CHAPTER 5

Study Guide and Review

Study Guide

KeyConcepts

Special Segments in Triangles (Lessons 5-1 and 5-2)

- The special segments of triangles are perpendicular bisectors, angle bisectors, medians, and altitudes.
- The intersection points of each of the special segments of a triangle are called the points of concurrency.
- The points of concurrency for a triangle are the circumcenter, incenter, centroid, and orthocenter.

Indirect Proof (Lesson 5-4)

- Writing an Indirect Proof:

  1. Assume that the conclusion is false.

  2. Show that this assumption leads to a contradiction.

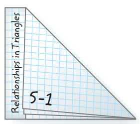
  3. Since the false conclusion leads to an incorrect statement, the original conclusion must be true.

Triangle Inequalities (Lessons 5-3, 5-5, and 5-6)

- The largest angle in a triangle is opposite the longest side, and the smallest angle is opposite the shortest side.
- The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- **SAS Inequality** (Hinge Theorem): In two triangles, if two sides are congruent, then the measure of the included angle determines which triangle has the longer third side.
- **SSS Inequality**: In two triangles, if two corresponding sides of each triangle are congruent, then the length of the third side determines which triangle has the included angle with the greater measure.

FOLDABLES® StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

altitude (p. 337)

centroid (p. 335)

circumcenter (p. 325)

concurrent lines (p. 325)

incenter (p. 328)

indirect proof (p. 355)

indirect reasoning (p. 355)

median (p. 335)

orthocenter (p. 337)

perpendicular bisector (p. 324)

point of concurrency (p. 325)

proof by contradiction (p. 355)

VocabularyCheck

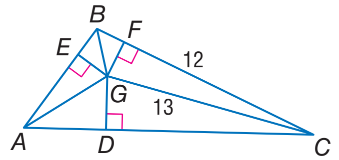
State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. The altitudes of a triangle intersect at the centroid.
2. The point of concurrency of the medians of a triangle is called the incenter.
3. The point of concurrency is the point at which three or more lines intersect.
4. The circumcenter of a triangle is equidistant from the vertices of the triangle.
5. To find the centroid of a triangle, first construct the angle bisectors.
6. The perpendicular bisectors of a triangle are concurrent lines.
7. To start a proof by contradiction, first assume that what you are trying to prove is true.
8. A proof by contradiction uses indirect reasoning.
9. A median of a triangle connects the midpoint of one side of the triangle to the midpoint of another side of the triangle.
10. The incenter is the point at which the angle bisectors of a triangle intersect.

Lesson-by-Lesson Review

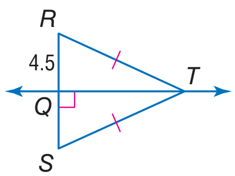
5-1 Bisectors of Triangles

11. Find EG if G is the incenter of $\triangle ABC$.

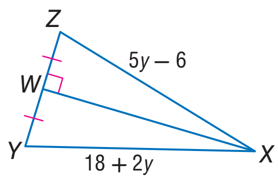


Find each measure.

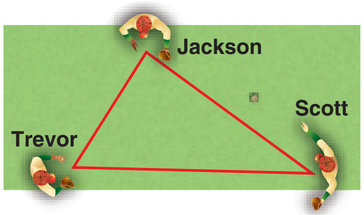
12. RS



13. XZ

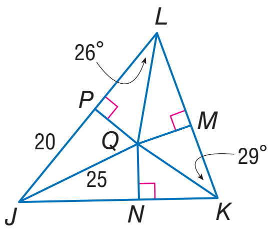


14. BASEBALL Jackson, Trevor, and Scott are warming up before a baseball game. One of their warm-up drills requires three players to form a triangle, with one player in the middle. Where should the fourth player stand so that he is the same distance from the other three players?



Example 1

Find each measure if Q is the incenter of $\triangle JKL$.



a. $\angle QJK$

$m\angle KLP + m\angle MKN + m\angle NJP = 180$ — $\triangle$ Sum Theorem

$2(26) + 2(29) + m\angle NJP = 180$ — Substitution

$110 + m\angle NJP = 180$ — Simplify.

$m\angle NJP = 70$ — Subtract.

Since $\overrightarrow{JQ}$ bisects $\angle NJP$, $2m\angle QJK = m\angle NJP$.

So, $m\angle QJK = \frac{1}{2}m\angle NJP$, so $m\angle QJK = \frac{1}{2}(70)$ or 35.

b. QP

$a^2 + b^2 = c^2$ — Pythagorean Theorem

$(QP)^2 + 20^2 = 25^2$ — Substitution

$(QP)^2 + 400 = 625$ — $20^2 = 400$ and $25^2 = 625$

$(QP)^2 = 225$ — Subtract.

$QP = 15$ — Simplify.

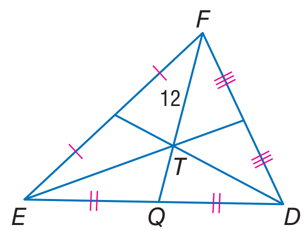
5-2 Medians and Altitudes of Triangles

15. The vertices of $\triangle DEF$ are $D(0, 0)$, $E(0, 7)$, and $F(6, 3)$. Find the coordinates of the orthocenter of $\triangle DEF$.

16. PROM Georgia is on the prom committee. She wants to hang a dozen congruent triangles from the ceiling so that they are parallel to the floor. She sketched out one triangle on a coordinate plane with coordinates (0, 4), (3, 8), and (6, 0). If each triangle is to be hung by one chain, what are the coordinates of the point where the chain should attach to the triangle?

Example 2

In $\triangle EDF$, T is the centroid and $FT = 12$. Find TQ.



$FT = \frac{2}{3}FQ$

$FT = \frac{2}{3}(FT + TQ)$

$12 = \frac{2}{3}(12 + TQ)$ — $FT = 12$

$12 = 8 + \frac{2}{3}TQ$ — Distributive Property

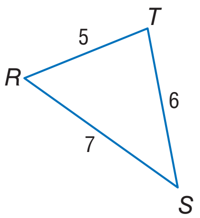
$4 = \frac{2}{3}TQ$ — Subtract.

$6 = TQ$ — Multiply.

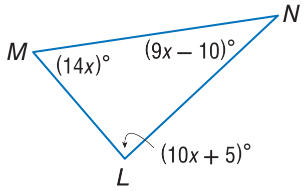
5-3 Inequalities in One Triangle

List the angles and sides of each triangle in order from smallest to largest.

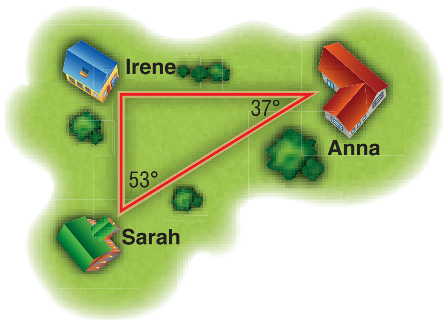
17.



18.

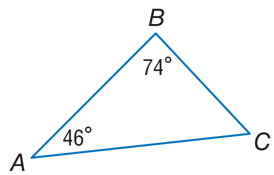


19. **NEIGHBORHOODS** Anna, Sarah, and Irene live at the intersections of the three roads that make the triangle shown. If the girls want to spend the afternoon together, is it a shorter path for Anna to stop and get Sarah and go onto Irene's house, or for Sarah to stop and get Irene and then go on to Anna's house?



Example 3

List the angles and sides of $\triangle ABC$ in order from smallest to largest.



a. First, find the missing angle measure using the Triangle Sum Theorem.

$m\angle C = 180 - (46 + 74)$ or 60

So, the angles from smallest to largest are $\angle A$, $\angle C$, and $\angle B$.

b. The sides from shortest to longest are $\overline{BC}$, $\overline{AB}$, and $\overline{AC}$.

5-4 Indirect Proof

State the assumption you would make to start an indirect proof of each statement.

20. $m\angle A \geq m\angle B$

21. $\triangle FGH \cong \triangle MNO$

22. $\triangle KLM$ is a right triangle.

23. If $3y < 12$, then $y < 4$.

24. Write an indirect proof to show that if two angles are complementary, neither angle is a right angle.

25. **MOVIES** Isaac bought two DVD's and spent over \$50. Use indirect reasoning to show that at least one of the DVD's he purchased was over \$25.

Example 4

State the assumption necessary to start an indirect proof of each statement.

a. $\overline{XY} \ncong \overline{JK}$

$\overline{XY} \cong \overline{JK}$

b. If $3x < 18$, then $x < 6$.

The conclusion of the conditional statement is $x < 6$. The negation of the conclusion is $x \geq 6$.

c. $\angle 2$ is an acute angle.

If $\angle 2$ *is an acute angle* is false, then $\angle 2$ *is not an acute angle* must be true. This means that $\angle 2$ *is an obtuse or right angle* must be true.

5-5 The Triangle Inequality

Is it possible to form a triangle with the given lengths? If not, explain why not.

26. 5, 6, 9

27. 3, 4, 8

Find the range for the measure of the third side of a triangle given the measure of two sides.

28. 5 ft, 7 ft

29. 10.5 cm, 4 cm

30. **BIKES** Leonard rides his bike to visit Josh. Since High Street is closed, he has to travel 2 miles down Main Street and turn to travel 3 miles farther on 5th Street. If the three streets form a triangle with Leonard and Josh's house as two of the vertices, find the range of the possible distance between Leonard and Josh's houses when traveling straight down High Street.

Example 5

Is it possible to form a triangle with the lengths 7, 10, and 9 feet? If not, explain why not.

Check each inequality.

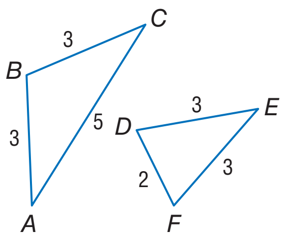
$7 + 10 > 9$	$7 + 9 > 10$	$10 + 9 > 7$
$17 > 9$ ✓	$16 > 10$ ✓	$19 > 7$ ✓

Since the sum of each pair of side lengths is greater than the third side length, sides with lengths 7, 10, and 9 feet will form a triangle.

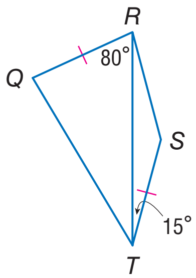
5-6 Inequalities in Two Triangles

Compare the given measures.

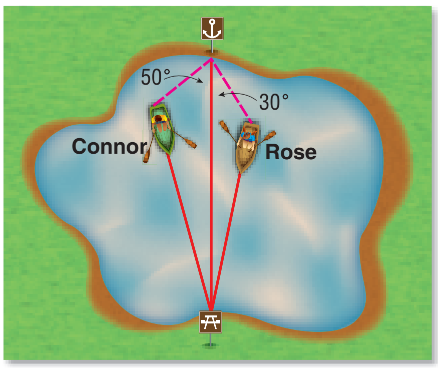
31. $m\angle ABC$, $m\angle DEF$



32. QT and RS

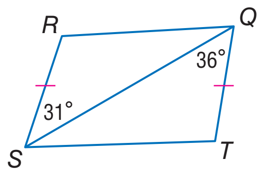


33. **BOATING** Rose and Connor each row across a pond heading to the same point. Neither of them has rowed a boat before, so they both go off course as shown in the diagram. After two minutes, they have each traveled 50 yards. Who is closer to their destination?



Example 6

Compare the given measures.

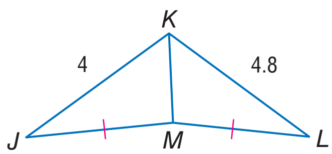


a. RQ and ST

In $\triangle QRS$ and $\triangle STQ$, $\overline{RS} \cong \overline{TQ}$, $\overline{QS} \cong \overline{QS}$, and $\angle SQT > \angle RSQ$. By the Hinge Theorem, $m\angle SQT < m\angle RSQ$, so $RQ < ST$.

b. $m\angle JKM$ and $m\angle LKM$

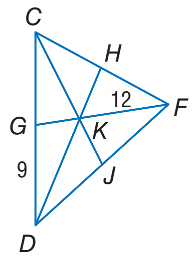
In $\triangle JKM$ and $\triangle LKM$, $\overline{JM} \cong \overline{LM}$, $\overline{KM} \cong \overline{KM}$, and $LK > JK$. By the Converse of the Hinge Theorem, $\angle LKM > \angle JKM$.



CHAPTER 5 Practice Test

1. **GARDENS** Maggie wants to plant a circular flower bed within a triangular area set off by three pathways. Which point of concurrency related to triangles would she use for the center of the largest circle that would fit inside the triangle?

In $\triangle CDF$, K is the centroid and $DK = 16$. Find each length.



2. KH
3. CD
4. FG

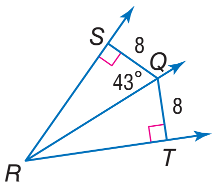
5. **PROOF** Write an indirect proof.

   Given: $5x + 7 \geq 52$

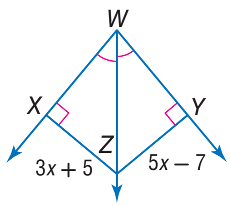
   Prove: $x \geq 9$

Find each measure.

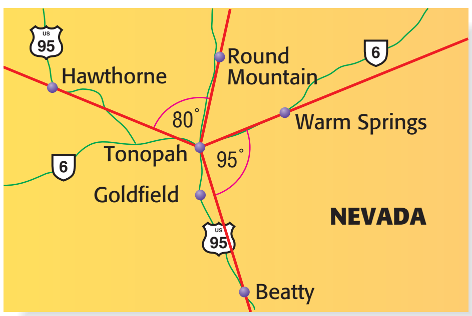
6. $m\angle TQR$



7. XZ



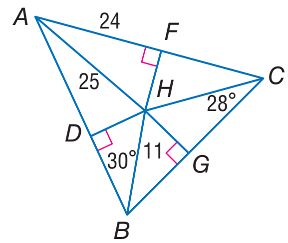
8. **GEOGRAPHY** The distance from Tonopah to Round Mountain is equal to the distance from Tonopah to Warm Springs. The distance from Tonopah to Hawthorne is the same as the distance from Tonopah to Beatty. Determine which distance is greater, Round Mountain to Hawthorne or Warm Springs to Beatty.



9. **MULTIPLE CHOICE** If the measures of two sides of a triangle are 3.1 feet and 4.6 feet, which is the *least* possible whole number measure for the third side?

   A 1.6 feet
   B 2 feet
   C 7.5 feet
   D 8 feet

Point H is the incenter of $\triangle ABC$. Find each measure.



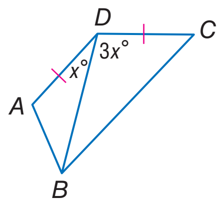
10. DH
11. BD
12. $m\angle HAC$
13. $m\angle DHG$

14. **MULTIPLE CHOICE** If the lengths of two sides of a triangle are 5 and 11, what is the range of possible lengths for the third side?

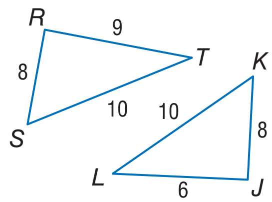
    F $6 < x < 10$
    G $5 < x < 11$
    H $6 < x < 16$
    J $x < 5$ or $x > 11$

Compare the given measures.

15. AB and BC



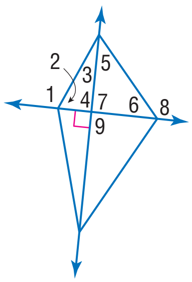
16. $\angle RST$ and $\angle JKL$



State the assumption necessary to start an indirect proof of each statement.

17. If 8 is a factor of n, then 4 is a factor of n.
18. $m\angle M > m\angle N$
19. If $3a + 7 \leq 28$, then $a \leq 7$.

Use the figure to determine which angle has the greatest measure.

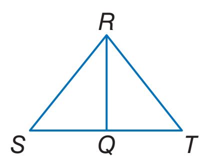


20. $\angle 1, \angle 5, \angle 6$
21. $\angle 9, \angle 8, \angle 3$
22. $\angle 4, \angle 3, \angle 2$

23. **PROOF** Write a two-column proof.

    Given: $\overline{RQ}$ bisects $\angle SRT$.

    Prove: $m\angle SQR > m\angle SRQ$



Find the range for the measure of the third side of a triangle given the measures of the two sides.

24. 10 ft, 16 ft
25. 23 m, 39 m