Study Guide

KeyConcepts

Circles and Circumference (Lesson 10-1)

- The circumference of a circle is equal to πd or $2\pi r$.

Angles, Arcs, Chords, and Inscribed Angles
(Lessons 10-2 to 10-4)

- The sum of the measures of the central angles of a circle is 360°.
- The length of an arc is proportional to the length of the circumference.
- Diameters perpendicular to chords bisect chords and intercepted arcs.
- The measure of an inscribed angle is half the measure of its intercepted arc.

Tangents, Secants, and Angle Measures
(Lessons 10-5 and 10-6)

- A line that is tangent to a circle intersects the circle in exactly one point and is perpendicular to a radius.
- Two segments tangent to a circle from the same exterior point are congruent.
- The measure of an angle formed by two secant lines is half the positive difference of its intercepted arcs.
- The measure of an angle formed by a secant and tangent line is half its intercepted arc.
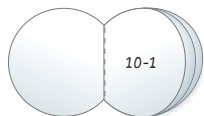
Special Segments and Equation of a Circle
(Lessons 10-7 and 10-8)

- The lengths of intersecting chords in a circle can be found by using the products of the measures of the segments.
- The equation of a circle with center (h, k) and radius r is $(x - h)^2 - (y - k)^2 = r^2$.

FOLDABLES® StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.

10-1

KeyVocabulary

adjacent arcs (p. 708)	external secant segment (p. 752)
arc (p. 706)	inscribed (p. 700)
arc length (p. 709)	inscribed angle (p. 723)
center (p. 697)	intercepted arc (p. 723)
central angle (p. 706)	major arc (p.707)
chord (p. 697)	minor arc (p. 707)
chord segment (p. 750)	pi (π) (p. 699)
circle (p. 697)	point of tangency (p. 732)
circumference (p. 699)	radius (p. 697)
circumscribed (p. 700)	secant (p. 741)
common tangent (p. 732)	secant segment (p. 752)
compound locus (p. 762)	semicircle (p. 707)
concentric circles (p. 698)	tangent (p. 732)
congruent arcs (p. 707)	
diameter (p. 697)	

VocabularyCheck

State whether each sentence is *true* or *false.* If *false,* replace the underlined word or phrase to make a true sentence.

1. Any segment with both endpoints on the circle is a <u>radius</u> of the circle.

2. A chord passing through the center of a circle is a <u>diameter</u>.

3. A <u>central angle</u> has the center as its vertex and its sides contain two radii of the circle.

4. An arc with a measure of less than 180° is a <u>major arc</u>.

5. An <u>intercepted arc</u> is an arc that has its endpoints on the sides of an inscribed angle and lies in the interior of the inscribed angle.

6. A <u>common tangent</u> is the point at which a line in the same plane as a circle intersects the circle.

7. A secant is a line that intersects a circle in exactly <u>one</u> point.

8. A secant segment is a segment of a <u>diameter</u> that has exactly one endpoint on the circle.

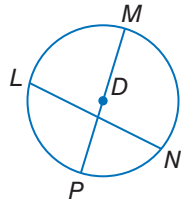9. Two circles are <u>concentric</u> circles if and only if they have congruent radii.

Lesson-by-Lesson Review

10-1 Circles and Circumference

For Exercises 10–12, refer to ⊙D.

10. Name the circle.

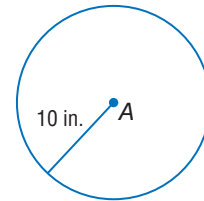11. Name a radius.

12. Name a chord that is not a diameter.

Find the diameter and radius of a circle with the given circumference. Round to the nearest hundredth.

13. $C = 43$ cm

14. $C = 26.7$ yd

15. $C = 108.5$ ft

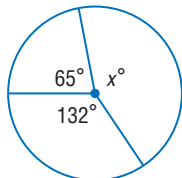16. $C = 225.9$ mm

Example 1

Find the circumference of ⊙A.

10 in.

$$C = 2\pi r \quad \text{Circumference formula}$$
$$= 2\pi(10) \quad \text{Substitution}$$
$$\approx 62.83 \quad \text{Use a calculator.}$$

The circumference of ⊙A is about 62.83 inches.

10-2 Measuring Angles and Arcs

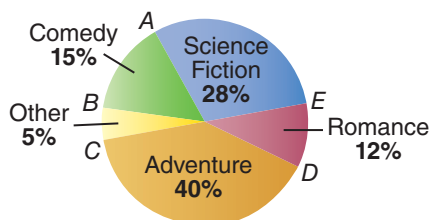Find the value of x.

17.

65° $x°$
132°

18.

30°
$x°$
110°
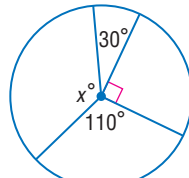
19. **MOVIES** The pie chart below represents the results of a survey taken by Mrs. Jameson regarding her students' favorite types of movies. Find each measure.
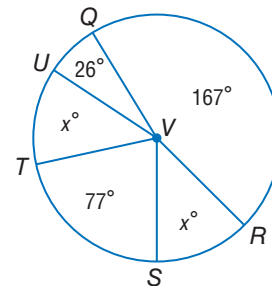
**Mrs. Jameson's Students'
Favorite Types of Movies**

Comedy 15% A
Science Fiction 28%
Other 5% B
C E
Adventure 40% D
Romance 12%

a. $m\widehat{AE}$

b. $m\widehat{BC}$

c. Describe the type of arc that the category Adventure represents.

Example 2

Find the value of x.

Q
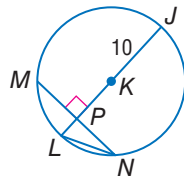U 26°
167°
$x°$ V
T
77°
$x°$ R
S

$$m\angle QVR + m\angle RVS + m\angle SVT + $$
$$m\angle TVU + m\angle UVQ = 360 \quad \text{Sum of Central Angles}$$
$$167 + x + 77 + x + 26 = 360 \quad \text{Substitution}$$
$$270 + 2x = 360 \quad \text{Simplify.}$$
$$2x = 90 \quad \text{Subtract.}$$
$$x = 45 \quad \text{Divide.}$$

10-3 Arcs and Chords

20. Find the value of *x*.

142°
3x + 7
5x − 9
142°
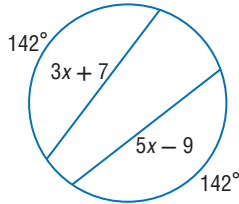
In ⊙*K*, *MN* = 16 and $m\widehat{MN}$ = 98. Find each measure. Round to the nearest hundredth.

21. $m\widehat{NJ}$ **22.** *LN*

J
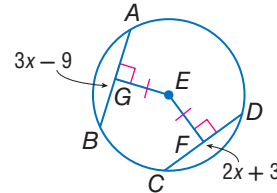10
M   K
P
L
N

23. GARDENING The top of the trellis shown is an arc of a circle in which $\overline{CD}$ is part of the diameter and $\overline{CD} \perp \overline{AB}$. If $\widehat{ACB}$ is about 28% of a complete circle, what is $m\widehat{CB}$?

C
A   D   B
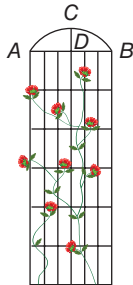
Example 3

ALGEBRA In ⊙*E*, *EG* = *EF*. Find *AB*.

A
3x − 9
G   E
B   F   D
C
2x + 3

Since chords $\overline{EG}$ and $\overline{EF}$ are congruent, they are equidistant from *E*. So, *AB* = *CD*.

$AB = CD$	Theorem 10.5
$3x - 9 = 2x + 3$	Substitution
$3x = 2x + 12$	Add.
$x = 12$	Simplify.

So, $AB = 3(12) - 9$ or 27.

10-4 Inscribed Angles

Find each measure.

24. *m∠1*   **25.** $m\widehat{GH}$

218°
1
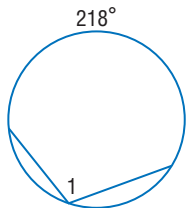
G
28°
H
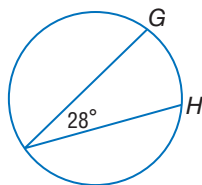
26. MARKETING In the logo at the right, *m∠1* = 42. Find *m∠5*.

1   2   3   4
5
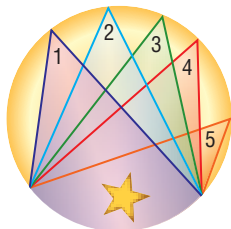
Example 4

Find *m∠D* and *m∠B*.

B   (21x − 8)°
A
(23x + 12)°   C
D

Since *ABCD* is inscribed in a circle, opposite angles are supplementary.
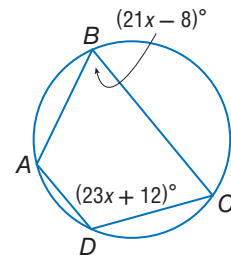
|	
$m\angle D + m\angle B = 180$	Definition of supplementary
$23x + 12 + 21x - 8 = 180$	Substitution
$44x + 4 = 180$	Simplify.
$44x = 176$	Subtract.
$x = 4$	Divide.

So, $m\angle D = 23(4) + 12$ or 104 and $m\angle B = 21(4) - 8$ or 76.

27. SCIENCE FICTION In a story Todd is writing, instantaneous travel between a two-dimensional planet and its moon is possible when the time-traveler follows a tangent. Copy the figures below and draw all possible travel paths.
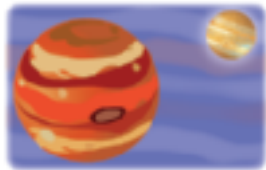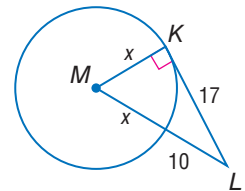

28. Find x and y. Assume that segments that appear to be tangent are tangent. Round to the nearest tenth if necessary.
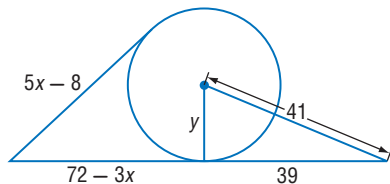
Example 5

In the figure, $\overline{KL}$ is tangent to $\odot M$ at K. Find the value of x.

By Theorem 10.9, $\overline{MK} \perp \overline{KL}$. So, $\triangle MKL$ is a right triangle.

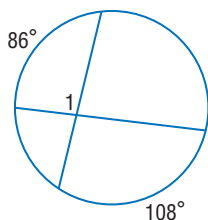|	
$KM^2 + KL^2 = ML^2$	Pythagorean Theorem
$x^2 + 17^2 = (x + 10)^2$	Substitution
$x^2 + 289 = x^2 + 20x + 100$	Multiply.
$289 = 20x + 100$	Simplify.
$189 = 20x$	Subtract.
$9.45 = x$	Divide.

Find each measure. Assume that segments that appear to be tangent are tangent.
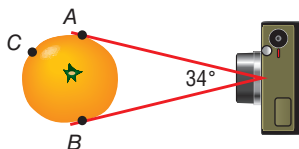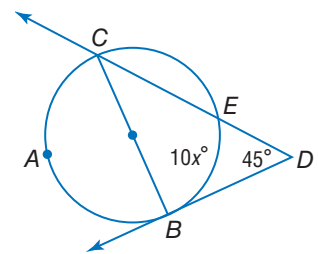
29. $m\angle 1$

30. $m\widehat{AC}$
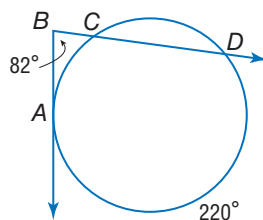
31. PHOTOGRAPHY Ahmed needs to take a close-up shot of an orange for his art class. He frames a shot of an orange as shown below, so that the lines of sight form tangents to the orange. If the measure of the camera's viewing angle is 34°, what is $m\widehat{ACB}$?

Example 6

Find the value of x.

$\widehat{CAB}$ is a semicircle because $\overline{CB}$ is a diameter.

So, $m\widehat{CAB} = 180$.

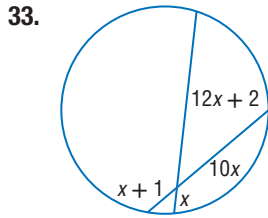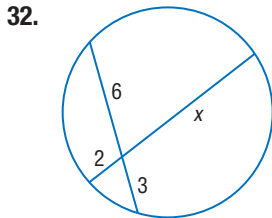|	
$m\angle D = \frac{1}{2}(m\widehat{CB} - m\widehat{EB})$	Theorem 10.14
$45 = \frac{1}{2}(180 - 10x)$	Substitution
$90 = 180 - 10x$	Multiply.
$-90 = -10x$	Subtract.
$9 = x$	Divide.

10-7 Special Segments in a Circle

Find *x*. Assume that segments that appear to be tangent are tangent.
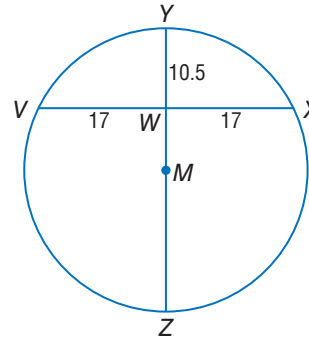
32.

6
x
2
3

33.

12x + 2
10x
x + 1 x

34. ARCHAEOLOGY While digging a hole to plant a tree, Henry found a piece of a broken saucer. What was the circumference of the original saucer? Round to the nearest hundredth.

0.75 in.
2 in.
2 in.
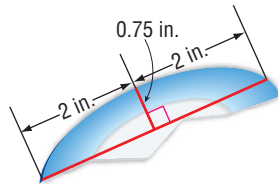
Example 7

Find the diameter of circle *M*.

Y
10.5
V   17   W   17   X
M
Z

$$VW \cdot WX = YW \cdot WZ \qquad \text{Theorem 10.14}$$
$$17 \cdot 17 = 10.5 \cdot WZ \qquad \text{Substitution}$$
$$289 = 10.5 \cdot WZ \qquad \text{Simplify.}$$
$$27.5 \approx WZ \qquad \text{Divide each side by 10.5.}$$

$$YZ = YW + WZ \qquad \text{Segment Addition Postulate}$$
$$YZ = 10.5 + 27.5 \qquad \text{Substitution}$$
$$YZ = 38 \qquad \text{Simplify.}$$

10-8 Equations of Circles

Write the equation of each circle.

35. center at $(-2, 4)$, radius 5

36. center at $(1, 2)$, diameter 14

37. FIREWOOD In an outdoor training course, Kat learns a wood-chopping safety check that involves making a circle with her arm extended, to ensure she will not hit anything overhead as she chops. If her reach is 19 inches, the hatchet handle is 15 inches, and her shoulder is located at the origin, what is the equation of Kat's safety circle?

15 in.
19 in.
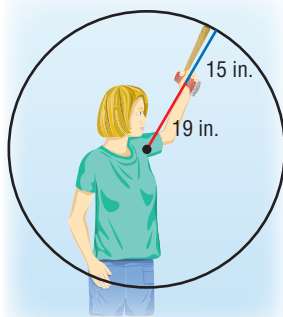
Example 8

Write the equation of the circle graphed below.

y
8
4
O   4   8   x
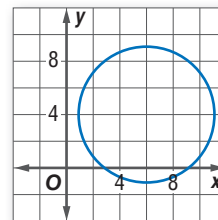
The center is at (6, 4) and the radius is 5.

$$(x - h)^2 + (y - k)^2 = r^2 \qquad \text{Equation of a circle}$$
$$(x - 6)^2 + (y - 4)^2 = 5^2 \qquad (h, k) = (6, 4) \text{ and } r = 5$$
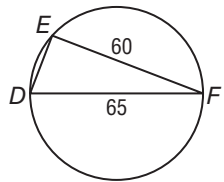$$(x - 6)^2 + (y - 4)^2 = 25 \qquad \text{Simplify.}$$

Practice Test

1. POOLS Amanda's family has a swimming pool that is 4 feet deep in their backyard. If the diameter of the pool is 25 feet, what is the circumference of the pool to the nearest foot?

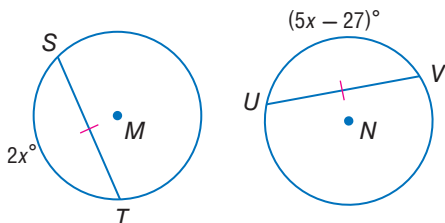**2.** Find the exact circumference of the circle below.

Find the value of x.

3.

4.

5.

6.
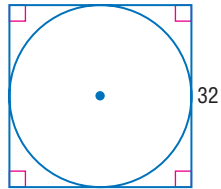
7. MULTIPLE CHOICE What is ED?

A 15

B 25

C 88.5

D not enough information

8. Find x if $\odot M \cong \odot N$.

9. MULTIPLE CHOICE How many points are shared by concentric circles?
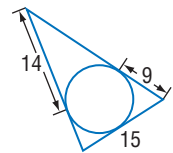
F 0

G 1

H 2

J infinite points

10. Determine whether $\overline{FG}$ is tangent to $\odot E$. Justify your answer.
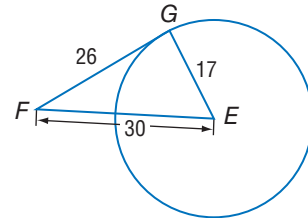
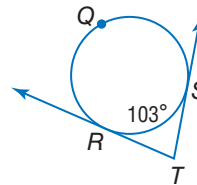**11. MULTIPLE CHOICE** Which of the figures below shows a polygon circumscribed about a circle?

A

C

B

D
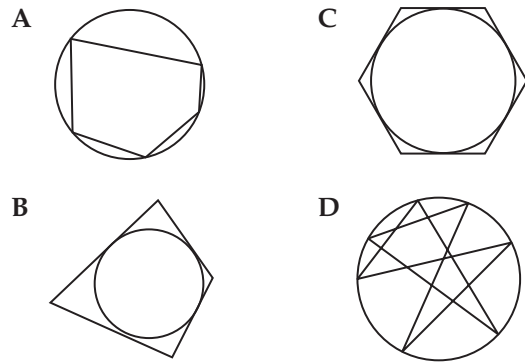
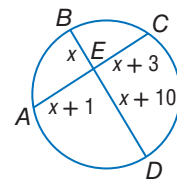**12.** Find the perimeter of the triangle at the right. Assume that segments that appear to be tangent are tangent.

Find each measure.

13. $m\angle T$

14. x

15. FLOWERS Hannah wants to encircle a tree trunk with a flower bed. If the center of the tree trunk is the origin and Hannah wants the flower bed to extend to 3 feet from the center of the tree, what is the equation that would represent the flower bed?