:: Then	:: Now	:: Why?
• You used similar triangles to measure distances indirectly.	**1** Solve problems involving angles of elevation and depression. **2** Use angles of elevation and depression to find the distance between two objects.	• To make a field goal, a kicker must kick the ball with enough force and at an appropriate angle of elevation to ensure that the ball will reach the goal post at a level high enough to make it over the horizontal bar. This angle must change depending on the initial placement of the ball away from the base of the goalpost.

NewVocabulary
angle of elevation
angle of depression

Common Core State Standards

Content Standards
G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ★

Mathematical Practices
4 Model with mathematics.
1 Make sense of problems and persevere in solving them.

1 **Angles of Elevation and Depression** An **angle of elevation** is the angle formed by a horizontal line and an observer's line of sight to an object above the horizontal line. An **angle of depression** is the angle formed by a horizontal line and an observer's line of sight to an object below the horizontal line.
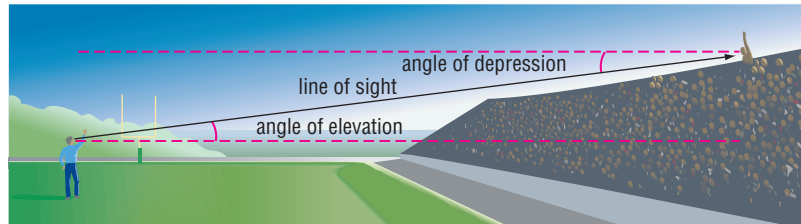

Horizontal lines are parallel, so the angle of elevation and the angle of depression in the diagram are congruent by the Alternate Interior Angles Theorem.

Example 1 Angle of Elevation

VACATION **Leah wants to see a castle in an amusement park. She sights the top of the castle at an angle of elevation of 38°. She knows that the castle is 190 feet tall. If Leah is 5.5 feet tall, how far is she from the castle to the nearest foot?**

Make a sketch to represent the situation.
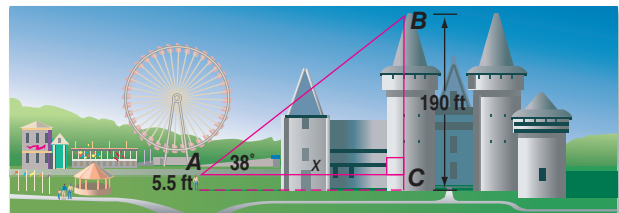
Since Leah is 5.5 feet tall, $BC = 190 - 5.5$ or 184.5 feet. Let x represent the distance from Leah to the castle, AC.

$$\tan A = \frac{BC}{AC} \qquad \tan = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 38° = \frac{184.5}{x} \qquad m\angle A = 38, BC = 184.5, AC = x$$

$$x = \frac{184.5}{\tan 38°} \qquad \text{Solve for } x.$$

$$x \approx 236.1 \qquad \text{Use a calculator.}$$

Leah is about 236 feet from the castle.

▶ **Guided**Practice

1. FOOTBALL The cross bar of a goalpost is 10 feet high. If a field goal attempt is made 25 yards from the base of the goalpost that clears the goal by 1 foot, what is the smallest angle of elevation at which the ball could have been kicked to the nearest degree?

Example 2 Angle of Depression

EMERGENCY **A search and rescue team is airlifting people from the scene of a boating accident when they observe another person in need of help. If the angle of depression to this other person is 42° and the helicopter is 18 feet above the water, what is the horizontal distance from the rescuers to this person to the nearest foot?**

Make a sketch of the situation.

Since $\overrightarrow{AB}$ and $\overline{DC}$ are parallel, $m\angle BAC = m\angle ACD$ by the Alternate Interior Angles Theorem.

Let x represent the horizontal distance from the rescuers to the person DC.

$$\tan C = \frac{AD}{DC} \qquad \tan = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 42° = \frac{18}{x} \qquad C = 42, AD = 18, \text{ and } DC = x$$

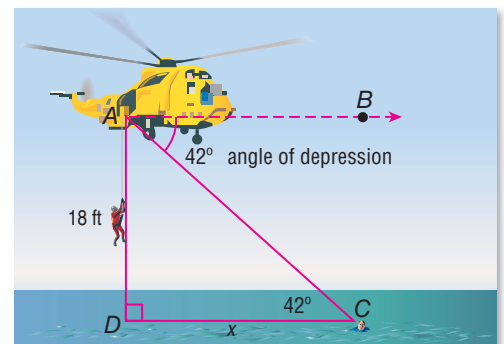$$x \tan 42° = 18 \qquad \text{Multiply each side by } x.$$

$$x = \frac{18}{\tan 42°} \qquad \text{Divide each side by } \tan 42°.$$

$$x \approx 20.0 \qquad \text{Use a calculator.}$$

The horizontal distance from the rescuers to the person is 20.0 feet.

Note: Art not drawn to scale.

▶ **Guided**Practice

2. LIFEGUARDING A lifeguard is watching a beach from a line of sight 6 feet above the ground. She sees a swimmer at an angle of depression of 8°. How far away from the tower is the swimmer?

WatchOut!

Angles of Elevation and Depression To avoid mislabeling, remember that angles of elevation and depression are always formed with a horizontal line and never with a vertical line.
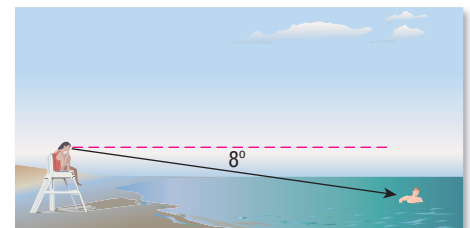

Math History Link

Eratosthenes (276–194 B.C.)
Eratosthenes was a mathematician and astronomer who was born in Cyrene, which is now Libya. He used the angle of elevation of the Sun at noon in the cities of Alexandria and Syene (now Egypt) to measure the circumference of Earth.

Source: *Encyclopaedia Britannica*

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2 **Two Angles of Elevation or Depression** Angles of elevation or depression to two different objects can be used to estimate the distance between those objects. Similarly, the angles from two different positions of observation to the same object can be used to estimate the object's height.
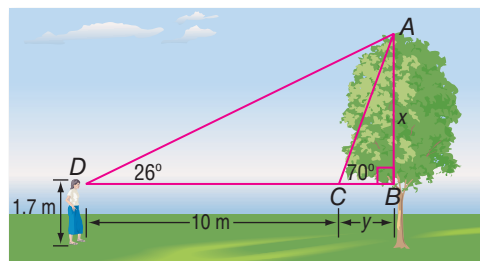
Example 3 Use Two Angles of Elevation or Depression

TREE REMOVAL To estimate the height of a tree she wants removed, Mrs. Long sights the tree's top at a 70° angle of elevation. She then steps back 10 meters and sights the top at a 26° angle. If Mrs. Long's line of sight is 1.7 meters above the ground, how tall is the tree to the nearest meter?

Understand $\triangle ABC$ and $\triangle ABD$ are right triangles. The height of the tree is the sum of Mrs. Long's height and AB.

Plan Since her initial distance from the tree is not given, write and solve a system of equations using both triangles. Let $AB = x$ and $CB = y$. So $DB = y + 10$ and the height of the tree is $x + 1.7$.

Solve Use $\triangle ABC$.

$$\tan 70° = \frac{x}{y} \qquad \tan = \frac{\text{opposite}}{\text{adjacent}}; m\angle ACB = 70$$

$$y \tan 70° = x \qquad \text{Multiply each side by } y.$$

Use $\triangle ABD$.

$$\tan 26° = \frac{x}{y + 10} \qquad \tan = \frac{\text{opposite}}{\text{adjacent}}; m\angle D = 26$$

$$(y + 10) \tan 26° = x \qquad \text{Multiply each side by } y + 10.$$

Substitute the value for x from $\triangle ABD$ in the equation for $\triangle ABC$ and solve for y.

$$y \tan 70° = x$$

$$y \tan 70° = (y + 10)\ \textcolor{red}{\tan 26°}$$

$$y \tan 70° = y\ \textcolor{red}{\tan 26°} + 10\ \textcolor{red}{\tan 26°}$$

$$\textcolor{blue}{y} \tan 70° - \textcolor{blue}{y} \tan 26° = 10 \tan 26°$$

$$\textcolor{blue}{y}(\tan 70° - \tan 26°) = 10 \tan 26°$$

$$y = \frac{10 \tan 26°}{\tan 70° - \tan 26°}$$

Use a calculator to find that $y \approx \textbf{2.16}$. Using the equation from $\triangle ABC$, $x = \textbf{2.16} \tan 70°$ or about $\textbf{5.9}$.

The height of the tree is $\textbf{5.9} + 1.7$ or 7.6, which is about 8 meters.

Check Substitute the value for y in the equation from $\triangle ABD$.

$x = (\textbf{2.16} + 10) \tan 26°$ or about 5.9. This is the same value found using the equation from $\triangle ABC$. ✓

▶ **Guided**Practice

3. **SKYSCRAPERS** Two buildings are sited from atop a 200-meter skyscraper. Building A is sited at a 35° angle of depression, while Building B is sighted at a 36° angle of depression. How far apart are the two buildings to the nearest meter?


Real-World Link

In the United States, lumber volume is measured in board-feet, which is defined as a piece of wood containing 144 cubic inches. Woodland owners often estimate the lumber volume of trees they own to determine how many to cut and sell.

Source: The Ohio State University School of Natural Resources

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StudyTip

Indirect Measurement
When using the angles of depression to two different objects to calculate the distance between them, it is important to remember that the two objects must lie in the same horizontal plane. In other words, one object cannot be higher or lower than the other.

Example 1   **1.** **BIKING** Lenora wants to build the bike ramp shown. Find the length of the base of the ramp.
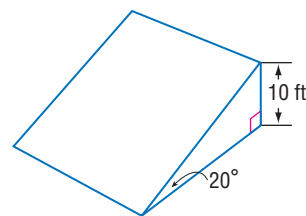
Example 2   **2.** **BASEBALL** A fan is seated in the upper deck of a stadium 200 feet away from home plate. If the angle of depression to the field is 62°, at what height is the fan sitting?

Example 3   **3.** **CCSS** **MODELING** Annabelle and Rich are setting up decorations for their school dance. Rich is standing 5 feet directly in front of Annabelle under a disco ball. If the angle of elevation from Annabelle to the ball is 40° and Rich to the ball is 50°, how high is the disco ball?
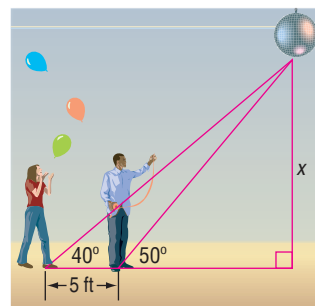
Practice and Problem Solving   **Extra Practice is on page R8.**

Example 1   **4.** **HOCKEY** A hockey player takes a shot 20 feet away from a 5-foot goal. If the puck travels at a 15° angle of elevation toward the center of the goal, will the player score?
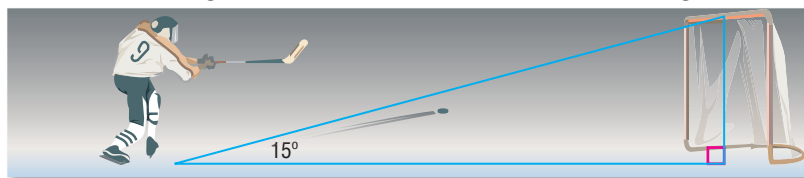
5 **MOUNTAINS** Find the angle of elevation to the peak of a mountain for an observer who is 155 meters from the mountain if the observer's eye is 1.5 meters above the ground and the mountain is 350 meters tall.

Example 2   **6.** **WATERPARK** Two water slides are 50 meters apart on level ground. From the top of the taller slide, you can see the top of the shorter slide at an angle of depression of 15°. If you know that the top of the other slide is approximately 15 meters above the ground, about how far above the ground are you? Round to the nearest tenth of a meter.
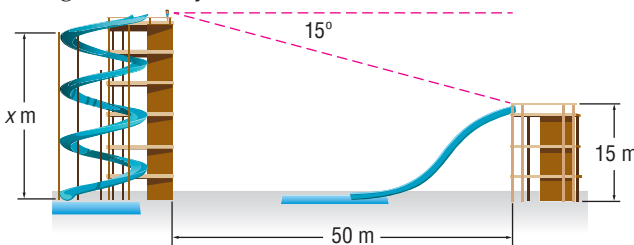
7. **AVIATION** Due to a storm, a pilot flying at an altitude of 528 feet has to land. If he has a horizontal distance of 2000 feet to land, at what angle of depression should he land?

Example 3   **8.** **PYRAMIDS** Miko and Tyler are visiting the Great Pyramid in Egypt. From where Miko is standing, the angle of elevation to the top of the pyramid is 48.6°. From Tyler's position, the angle of elevation is 50°. If they are standing 20 feet apart, and they are each 5 feet 6 inches tall, how tall is the pyramid?
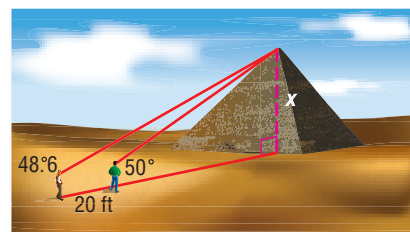
9 **DIVING** Austin is standing on the high dive at the local pool. Two of his friends are in the water as shown. If the angle of depression to one of his friends is 40°, and 30° to his other friend who is 5 feet beyond the first, how tall is the platform?
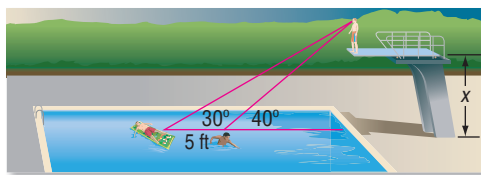
10. **BASKETBALL** Claire and Marisa are both waiting to get a rebound during a basketball game. If the height of the basketball hoop is 10 feet, the angle of elevation between Claire and the goal is 35°, and the angle of elevation between Marisa and the goal is 25°, how far apart are they standing?
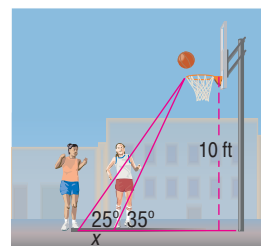

11. **RIVERS** Hugo is standing in the top of St. Louis' Gateway Arch, looking down on the Mississippi River. The angle of depression to the closer bank is 45° and the angle of depression to the farther bank is 18°. The arch is 630 feet tall. Estimate the width of the river at that point.
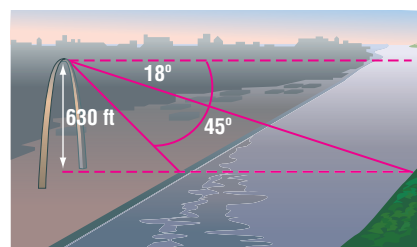

12. **CCSS MODELING** The Unzen Volcano in Japan has a magma reservoir located 15 kilometers beneath the Chijiwa Bay, located east of the volcano. A magma channel, which connects the reservoir to the volcano, rises at a 40° angle of elevation toward the volcano. What length of magma channel is below sea level?

13. **BRIDGES** Suppose you are standing in the middle of the platform of the world's longest suspension bridge, the Akashi Kaikyo Bridge. If the height from the top of the platform holding the suspension cables is 297 meters, and the length from the platform to the center of the bridge is 995 meters, what is the angle of depression from the center of the bridge to the platform?
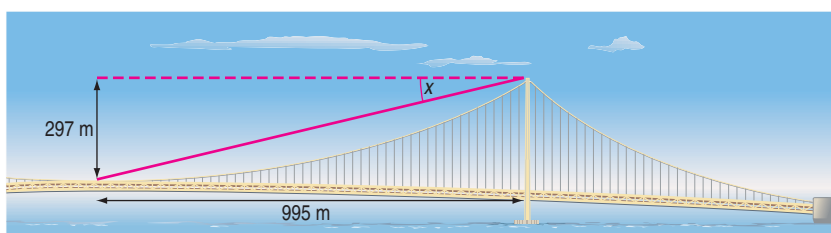

14. **LIGHTHOUSES** Little Gull Island Lighthouse shines a light from a height of 91 feet with a 6° angle of depression. Plum Island Lighthouse, 1800 feet away, shines a light from a height of 34 feet with a 2° angle of depression. Which light will reach a boat that sits exactly between Little Gull Island Lighthouse and Plum Island Lighthouse?

15. **TOURISM** From the position of the bus on the street, the L'arc de Triomphe is at an angle of 34°. If the arc is 162 feet tall, how far away is the bus? Round to the nearest tenth.
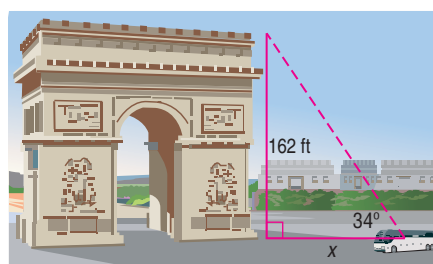
16. MAINTENANCE Two telephone repair workers arrive at a location to restore electricity after a power outage. One of the workers climbs up the telephone pole while the other worker stands 10 feet to left of the pole. If the terminal box is located 30 feet above ground on the pole and the angle of elevation from the truck to the repair worker is 70°, how far is the worker on the ground standing from the truck?

30 ft
70°
10 ft   x
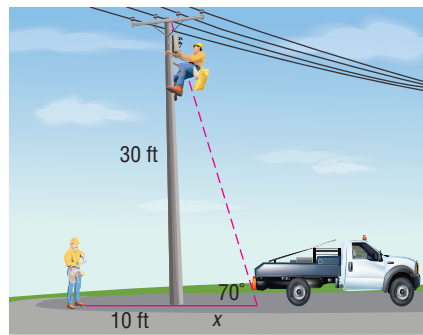
17 **PHOTOGRAPHY** A digital camera with a panoramic lens is described as having a view with an angle of elevation of 38°. If the camera is on a 3-foot tripod aimed directly at a 124-foot-tall monument, how far from the monument should you place the tripod to see the entire monument in your photograph?

18. **CCSS** **MODELING** As a part of their weather unit, Anoki's science class took a hot air balloon ride. As they passed over a fenced field, the angle of depression of the closer side of the fence was 32°, and the angle of depression of the farther side of the fence was 27°. If the height of the balloon was 800 feet, estimate the width of the field.


27° 32°
800 ft
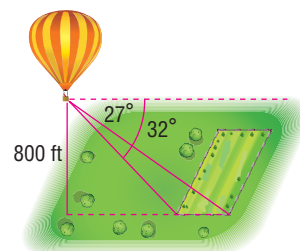
19. MARATHONS The Badwater Ultramarathon is a race that begins at the lowest point in California, Death Valley, and ends at the highest point of the state, Mount Whitney. The race starts at a depth of 86 meters below sea level and ends 2530 meters above sea level.

a. Determine the angle of elevation to Mount Whitney if the horizontal distance from the base to the peak is 1200 meters.


2530 m
x
1200 m

b. If the angle of depression to Death Valley is 38°, what is the horizontal distance from sea level?

x   38°
86 m
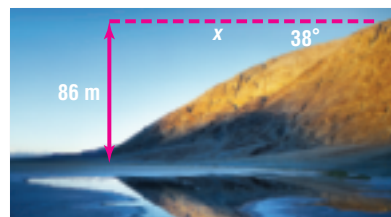
**20. AMUSEMENT PARKS** India, Enrique, and Trina went to an amusement park while visiting Japan. They went on a Ferris wheel that was 100 meters in diameter and on an 80-meter cliff-dropping slide.

a. When Enrique and Trina are at the topmost point on the Ferris wheel shown below, how far are they from India?

57°
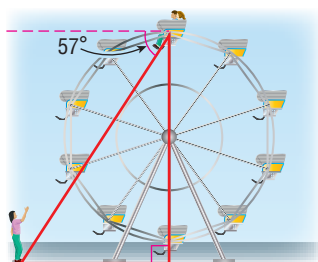
b. If the cliff-dropping ride has an angle of depression of 46°, how long is the slide?

46°
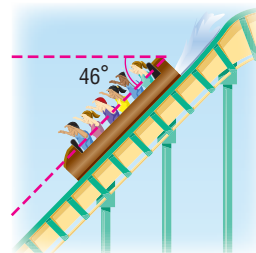
21 **DARTS** Kelsey and José are throwing darts from a distance of 8.5 feet. The center of the bull's-eye on the dartboard is 5.7 feet from the floor. José throws from a height of 6 feet, and Kelsey throws from a height of 5 feet. What are the angles of elevation or depression from which each must throw to get a bull's-eye? Ignore other factors such as air resistance, velocity, and gravity.


5.7 ft

8.5 ft
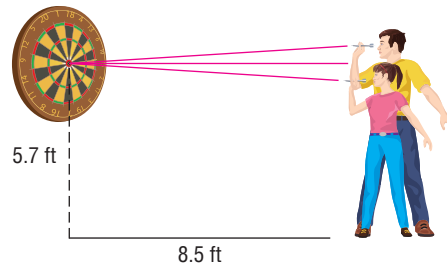
22. ⟳ **MULTIPLE REPRESENTATIONS** In this problem, you will investigate relationships between the sides and angles of triangles.

a. Geometric Draw three triangles. Make one acute, one obtuse, and one right. Label one triangle *ABC*, a second *MNP*, and the third *XYZ*. Label the side lengths and angle measures of each triangle.

b. Tabular Copy and complete the table below.

Triangle	Ratios		
ABC	$\dfrac{\sin A}{BC} =$	$\dfrac{\sin B}{CA} =$	$\dfrac{\sin C}{AB} =$
MNP	$\dfrac{\sin M}{NP} =$	$\dfrac{\sin N}{PM} =$	$\dfrac{\sin P}{MN} =$
XYZ	$\dfrac{\sin X}{YZ} =$	$\dfrac{\sin Y}{ZX} =$	$\dfrac{\sin Z}{XY} =$

c. Verbal Make a conjecture about the ratio of the sine of an angle to the length of the leg opposite that angle for a given triangle.

H.O.T. Problems   Use Higher-Order Thinking Skills

23. ERROR ANALYSIS Terrence and Rodrigo are trying to determine the relationship between angles of elevation and depression. Terrence says that if you are looking up at someone with an angle of elevation of 35°, then they are looking down at you with an angle of depression of 55°, which is the complement of 35°. Rodrigo disagrees and says that the other person would be looking down at you with an angle of depression equal to your angle of elevation, or 35°. Is either of them correct? Explain.

24. CHALLENGE Find the value of *x*. Round to the nearest tenth.

15°   x   70°

2
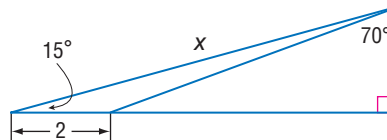
25. CCSS **REASONING** Classify the statement below as *true* or *false*. Explain.

As a person moves closer to an object he or she is sighting, the angle of elevation increases.

26. WRITE A QUESTION A classmate finds the angle of elevation of an object, but she is trying to find the angle of depression. Write a question to help her solve the problem.

27. WRITING IN MATH Describe a way that you can estimate the height of an object without using trigonometry by choosing your angle of elevation. Explain your reasoning.

28. Ryan wanted to know the height of a cell-phone tower neighboring his property. He walked 80 feet from the base of the tower and measured the angle of elevation to the top of the tower at 54°. If Ryan is 5 feet tall, what is the height of the cell-phone tower?

A 52 ft   **C** 110 ft
B 63 ft   **D** 115 ft

29. SHORT RESPONSE A searchlight is 6500 feet from a weather station. If the angle of elevation to the spot of light on the clouds above the station is 45°, how high is the cloud ceiling?

30. ALGEBRA What is the solution of this system of equations?

$$2x - 4y = -12$$
$$-x + 4y = 8$$

F $(4, 4)$ **H** $(-4, -4)$
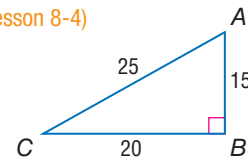**G** $(-4, 1)$ **J** $(1, -4)$

31. SAT/ACT A triangle has sides in the ratio of $5:12:13$. What is the measure of the triangle's smallest angle in degrees?

A 13.34   **D** 42.71
**B** 22.62   **E** 67.83
C 34.14

Spiral Review

Express each ratio as a fraction and as a decimal to the nearest hundredth. (Lesson 8-4)

32. $\sin C$ **33.** $\tan A$ **34.** $\cos C$

35. $\tan C$ **36.** $\cos A$ **37.** $\sin A$

38. LANDSCAPING Imani needs to determine the height of a tree. Holding a drafter's 45° triangle so that one leg is horizontal, she sights the top of the tree along the hypotenuse, as shown at the right. If she is 6 yards from the tree and her eyes are 5 feet from the ground, find the height of the tree. (Lesson 8-3)
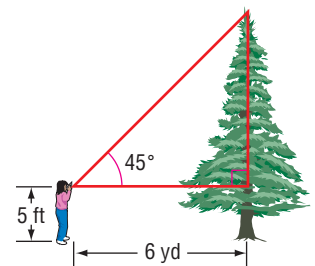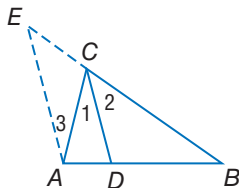

PROOF **Write a two-column proof.** (Lesson 7-5)

39. Given: $\overline{CD}$ bisects $\angle ACB$.
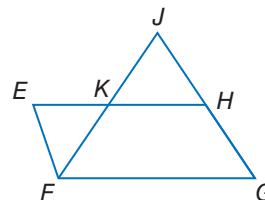
By construction, $\overline{AE} \parallel \overline{CD}$.

Prove: $\dfrac{AD}{DB} = \dfrac{AC}{BC}$

40. Given: $\overline{JF}$ bisects $\angle EFG$.

$\overline{EH} \parallel \overline{FG}, \overline{EF} \parallel \overline{HG}$

Prove: $\dfrac{EK}{KF} = \dfrac{GJ}{JF}$

COORDINATE GEOMETRY **Find the coordinates of the centroid of each triangle.** (Lesson 5-2)

41. $A(2, 2), B(7, 8), C(12, 2)$

42. $X(-3, -2), Y(1, -12), Z(-7, -7)$

43. $A(-1, 11), B(-5, 1), C(-9, 6)$

44. $X(4, 0), Y(-2, 4), Z(0, 6)$

Skills Review

Solve each proportion.

45. $\dfrac{1}{5} = \dfrac{x}{10}$

46. $\dfrac{2x}{11} = \dfrac{3}{8}$

47. $\dfrac{4x}{16} = \dfrac{62}{118}$

48. $\dfrac{12}{21} = \dfrac{45}{10x}$