONSE What is the scale factor of the dilation shown below?



30. In the figure below, $\angle A \cong \angle C$.



Which additional information would *not* be enough to prove that $\triangle ADB \sim \triangle CEB$?

F $\frac{AB}{DB} = \frac{CB}{EB}$ **H** $\overline{ED} \cong \overline{DB}$

G $\angle ADB \cong \angle CEB$ **J** $\overline{EB} \perp \overline{AC}$

31. SAT/ACT $x = \frac{6}{4p + 3}$ and $xy = \frac{3}{4p + 3}$. $y =$

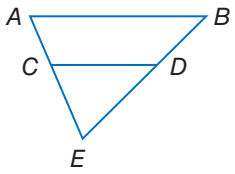
A 4 **C** 1 **E** $\frac{1}{2}$

B 2 **D** $\frac{3}{4}$

Spiral Review

32. LANDSCAPING Shea is designing two gardens shaped like similar triangles. One garden has a perimeter of 53.5 feet, and the longest side is 25 feet. She wants the second garden to have a perimeter of 32.1 feet. Find the length of the longest side of this garden. (Lesson 7-5)

Determine whether $\overline{AB} \parallel \overline{CD}$. Justify your answer. (Lesson 7-4)



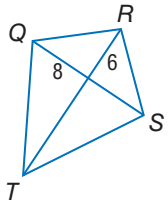
33. $AC = 8.4$, $BD = 6.3$, $DE = 4.5$, and $CE = 6$

34. $AC = 7$, $BD = 10.5$, $BE = 22.5$, and $AE = 15$

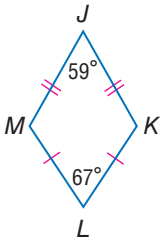
35. $AB = 8$, $AE = 9$, $CD = 4$, and $CE = 4$

If each figure is a kite, find each measure. (Lesson 6-6)

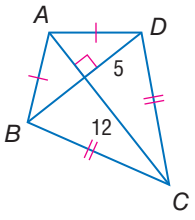
36. QR



37. $m\angle K$



38. BC



39. PROOF Write a coordinate proof for the following statement. (Lesson 4-8)

If a line segment joins the midpoints of two sides of a triangle, then it is parallel to the third side.

Skills Review

Solve each equation.

40. $145 = 29 \cdot t$

41. $216 = d \cdot 27$

42. $2r = 67 \cdot 5$

43. $100t = \frac{70}{240}$

44. $\frac{80}{4} = 14d$

45. $\frac{2t + 15}{t} = 92$