LESSON 8-3

Special Right Triangles

Then

- You used properties of isosceles and equilateral triangles.

Now

1. Use the properties of 45°-45°-90° triangles.
2. Use the properties of 30°-60°-90° triangles.

Why?

- As part of a packet for students attending a regional student council meeting, Lyndsay orders triangular highlighters. She wants to buy rectangular boxes for the highlighters and other items, but she is concerned that the highlighters will not fit in the box she has chosen. If she knows the length of a side of the highlighter, Lyndsay can use the properties of special right triangles to determine if it will fit in the box.

Common Core State Standards

Content Standards

G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

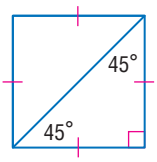
Mathematical Practices

1 Make sense of problems and persevere in solving them.

7 Look for and make use of structure.

1 Properties of 45°-45°-90° Triangles

The diagonal of a square forms two congruent isosceles right triangles. Since the base angles of an isosceles triangle are congruent, the measure of each acute angle is 90 ÷ 2 or 45. Such a triangle is also known as a 45°-45°-90° triangle.



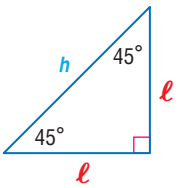
You can use the Pythagorean Theorem to find a relationship among the side lengths of a 45°-45°-90° right triangle.

$\ell^2 + \ell^2 = h^2$ Pythagorean Theorem

$2\ell^2 = h^2$ Simplify.

$\sqrt{2\ell^2} = \sqrt{h^2}$ Take the positive square root of each side.

$\ell\sqrt{2} = h$ Simplify.

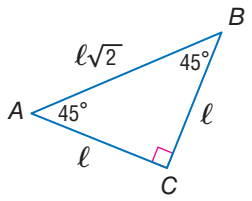


This algebraic proof verifies the following theorem.

Theorem 8.8 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the legs ℓ are congruent and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg.

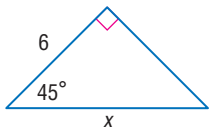
Symbols In a 45°-45°-90° triangle, $\ell = \ell$ and $h = \ell\sqrt{2}$.



Example 1 Find the Hypotenuse Length in a 45°-45°-90° Triangle

Find x.

a.

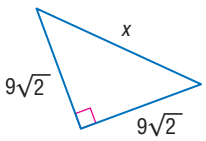


The acute angles of a right triangle are complementary, so the measure of the third angle is 90 − 45 or 45. Since this is a 45°-45°-90° triangle, use Theorem 8.8.

$h = \ell\sqrt{2}$ Theorem 8.8

$x = 6\sqrt{2}$ Substitution

b.



The legs of this right triangle have the same measure, so it is isosceles. Since this is a 45°-45°-90° triangle, use Theorem 8.8.

$h = \ell\sqrt{2}$ Theorem 8.8

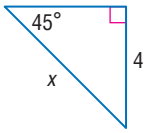
$x = 9\sqrt{2} \cdot \sqrt{2}$ Substitution

$x = 9 \cdot 2$ or 18 $\sqrt{2} \cdot \sqrt{2} = 2$

GuidedPractice

Find x.

1A.



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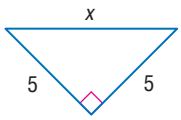

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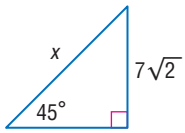
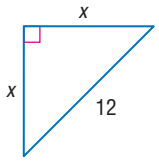


You can also work backward using Theorem 8.8 to find the lengths of the legs of a 45°-45°-90° triangle given the length of its hypotenuse.

Example 2 Find the Leg Lengths in a 45°-45°-90° Triangle

Find x.

The legs of this right triangle have the same measure, x, so it is a 45°-45°-90° triangle. Use Theorem 8.8 to find x.



$h = \ell\sqrt{2}$ — 45°-45°-90° Triangle Theorem

$12 = x\sqrt{2}$ — Substitution

$\frac{12}{\sqrt{2}} = x$ — Divide each side by $\sqrt{2}$.

$\frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = x$ — Rationalize the denominator.

$\frac{12\sqrt{2}}{2} = x$ — Multiply.

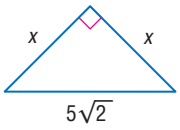
$6\sqrt{2} = x$ — Simplify.

ReviewVocabulary

rationalizing the denominator a method used to eliminate radicals from the denominator of a fraction

GuidedPractice

2A.



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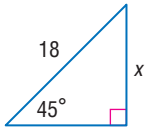


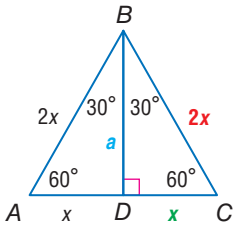
2 Properties of 30°-60°-90° Triangles

A 30°-60°-90° triangle is another *special* right triangle or right triangle with side lengths that share a special relationship. You can use an equilateral triangle to find this relationship.

StudyTip

Altitudes of Isosceles Triangles Notice that an altitude of an isosceles triangle is also a median of the triangle. In the figure at the right, $\overline{BD}$ bisects $\overline{AC}$.

When an altitude is drawn from any vertex of an equilateral triangle, two congruent 30°-60°-90° triangles are formed. In the figure shown, $\triangle ABD \cong \triangle CBD$, so $\overline{AD} \cong \overline{CD}$. If $AD = x$, then $CD = x$ and $AC = 2x$. Since $\triangle ABC$ is equilateral, $AB = 2x$ and $BC = 2x$.



Use the Pythagorean Theorem to find a, the length of the altitude $\overline{BD}$, which is also the longer leg of $\triangle BDC$.

$a^2 + x^2 = (2x)^2$ — Pythagorean Theorem

$a^2 + x^2 = 4x^2$ — Simplify.

$a^2 = 3x^2$ — Subtract x^2 from each side.

$a = \sqrt{3x^2}$ — Take the positive square root of each side.

$a = x\sqrt{3}$ — Simplify.

StudyTip

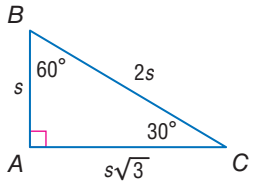
Use Ratios The lengths of the sides of a 30°-60°-90° triangle are in a ratio of 1 to $\sqrt{3}$ to 2 or $1 : \sqrt{3} : 2$.

This algebraic proof verifies the following theorem.

Theorem 8.9 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the length of the hypotenuse h is 2 times the length of the shorter leg s, and the length of the longer leg ℓ is $\sqrt{3}$ times the length of the shorter leg.

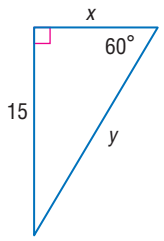
Symbols In a 30°-60°-90° triangle, $h = 2s$ and $\ell = s\sqrt{3}$.



Remember, the shortest side of a triangle is opposite the smallest angle. So the shorter leg in a 30°-60°-90° triangle is opposite the 30° angle, and the longer leg is opposite the 60° angle.

Example 3 Find Lengths in a 30°-60°-90° Triangle

Find x and y.



The acute angles of a right triangle are complementary, so the measure of the third angle in this triangle is $90 - 60$ or 30. This is a 30°-60°-90° triangle.

Use Theorem 8.9 to find x, the length of the shorter side.

$\ell = s\sqrt{3}$ Theorem 8.9

$15 = x\sqrt{3}$ Substitution

$\frac{15}{\sqrt{3}} = x$ Divide each side by $\sqrt{3}$.

$\frac{15}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$ Rationalize the denominator.

$\frac{15\sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = x$ Multiply.

$\frac{15\sqrt{3}}{3} = x$ $\sqrt{3} \cdot \sqrt{3} = 3$

$5\sqrt{3} = x$ Simplify.

Now use Theorem 8.9 to find y, the length of the hypotenuse.

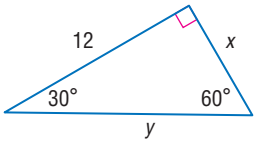
$h = 2s$ Theorem 8.9

$y = 2(5\sqrt{3})$ or $10\sqrt{3}$ Substitution

GuidedPractice

Find x and y.

3A.



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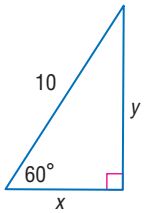

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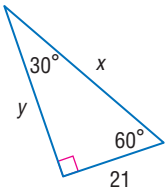
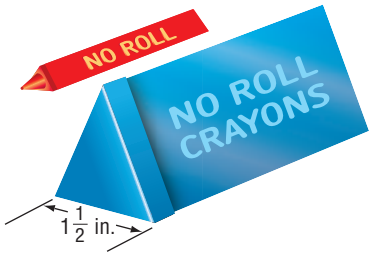


You can use the properties of 30°-60°-90° and 45°-45°-90° triangles to solve real-world problems.

Real-World Example 4 Use Properties of Special Right Triangles

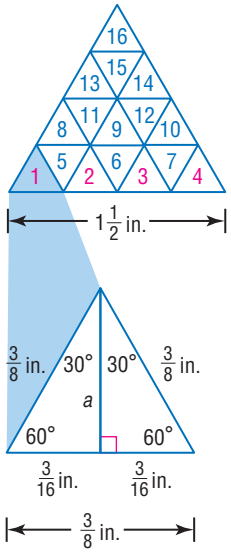
INVENTIONS **A company makes crayons that "do not roll off tables" by shaping them as triangular prisms with equilateral bases. Sixteen of these crayons fit into a box shaped like a triangular prism that is $1\frac{1}{2}$ inches wide. The crayons stand on end in the box and the base of the box is equilateral. What are the dimensions of each crayon?**



Understand You know that 16 crayons with equilateral triangular bases fit into a prism. You need to find the base length and height of each crayon.

Plan Guess and check to determine the arrangement of 16 crayons that would stack to fill the box. Find the width of one crayon and use the 30°-60°-90° Triangle Theorem to find its altitude.

Solve Make a guess that 4 equilateral crayons will fit across the base of the box. A sketch shows that the total number of crayons it takes to fill the box using 4 crayons across the base is 16. ✓



The width of the box is $1\frac{1}{2}$ inches, so the width of one crayon is $1\frac{1}{2} \div 4$ or $\frac{3}{8}$ inch.

Draw an equilateral triangle representing one crayon. Its altitude forms the longer leg of two 30°-60°-90° triangles. Use Theorem 8.9 to find the approximate length of the altitude a.

$$\text{longer leg length} = \text{shorter leg length} \cdot \sqrt{3}$$

$$a = \frac{3}{16} \cdot \sqrt{3} \text{ or about } 0.3$$

Each crayon is $\frac{3}{8}$ or about 0.4 inch by about 0.3 inch.

Check Find the height of the box using the 30°-60°-90° Triangle Theorem. Then divide by four, since the box is four crayons high. The result is a crayon height of about 0.3 inch. ✓

Problem-Solving Tip

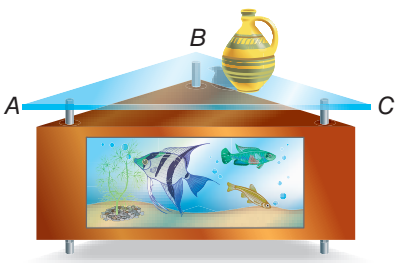
Guess and Check When using the guess and check strategy, it can be helpful to keep a list of those guesses that you have already tried and know do not work.

In Example 4, suppose your first guess had been that the box was 5 crayons wide.

The sketch of this possibility reveals that this leads to a stack of 25, not 16 crayons.

Guided Practice

4. FURNITURE The top of the aquarium coffee table shown is an isosceles right triangle. The table's longest side, $\overline{AC}$, measures 107 centimeters. What is the distance from vertex B to side $\overline{AC}$? What are the lengths of the other two sides?



Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Examples 1-2 Find x.

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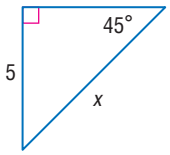

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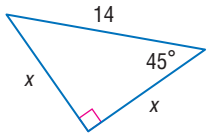

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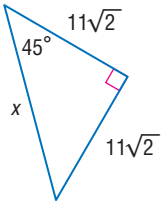
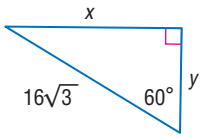


Example 3 Find x and y.

4.



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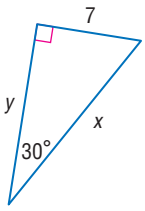

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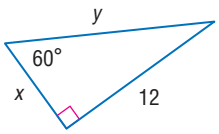


Example 4

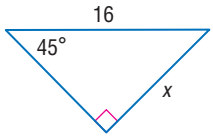
7. **ART** Paulo is mailing an engraved plaque that is $3\frac{1}{4}$ inches high to the winner of a chess tournament. He has a mailer that is a triangular prism with 4-inch equilateral triangle bases as shown in the diagram. Will the plaque fit through the opening of the mailer? Explain.

Practice and Problem Solving

Extra Practice is on page R8.

Examples 1-2 CCSS **SENSE-MAKING** Find x.

8.



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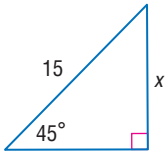

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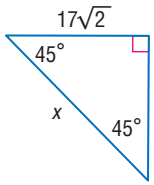

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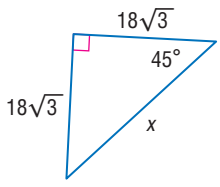

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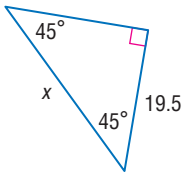

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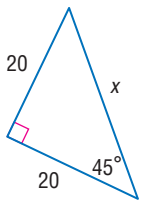


14. If a 45°-45°-90° triangle has a hypotenuse length of 9, find the leg length.

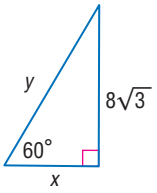
15. Determine the length of the leg of a 45°-45°-90° triangle with a hypotenuse length of 11.

16. What is the length of the hypotenuse of a 45°-45°-90° triangle if the leg length is 6 centimeters?

17. Find the length of the hypotenuse of a 45°-45°-90° triangle with a leg length of 8 centimeters.

Example 3 Find x and y.

18.



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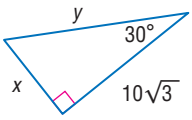

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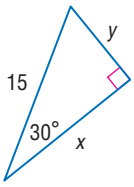

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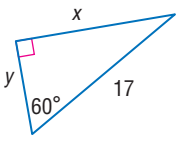

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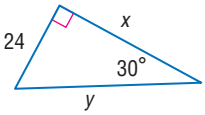

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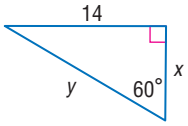
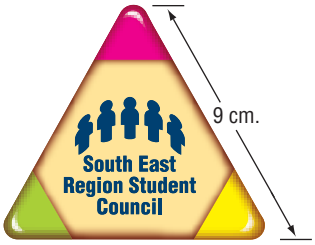


24. An equilateral triangle has an altitude length of 18 feet. Determine the length of a side of the triangle.

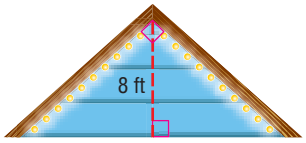
25. Find the length of the side of an equilateral triangle that has an altitude length of 24 feet.

Example 4

26. CCSS **MODELING** Refer to the beginning of the lesson. Each highlighter is an equilateral triangle with 9-centimeter sides. Will the highlighter fit in a 10-centimeter by 7-centimeter rectangular box? Explain.

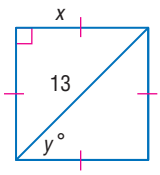


27. **EVENT PLANNING** Grace is having a party, and she wants to decorate the gable of the house as shown. The gable is an isosceles right triangle and she knows that the height of the gable is 8 feet. What length of lights will she need to cover the gable below the roof line?



Find x and y.

28.



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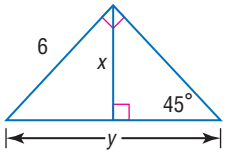

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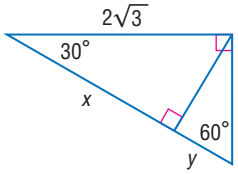

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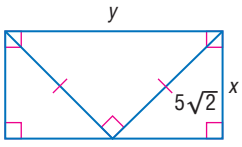

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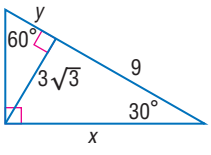

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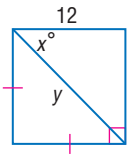
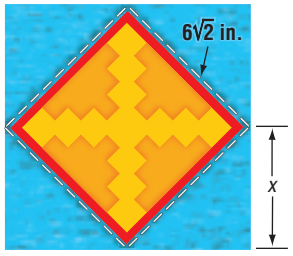
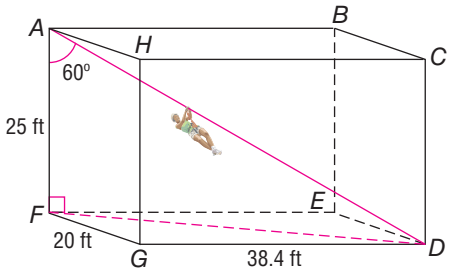


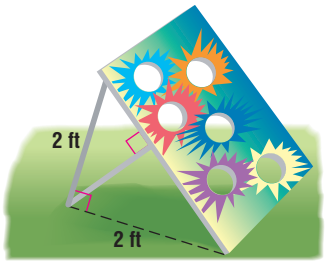
34. **QUILTS** The quilt block shown is made up of a square and four isosceles right triangles. What is the value of x? What is the side length of the entire quilt block?



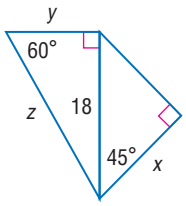
35 **ZIP LINE** Suppose a zip line is anchored in one corner of a course shaped like a rectangular prism. The other end is anchored in the opposite corner as shown. If the zip line makes a 60° angle with post $\overline{AF}$, find the zip line's length, AD.



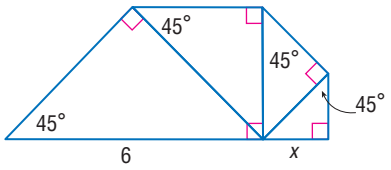
36. **GAMES** Kei is building a bean bag toss for the school carnival. He is using a 2-foot back support that is perpendicular to the ground 2 feet from the front of the board. He also wants to use a support that is perpendicular to the board as shown in the diagram. How long should he make the support?



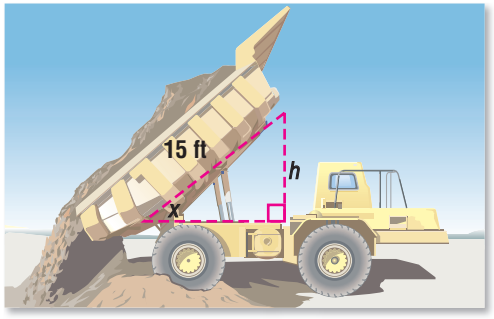
37. Find x, y, and z.



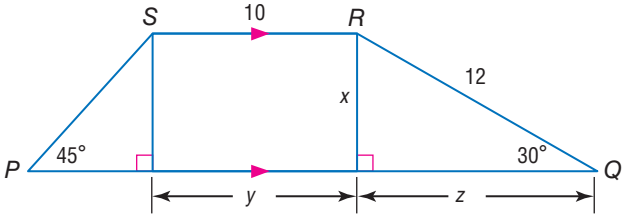
38. Each triangle in the figure is a 45°-45°-90° triangle. Find x.



39. CCSS **MODELING** The dump truck shown has a 15-foot bed length. What is the height of the bed h when angle x is 30°? 45°? 60°?



40. Find x, y, and z, and the perimeter of trapezoid $PQRS$.

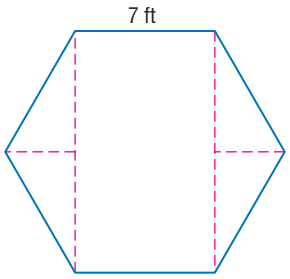


41. **COORDINATE GEOMETRY** $\triangle XYZ$ is a 45°-45°-90° triangle with right angle Z. Find the coordinates of X in Quadrant I for $Y(-1, 2)$ and $Z(6, 2)$.

42. **COORDINATE GEOMETRY** $\triangle EFG$ is a 30°-60°-90° triangle with $m\angle F = 90$. Find the coordinates of E in Quadrant III for $F(-3, -4)$ and $G(-3, 2)$. $\overline{FG}$ is the longer leg.

43. **COORDINATE GEOMETRY** $\triangle JKL$ is a 45°-45°-90° triangle with right angle K. Find the coordinates of L in Quadrant IV for $J(-3, 5)$ and $K(-3, -2)$.

44. EVENT PLANNING Eva has reserved a gazebo at a local park for a party. She wants to be sure that there will be enough space for her 12 guests to be in the gazebo at the same time. She wants to allow 8 square feet of area for each guest. If the floor of the gazebo is a regular hexagon and each side is 7 feet, will there be enough room for Eva and her friends? Explain. (*Hint:* Use the Polygon Interior Angle Sum Theorem and the properties of special right triangles.)



45. MULTIPLE REPRESENTATIONS In this problem, you will investigate ratios in right triangles.

a. Geometric Draw three similar right triangles with a 50° angle. Label one triangle ABC where angle A is the right angle and B is the 50° angle. Label a second triangle MNP where M is the right angle and N is the 50° angle. Label the third triangle XYZ where X is the right angle and Y is the 50° angle.

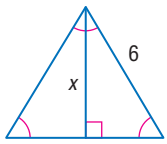
b. Tabular Copy and complete the table below.

Triangle	Length				Ratio	
ABC	AC		BC		$\frac{AC}{BC}$	
MNP	MP		NP		$\frac{MP}{NP}$	
XYZ	XZ		YZ		$\frac{XZ}{YZ}$	

c. Verbal Make a conjecture about the ratio of the leg opposite the 50° angle to the hypotenuse in any right triangle with an angle measuring 50°.

H.O.T. Problems Use Higher-Order Thinking Skills

46. CCSS CRITIQUE Carmen and Audrey want to find x in the triangle shown. Is either of them correct? Explain.



Carmen

$x = \frac{6\sqrt{3}}{2}$

$x = 3\sqrt{3}$

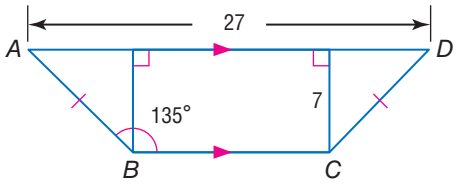
Audrey

$x = \frac{6\sqrt{2}}{2}$

$x = 3\sqrt{2}$

47. OPEN ENDED Draw a rectangle that has a diagonal twice as long as its width. Then write an equation to find the length of the rectangle.

48. CHALLENGE Find the perimeter of quadrilateral $ABCD$.



49. REASONING The ratio of the measure of the angles of a triangle is 1:2:3. The length of the shortest side is 8. What is the perimeter of the triangle?

50. WRITING IN MATH Why are some right triangles considered *special*?

Standardized Test Practice

51. If the length of the longer leg in a 30°-60°-90° triangle is $5\sqrt{3}$, what is the length of the shorter leg?

A 3 **C** $5\sqrt{2}$

B 5 **D** 10

52. ALGEBRA Solve $\sqrt{5 - 4x} - 6 = 7$.

F −44 **H** 41

G −41 **J** 44

53. SHORT RESPONSE $\triangle XYZ$ is a 45°-45°-90° triangle with right angle Y. Find the coordinates of X in Quadrant III for $Y(-3, -3)$ and $Z(-3, 7)$.

54. SAT/ACT In the figure, below, square $ABCD$ is attached to $\triangle ADE$ as shown. If $m\angle EAD$ is 30° and AE is equal to $4\sqrt{3}$, then what is the area of square $ABCD$?

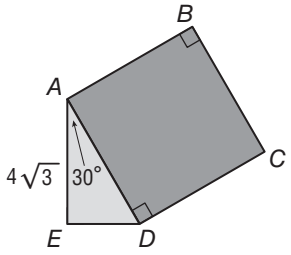
A $8\sqrt{3}$

B 16

C 64

D 72

E $64\sqrt{2}$

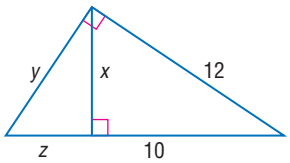


Spiral Review

55. SPORTS Dylan is making a ramp for bike jumps. The ramp support forms a right angle. The base is 12 feet long, and the height is 9 feet. What length of plywood does Dylan need for the ramp? (Lesson 8-2)

Find x, y, and z. (Lesson 8-1)

56.



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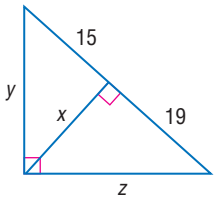
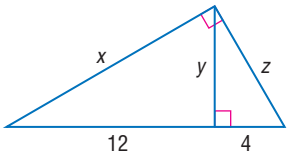


58.



Find the measures of the angles of each triangle. (Lesson 7-1)

59. The ratio of the measures of the three angles is 2:5:3.

60. The ratio of the measures of the three angles is 6:9:10.

61. The ratio of the measures of the three angles is 5:7:8.

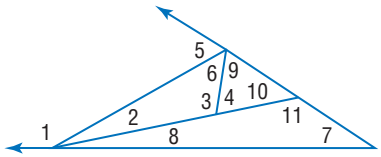
Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition. (Lesson 5-3)

62. measures less than $m\angle 5$

63. measures greater than $m\angle 6$

64. measures greater than $m\angle 10$

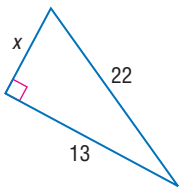
65. measures less than $m\angle 11$



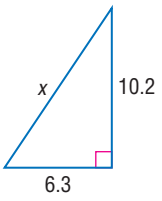
Skills Review

Find x.

66.



67.



68.